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Part 29:
Linear profile filters: wavelets

Spécification géométrique des produits (GPS) — Filtrage —

Partie 29: Filtres de profil linéaires: ondelettes

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 290, Dimensional and geometrical product specification and verification, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

This second edition cancels and replaces the first edition (ISO 16610-29:2015), which has been technically revised.

The main changes compared to the previous edition are as follows:

— The terminology and requirements around wavelets have been clarified and expanded to cover biorthogonal wavelets more fully.

— The requirements for cubic prediction wavelets are set out in Annex A.

— The requirements for cubic b-spline wavelets are given in Annex B.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences chain links C and F of the chains of standards on profile and areal surface texture.

The ISO GPS matrix model given in ISO 14638 gives an overview of the ISO GPS system of which this document is a part. The fundamental rules of ISO GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this document, unless otherwise indicated.

For more detailed information on the relation of this document to other standards and the GPS matrix model, see Annex D.

This document develops the terminology and concepts for wavelets.

Geometrical product specifications (GPS) — Filtration —

Part 29:
Linear profile filters: wavelets

1 Scope

This document specifies biorthogonal wavelets for profiles and contains the relevant concepts. It gives the basic terminology for biorthogonal wavelets of compact support, together with their usage.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO 16610-22, *Geometrical product specifications (GPS) — Filtration — Part 22: Linear profile filters: Spline filters*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1, ISO 16610-20, ISO 16610-22 and ISO/IEC Guide 99 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1
mother wavelet
function of one or more variables which forms the basic building block for wavelet analysis, i.e. an expansion of a signal/profile as a linear combination of wavelets

Note 1 to entry: A mother wavelet, which usually integrates to zero, is localized in space and has a finite bandwidth. Figure 1 provides an example of a real-valued mother wavelet.

3.1.1
biorthogonal wavelet
wavelet where the associated *wavelet transform* (3.3) is invertible but not necessarily orthogonal

Note 1 to entry: The merit of the biorthogonal wavelet is the possibility to construct symmetric wavelet functions, which allows a linear phase filter.

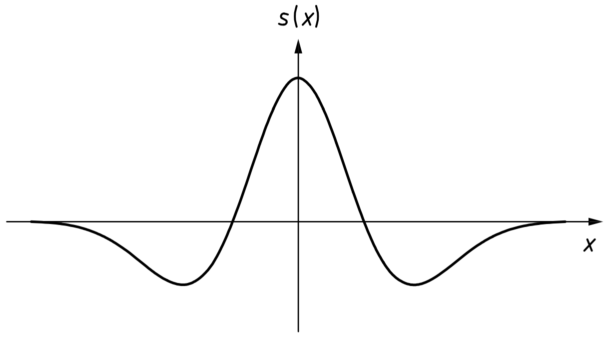


Figure 1 — Example of a real-valued mother wavelet

3.2
wavelet family
$g_{\alpha,b}$
family of functions generated from the *mother wavelet* (3.1) by *dilation* (3.2.1) and *translation* (3.2.2)

Note 1 to entry: If $g(x)$ is the *mother wavelet* (3.1), then the wavelet family $g_{\alpha,b}(x)$ is generated as shown in Formula (1):

$$g_{\alpha,b}(x)=\alpha^{-0,5}\times g\left(\frac{x-b}{\alpha}\right) \tag{1}$$

where

α is the dilation parameter for the wavelet of frequency band $[1/\alpha, 2/\alpha]$;

b is the translation parameter.

3.2.1
dilation
<wavelet> transformation which scales the spatial variable x by a factor α

Note 1 to entry: This transformation takes the function $g(x)$ to $\alpha^{-0,5}g(x/\alpha)$ for an arbitrary positive real number α.

Note 2 to entry: The factor $\alpha^{-0,5}$ keeps the area under the function constant.

3.2.2
translation
transformation which shifts the spatial position of a function by a real number b

Note 1 to entry: This transformation takes the function $g(x)$ to $g(x - b)$ for an arbitrary real number b.

3.3
wavelet transform
unique decomposition of a profile into a linear combination of a *wavelet family* (3.2)

3.4
discrete wavelet transform
DWT
unique decomposition of a profile into a linear combination of a *wavelet family* (3.2) where the *translation* (3.2.2) parameters are integers and the *dilation* (3.2.1) parameters are powers of a fixed positive integer greater than 1

Note 1 to entry: The dilation parameters are usually powers of 2.

3.5
multiresolution analysis
decomposition of a profile by a filter bank into portions of different scales

Note 1 to entry: The portions at different scales are also referred to as resolutions (see ISO 16610-20).

Note 2 to entry: Multiresolution is also called multiscale.

Note 3 to entry: See Figure 2.

Note 4 to entry: Since by definition there is no loss of information, it is possible to reconstruct the original profile from the *multiresolution ladder structure* (3.5.3).

3.5.1
low-pass component
smoothing component
component of the *multiresolution analysis* (3.5) obtained after convolution with a smoothing filter (low-pass) and a *decimation* (3.5.6)

3.5.2
high-pass component
difference component
component of the *multiresolution analysis* (3.5) obtained after convolution with a difference filter (high-pass) and a *decimation* (3.5.6)

Note 1 to entry: The weighting function of the difference filter is defined by the wavelet from a particular family of wavelets, with a particular *dilation* (3.2.1) parameter and no *translation* (3.2.2).

Note 2 to entry: The filter coefficients require the evaluation of an integral over a continuous space unless there exists a complementary function to form the basis expanding the signal/profile.

3.5.3
multiresolution ladder structure
structure consisting of all the orders of the difference components and the highest order smooth component

3.5.4
scaling function
function which defines the weighting function of the smoothing filter used to obtain the smooth component

Note 1 to entry: In order to avoid loss of information on the *multiresolution ladder structure* (3.5.3), the wavelet and scaling function are matched.

Note 2 to entry: The *low-pass component* (3.5.1) is obtained by convolving the input data with the scaling function.

3.5.5
wavelet function
function which defines the weighting function of the difference filter used to obtain the detail component

Note 1 to entry: The *high-pass component* (3.5.2) is obtained by convolving the input data with the wavelet function.

3.5.6
decimation
<wavelet> action which samples every *k*-th point in a sampled profile, where *k* is a positive integer

Note 1 to entry: Typically, *k* is equal to 2.

3.6
lifting scheme
fast *wavelet transform* (3.3) that uses *splitting*, *prediction* and *updating stages* (3.6.1), (3.6.2), (3.6.3)

3.6.1
splitting stage
partition of a profile into “even” and “odd” subsets, in which each sequence contains half as many samples as the original profile

3.6.2
prediction stage
calculation which predicts the odd subset from the even subset and then removes the predicted value from the odd subset value

3.6.3
updating stage
calculation which updates the even subset from the odd subset, in order to preserve as many profile moments as possible

4 General wavelet description

4.1 General

A cubic prediction wavelet claiming to conform with this document shall satisfy the procedure given in Annex A.

A cubic spline wavelet claiming to conform with this document shall satisfy the procedure given in Annex B.

NOTE The relationship to the filtration matrix model is given in Annex C.

4.2 Basic usage of wavelets

Wavelet analysis consists of decomposing a profile into a linear combination of wavelets $g_{a,b}(x)$, all generated from a single mother wavelet[4]. This is similar to Fourier analysis, which decomposes a profile into a linear combination of sinewaves, but unlike Fourier analysis, wavelets are finite in both spatial and frequency domain. Therefore, they can identify the location as well as the scale of a feature in a profile. As a result, they can decompose profiles where the small-scale structure in one portion of the profile is unrelated to the structure in a different portion, such as localized changes (i.e. scratches, defects or other irregularities). Wavelets are also ideally suited for non-stationary profiles. Basically, wavelets decompose a profile into building blocks of constant shape, but of different scales.

4.3 Wavelet transform

The discrete wavelet transform[5] of a profile, $s(x)$, given as height values, $s(x_i)$, at uniformly sampled positions, $x_i = (i{-}1)\,\Delta x$ (where Δx is the sampling interval, $i = 1, ..., n$ and n being the number of sampling points), with the wavelet function $g((x{-}b)/a)$, is given by the differences (or details), $d_k(i)$, and the smoothed data, $s_k(i)$, and a subsequent decimation (down-sampling) for each level or rung, k, of decomposition. The smoothed data and differences are obtained by convolving the signal with the scaling function, h, and the wavelet, g, as shown in Formula (2a) and Formula (2b):

$$s_k(i) = \sum_j h_j s_{k-1}(i-j) \tag{2a}$$

$$d_k(i) = \sum_j g_j s_{k-1}(i-j) \tag{2b}$$

where $j = -m, ..., -2, -1, 0, 1, 2, ..., m$; ($m$ is the number of coefficients of the filter on one side from the centre).

The dilation parameter, a, is determined by the level of decomposition, k, and by down-sampling the smoothed data commonly by a factor of two, i.e. $a = 2^{-k}$, respectively. $a = 1/(2^k\,\Delta x)$, such that for each step of the decomposition ladder the number of smoothed data points reduces by a factor of two.

The decomposition starts with the original signal values, $s(x_i)$, denoted as $s_0(i)$.

The mother wavelet of the discrete wavelet transform is defined as a set of discrete high-pass filter coefficients, g_j, and the scaling function as a set of discrete low-pass filter coefficients, h_j. As the decimation is carried out by keeping every second value of the smooth and every second of the difference signal, the total number of data points is conserved, such that $n/2$ of the $s_1(i)$ are saved and $n/2$ of the $d_1(i)$ and the distance between the i-th and the (i+1)-th is then $2\Delta x$.

For the second decomposition step, the set of $n/2$ differences, $d_1(i)$, will be kept until termination but the set of the $s_1(i)$ is subdivided half and half, such that $n/4$ values $s_2(i)$ and $n/4$ values $d_2(i)$ are obtained. For the k-th step of decomposition and decimation $n/2^k$ of $s_k(i)$ and $n/2^k$ values $d_k(i)$ are evaluated and the distance between the i-th and the (i+1)-th is then $2^k\Delta x$.

Therefore, the dilation is done by down-sampling, i.e. managing the indices of the signal rather than changing the wavelet and scaling functions. Thus, for discrete wavelet transformations only the two sets of filter coefficients, the set $\{h_j, j = -m,..,0,..m\}$ for the low-pass and $\{g_j, j = -m,..,0,..m\}$ for the high-pass, define the analysis filter.

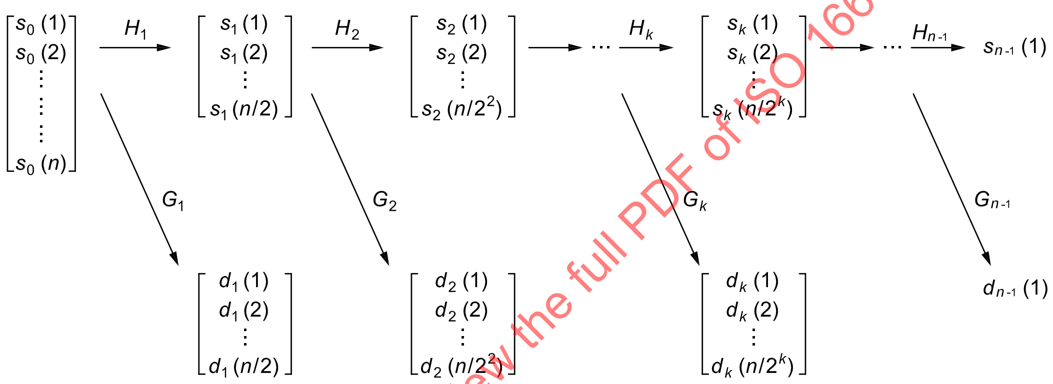


Figure 2 — Ladder structure of multiresolution separation using a discrete wavelet transform

Figure 2 illustrates the ladder structure of the consecutive steps with action of the low-pass (smoothing) filter with subsequent decimation, $H_k = \{2^{-k}, h_j, j = -m, .., 0, ..., m\}$, and the high-pass filter, $G_k = \{2^{-k}, g_j, j = -m, ..., 0, ..., m\}$, with decimation reducing the number of smooth profile points by half for each rung.

The reconstruction is performed by up-sampling and the subsequent application of the matching synthesis filters. The original profile can be regained if all difference signals are included to the recovery.

The multiresolution form of the wavelet transform consists of constructing a ladder of smooth approximations to the profile (see Figure 2). The first rung, i.e. rung number 0, is the original profile. Each rung in the ladder consists of a filter bank. To apply discrete wavelet transformations to the multiresolution concept in the sense of ISO 16610-20, a decomposition is performed until a desired level k, i.e. rung or resolution, is achieved. Then the signal is reconstructed without the details $d_1(i)$ ... $d_k(i)$, i.e. the up-sampling is done for $s_k(i)$ and thereafter the convolution with the synthesis low-pass yielding the desired smoothed signal.

4.4 Biorthogonal wavelets

4.4.1 General

The application addressed with this document is to recognize features of differing scales (resolutions) by smoothing accordingly. The biorthogonal wavelets specified in this document are all symmetrical wavelets and the decomposed signal can be reconstructed without loss.

4.4.2 Cubic prediction wavelets

A fast implementation of the wavelet decomposition and reconstruction has been employed using a lifting scheme with three stages: splitting, prediction and updating, originally introduced by Sweldens, in which the Neville polynomials are employed to implement the prediction stage by interpolating between sampling positions[6,7]. The cubic prediction wavelets in Annex A using Sweldens' lifting scheme[6] has been validated as an efficient tool for fast and in-place wavelet transform for geometrical products applications, for example surface metrology[9].

4.4.3 Cubic b-spline wavelets

Spline wavelets are based on the spline function. In this document a cubic b-spline function is used, which has a compact support. The particular cubic spline wavelets used are the biorthogonal wavelets CDF 9/7 with four vanishing moments, detailed in Annex B. This was original introduced by Cohen et al.[8] and has been used in geometrical products applications, for example multiscale analysis. The cubic spline wavelet transform can be implemented using both the Fourier method and the lifting scheme (however, it is a five-stage process) with relevant precision.

5 Filter designation

Lifting schemes using cubic interpolation for the wavelet transform in conformity with this document are designated:

FPLWCP

CDF 9/7 Spline wavelets in conformity with this document are designated:

FPLWCS

See also ISO 16610-1:2015, Clause 5.

Annex A
(normative)

Cubic prediction wavelets

A.1 General

The lifting scheme is used to define a fast in-place wavelet transform (see References [7], [9]). Starting with the original profile, each rung in the multiresolution ladder is calculated from the previous rung in three stages. These stages are called:

— splitting;

— prediction;

— updating.

The lifting scheme using cubic polynomial interpolation for the prediction stage described in this annex was introduced by Sweldens in 1996[7] for image processing purposes. Jiang et al. have applied the method to surface metrology[9] (see Figure A.1).

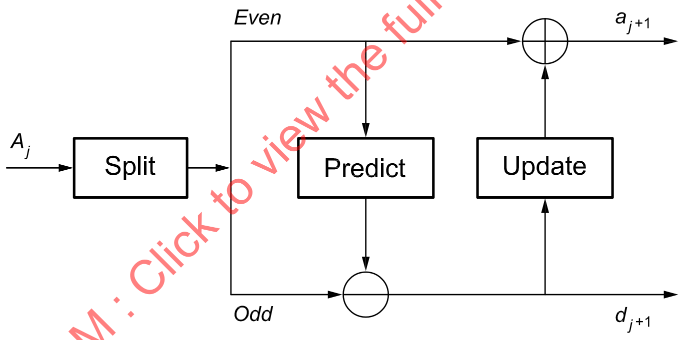


Figure A.1 — Forward transform using the lifting scheme for wavelet defined in this annex

A.2 Splitting

The lifting algorithm of the wavelet transform first of all divides the smoothed profile from the jth rung, $A_{j,k}$, into "even" and "odd" subsets, in which each sequence contains half as many samples as $A_{j,k}$. The operator is given by Formula (A.1):

$$\begin{cases} a_{j+1,k} = A_{j,2k} \\ d_{j+1,k} = A_{j,2k+1} \end{cases} \quad \text{(A.1)}$$

where $A_{0,k} = Z_k$, the original profile.

A.3 Prediction

The prediction of the wavelet algorithm consists of predicting the odd subset from the even subset and then removing the predicted value from the odd subset value. The operator is given by Formula (A.2):

$$d_{j+1,k} = d_{j+1,k} - \rho\left(a_{j+1,k}\right) \tag{A.2}$$

For the family of cubic interpolating wavelets, linear polynomials are used for the prediction. $\rho\left(a_{j+1,k}\right)$ is a weighted prediction of a wavelet coefficient point given by Formula (A.3):

$$\rho\left(a_{j+1,k}\right) = \sum_{i=1}^{N} f_i\left(a_{j+1,k}\right) \tag{A.3}$$

The value of $\rho\left(a_{j+1,k}\right)$ is based on the even set, where N denotes how many data points will attend the weighted prediction. f_i are a set of filtering factors (weighting function) of one wavelet coefficient point, and can be found by employing a “Neville's polynomial interpolation” (see References [6] to [8]) with a degree (N–1), with the recursion shown in Formula (A.4):

$$f_i = f_{1,2,\ldots,N}(x) = \frac{(x - x_1) f_{2,\ldots,N}(x) - (x - x_N) f_{1,2,\ldots,N-1}}{(x_N - x_1)} \tag{A.4}$$

Initial coefficients, $f_1, f_2, \ldots, f_N$, are a set of Bezier coefficients of a spline interpolation, with degree (N–1).

For example, if cubic interpolation (N = 4) is employed to create a weighting function, four neighbouring values will attend a weighted prediction. Five cases should be taken into account:

a) two neighbouring points on either side of an interval;

b) one sample point on the left and three on the right at the left boundary of an interval;

c) vice versa at the right boundary;

d) four sample points on the left;

e) four sample points on the right.

These cases are considered in order to guarantee boundary “naturalness”, without including any artefacts (all filtering factors are indicated in Table A.1). The result of this is that running-in and running-out lengths of normal filtering techniques are not needed.

Table A.1 — Filter coefficients for cubic polynomial interpolation

Number of samples		k − 7	k − 5	k − 3	k − 1	k + 1	k + 3	k + 5	k + 7
on left	**on right**								
0	4					2,187 5	−2,187 5	1,312 5	−0,312 5
1	3				0,312 5	0,937 5	−0,312 5	0,062 5	
2	2			−0,062 5	0,562 5	0,562 5	−0,062 5		
3	1		0,062 5	−0,312 5	0,937 5	0,312 5			
4	0	−0,312 5	1,312 5	−2,187 5	2,187 5				

For example, when there are two samples on the left and two samples on the right, the lifting factors are as shown in Formula (A.5):

$$f=\left(-\frac{1}{16},\frac{9}{16},\frac{9}{16},-\frac{1}{16}\right) \tag{A.5}$$

and the wavelet coefficients can be updated as shown in Formula (A.6):

$$d_{j+1,k}=d_{j+1,k}-\tfrac{1}{16}(-a_{j+1,k-2}+9a_{j+1,k-1}+9a_{j+1,k}-a_{j+1,k+1}) \tag{A.6}$$

A.4 Updating

For every level of the multiresolution ladder, the resulting smoother profiles should preserve some of the properties of the original profile, for example the same average value and other higher moments. This is achieved in the updating stage.

The updating stage of the wavelet algorithm consists of updating the even subset from the odd subset, in order to preserve as many profile moments as possible. The operator is given by Formula (A.7):

$$A_{j+1,k}=a_{j+1,k}+\mu(d_{j+1,k}) \tag{A.7}$$

where $\mu(d_{j+1,k})$ is a weighting update given by Formula (A.8):

$$\mu(d_{j+1,k})=\sum_{i=1}^{\tilde{N}} l_i(d_{j+1,k}) \tag{A.8}$$

$\mu(d_{j+1,k})$ is based on the real wavelet coefficients, where $\tilde{N}$ indicates how many wavelet coefficient points will attend the weighting update. The larger $\tilde{N}$ is, the more profile moments are preserved. The l_i are referred to as lifting factors.

The lifting factors can be calculated by the following algorithm. Firstly, an initial moment matrix is defined for all coefficients at the first level of the multiresolution ladder. The moment matrix M is defined by the number of points in the profile, s, and the value of $\tilde{N}$, as shown in Formula (A.9):

$$M[a,b]=\begin{bmatrix} m_{1,1} & \cdots & m_{1,s} \\ \vdots & m_{a,b} & \vdots \\ m_{\tilde{N},1} & \cdots & m_{\tilde{N},s} \end{bmatrix}=\begin{bmatrix} 1^0 & \cdots & s^0 \\ \vdots & b^{a-1} & \vdots \\ 1^{\tilde{N}-1} & \cdots & s^{\tilde{N}-1} \end{bmatrix} \quad \begin{matrix} 1\le a\le \tilde{N} \\ 1\le b\le s \end{matrix} \tag{A.9}$$

Updating the moment matrix requires an indication of how many filtering factors of corresponding wavelet coefficients will contribute to the update. When neighbouring point numbers on each side are the same, the moments can be expressed as shown in Formula (A.10):

$$m_{2p,q}=m_{2p,q}+\sum_{t,j} f_i m_{t,q} \tag{A.10}$$

where

$$t=2p-N+1,\, 2p-N+3,\cdots,2p+N-1$$

$i = 1, \cdots, N$

The lifting factors are the solution of the linear system in Formula (A.11).

$$\begin{bmatrix} m_{1,2p-\tilde{N}+2} & \cdots & m_{1,2p+\tilde{N}} \\ \vdots & m_{q,2p} & \vdots \\ m_{\tilde{N},2p-\tilde{N}+2} & \cdots & m_{\tilde{N},2p+\tilde{N}} \end{bmatrix}_{\tilde{N},\tilde{N}} \begin{bmatrix} l_1 \\ \vdots \\ l_q \\ \vdots \\ l_{\tilde{N}} \end{bmatrix} = \begin{bmatrix} m_{1,2p+1} \\ \vdots \\ m_{q,2p+1} \\ \vdots \\ m_{\tilde{N},2p+1} \end{bmatrix} \tag{A.11}$$

For example, when a weighting update of a scalar coefficient is considered to be a cubic interpolation, the update can be calculated by using four neighbour wavelet coefficients. In this case, the lifting factors are $l = \left(-\frac{1}{32}, \frac{9}{32}, \frac{9}{32}, -\frac{1}{32}\right)$ and the scalar coefficients can be updated as shown in Formula (A.12).

$$A_{j+1,k} = a_{j+1,k} + \frac{1}{32}\left(-d_{j+1,k-2} + 9d_{j+1,k-1} + 9d_{j+1,k} - d_{j+1,k+1}\right) \tag{A.12}$$

A.5 Forward and inverse transforms

To summarize, the forward transform is shown in Formula (A.13).

$$\begin{aligned} &\text{Split} \begin{cases} a_{j+1,k} = A_{j,2k} \\ d_{j+1,k} = A_{j,2k+1} \end{cases} \\ &\text{Predict } d_{j+1,k} = d_{j+1,k} - \rho(a_{j+1,k}) \\ &\text{Update } A_{j+1,k} = a_{j+1,k} + \mu(d_{j+1,k}) \end{aligned} \tag{A.13}$$

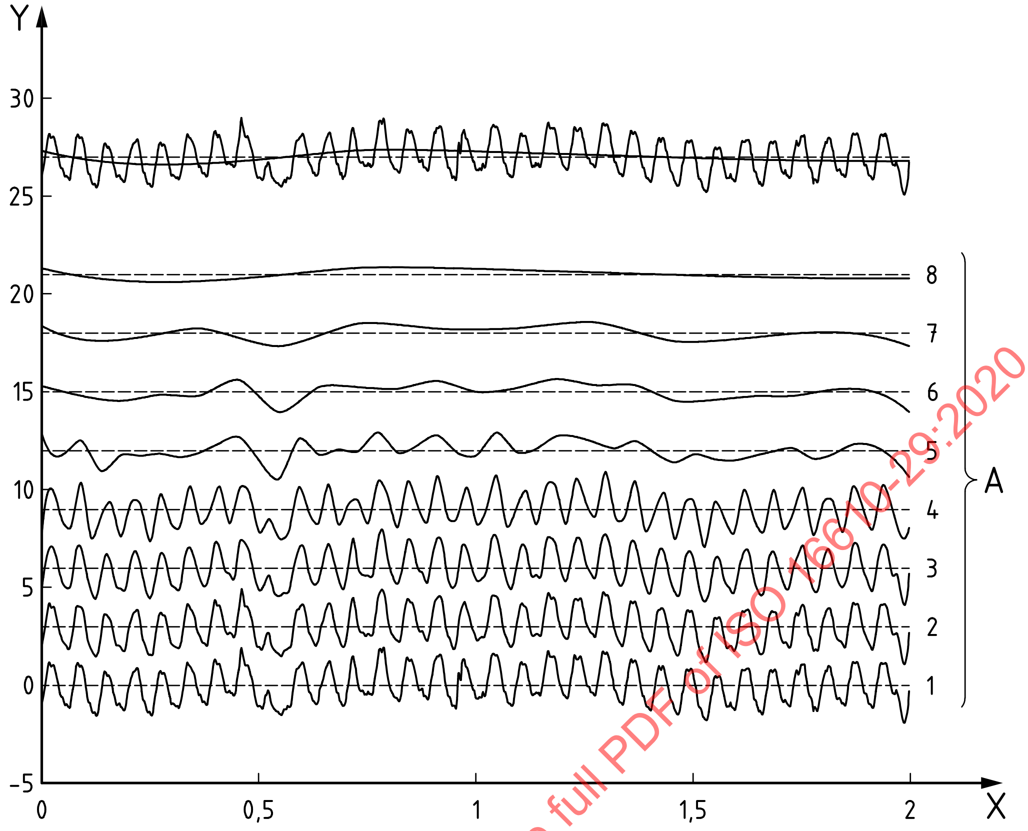
One important property of the lifting scheme is that once the forward transform is defined, the inverse transform can immediately be obtained. The operations are just reversed and the + and − toggled. This leads to the algorithm for the inverse transform shown in Formula (A.14).

$$\begin{aligned} &\text{Update } a_{j+1,k} = A_{j+1,k} - \mu(d_{j+1,k}) \\ &\text{Predict } d_{j+1,k} = d_{j+1,k} + \rho(a_{j+1,k}) \\ &\text{Combine } = \begin{cases} A_{j,2k} = a_{j+1,k} \\ A_{j,2k+1} = d_{j+1,k} \end{cases} \end{aligned} \tag{A.14}$$

A.6 Examples of the application of cubic prediction wavelets

A.6.1 Profile from a milled surface

The profile is from a milled surface and is measured with a 5 µm tip stylus. Figure A.2 shows the successively "smoothed" profiles, together with the original profile at the top, with the smoothest profile superimposed on top.



Key

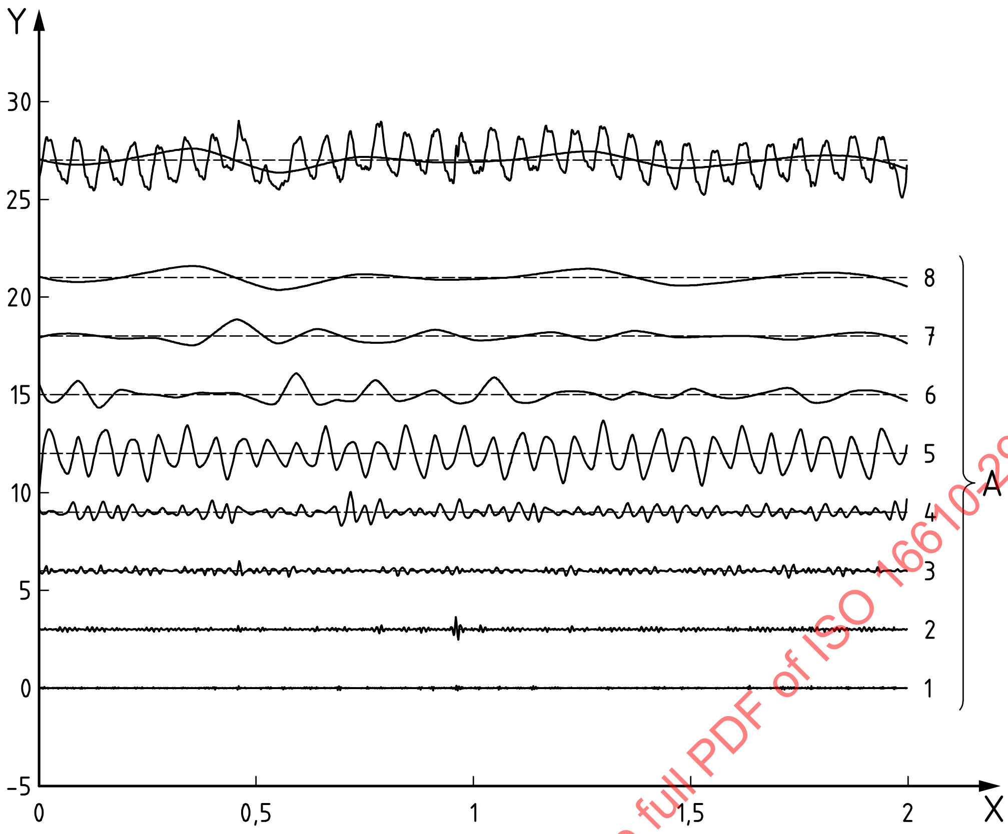
X distance, mm

Y height, µm

A levels

Figure A.2 — Successively smoothed profiles of a milled surface using cubic interpolating wavelets

Figure A.3 shows the differences (details) between successive smoothings. The milling marks are easily identifiable at level 5.



Key

X distance, mm

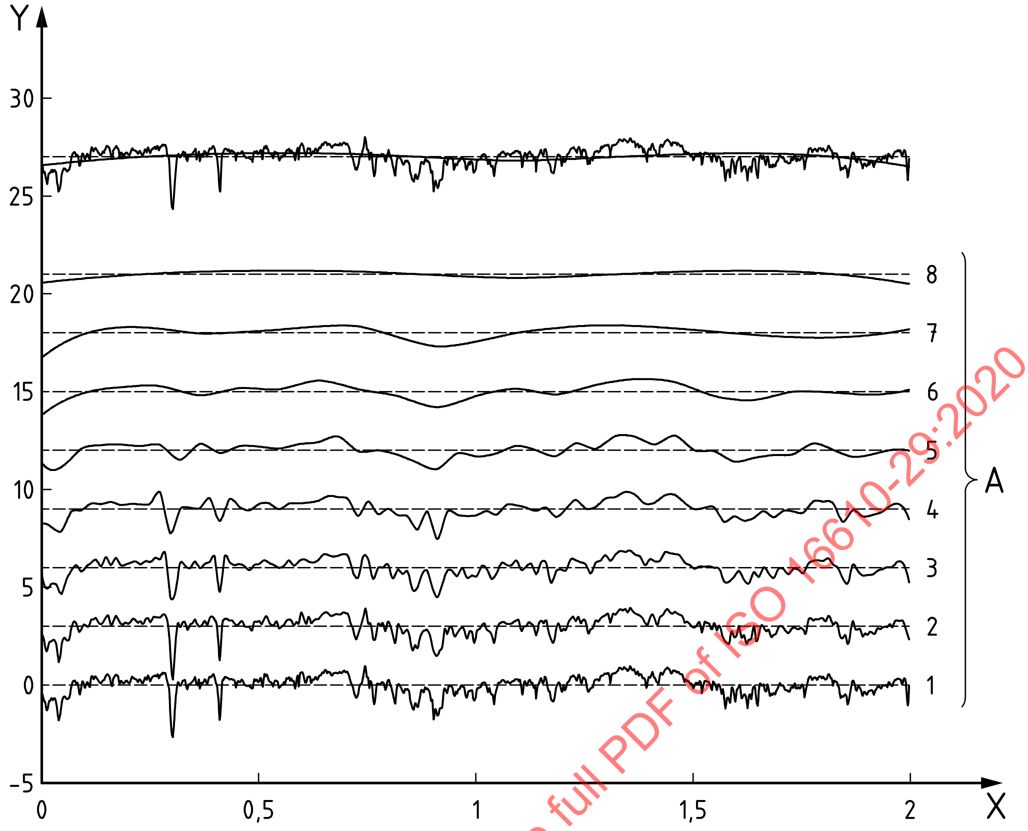
Y height, µm

A levels

Figure A.3 — Differences on a milled surface using cubic interpolating wavelets

A.6.2 Profile from a ceramic surface

The profile is from a rough ceramic surface and is measured with a 5 µm tip stylus. Figure A.4 shows the successively "smoothed" profiles, together with the original profile at the top.



Key

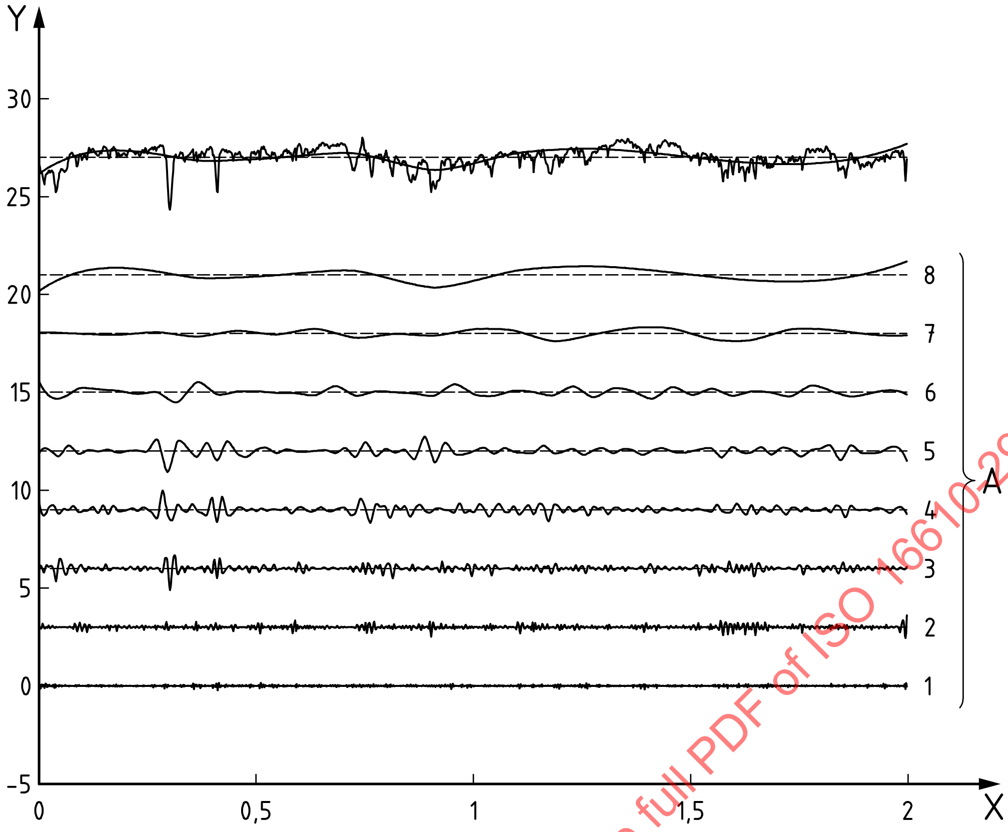
X distance, mm

Y height, µm

A levels

Figure A.4 — Successively smoothed profiles of a ceramic surface using cubic interpolating wavelets

Figure A.5 shows the differences (details) between successive smoothings. The deep valleys are easily identifiable at levels 3 and 4, and various asperities can be seen at levels 1 and 2.



Key

X distance, mm

Y height, µm

A levels

Figure A.5 — Differences on a ceramic surface using cubic interpolating wavelets

Annex B
(normative)

Cubic b-spline wavelets

B.1 Implementation of b-spline wavelets transform

The discrete wavelet filters of the BIOR 4.4, respectively CDF 9/7 family define the filter coefficients shown in Table B.1.

Table B.1 — Normalized coefficients for the direct wavelet transform (DWT)

	Decomposition/analysis		**Reconstruction/synthesis**	
	Filter coefficients of DWT		Filter coefficients of iDWT	
	The decomposition low-pass filter	The decomposition high-pass filter	The reconstruction low-pass filter	The reconstruction high-pass filter
j	g_j	h_j	$\widetilde{h}_j$	$\widetilde{g}_j$
−4	0,037 828 455 507			−0,037 828 455 507
−3	−0,023 849 465 020	−0,064 538 882 629	−0,064 538 882 629	−0,023 849 465 020
−2	−0,110 624 404 418	0,040 689 417 609	−0,040 689 417 609	0,110 624 404 418
−1	0,377 402 855 613	0,418 092 273 222	0,418 092 273 222	0,377 402 855 613
0	0,852 698 679 009	−0,788 485 616 406	0,788 485 616 406	−0,852 698 679 009
1	0,377 402 855 613	0,418 092 273 222	0,418 092 273 222	0,377 402 855 613
2	−0,110 624 404 418	0,040 689 417 609	−0,040 689 417 609	0,110 624 404 418
3	−0,023 849 465 020	−0,064 538 882 629	−0,064 538 882 629	−0,023 849 465 020
4	0,037 828 455 507			−0,037 828 455 507

The abbreviations for this wavelet family either denote the number of filter coefficients or the number of vanishing moments. CDF is the abbreviation for Cohen, Daubechies, Feauveau, here with nine coefficients for g_j and $\widetilde{g}_j$ and seven coefficients for h_j and $\widetilde{h}_j$. The name BIOR 4.4 denotes the number of vanishing moments, which is four for both high-pass filters h_j and $\widetilde{g}_j$ and with the abbreviation BIOR for biorthogonal.

For CDF 9/7 the lifting scheme uses two predict and two update steps with the following coefficients (the pseudocode applies for data arrays starting from 0 as, for instance, in C programming language):

Decomposition

```
Predict 1
   a = -1.586134342
   for i from 1 to n-3 by step = 2
     x(i) = x(i) + a*(x(i-1)+x(i+1))
   endfor
   x(n-1) = x(n-1) + 2*a*x(n-2)
```

Reconstruction

```
Undo scale
   a = 1.149604398
   for i from 0 to n-1 by step = 1
     if modulo(i,2) =  = 1 then
       x(i) = x(i)*a
     else
       x(i) = x(i)/a
   endfor
```

Decomposition	Reconstruction
Update 1 a = −0.05298011854 for i from 2 to n-1 by step = 2 x(i) = x(i) + a*(x(i-1)+x(i+1)) endfor x(0) = x(0) + 2*a*x(1)	Undo update 2 a = −0.4435068522 for i from 2 to n-1 by step = 2 x(i) = x(i) + a*(x(i-1)+x(i+1)) endfor x(0) = x(0) + 2*a*x(1)
Predict 2 a = 0.8829110762 for i from 1 to n-3 by step = 2 x(i) = x(i) + a*(x(i-1)+x(i+1)) endfor x(n-1) = x(n-1) + 2*a*x(n-2)	Undo predict 2 a = −0.8829110762 for i from 1 to n-3 by step = 2 x(i) = x(i) + a*(x(i-1)+x(i+1)) endfor x(n-1) = x(n-1) + 2*a*x(n-2)
Update 2 a = 0.4435068522 for i from 2 to n-1 by step = 2 x(i) = x(i) + a*(x(i-1)+x(i+1)) endfor x(0) = x(0) + 2*a*x(1)	Undo update 1 a = 0.05298011854 for i from 2 to n-1 by step = 2 x(i) = x(i) + a*(x(i-1)+x(i+1)) endfor x(0) = x(0) + 2*a*x(1)

Scale
a = 1.149604398
for i from 0 to n-1 by step = 1
if modulo(i,2) = = 1 then
x(i) = x(i)/a
else
x(i) = x(i)*a
endfor
	Undo predict 1
	a = 1.586134342
	for i from 1 to n-3 by step = 2
	x(i) = x(i) + a*(x(i-1)+x(i+1))
	endfor
	x(n-1) = x(n-1) + 2*a*x(n-2)

B.2 Examples of the application of cubic b-spline wavelets

B.2.1 Profile from a bored surface

Figure B.1 shows the successively "smoothed" profiles on the left, together with the original profile at the top, with the smoothest profile superimposed on top. The profile is from a bored surface and is measured with a 5 µm tip stylus.