INTERNATIONAL STANDARD

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Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment —

Part 2: Guidance for the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification

Spécification géométrique des produits (GPS) — Vérification par la mesure des pièces et des équipements de mesure —

Partie 2: Lignes directrices pour l'estimation de l'incertitude dans les mesures GPS, dans l'étalonnage des équipements de mesure et dans la vérification des produits

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Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 14253-2 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition of ISO 14253-2 cancels and replaces ISO/TS 14253-2:1999, which has been technically revised. It also incorporates the Technical Corrigendum ISO/TS 14253-2:1999/Cor.1:2007.

ISO 14253 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment*:

— *Part 1: Decision rules for proving conformance or non-conformance with specifications*

— *Part 2: Guidance for the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification*

— *Part 3: Guidelines for achieving agreements on measurement uncertainty statements*

— *Part 4: Background on functional limits and specification limits in decision rules* [Technical Specification]

Introduction

This part of ISO 14253 is a global GPS standard (see ISO/TR 14638:1995). This global GPS standard influences chain links 4, 5 and 6 in all chains of standards.

The ISO/GPS Masterplan given in ISO/TR 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

For more detailed information on the relation of this International Standard to other standards and to the GPS matrix model, see Annex D.

This part of ISO 14253 has been developed to support ISO 14253-1. This part of ISO 14253 establishes a simplified, iterative procedure of the concept and the way to evaluate and determine uncertainty (standard uncertainty and expanded uncertainty) of measurement, and the recommendations of the format to document and report the uncertainty of measurement information as given in the *Guide to the expression of uncertainty in measurement* (GUM). In most cases, only very limited resources are necessary to estimate uncertainty of measurement by this simplified, iterative procedure, but the procedure may lead to a slight overestimation of the uncertainty of measurement. If a more accurate estimation of the uncertainty of measurement is needed, the more elaborated procedures of the GUM need to be applied.

This simplified, iterative procedure of the GUM methods is intended for GPS measurements, but may be used in other areas of industrial (applied) metrology.

The uncertainty of measurement and the concept of handling uncertainty of measurement are important to all the technical functions within a company. This part of ISO 14253 is relevant to several technical functions, including management, design and development, manufacturing, quality assurance and metrology.

This part of ISO 14253 is of special importance in relation to ISO 9000 quality assurance systems, e.g. it is a requirement that methods for monitoring and measurement of the quality management system processes are suitable. The measurement uncertainty is a measure of the process suitability.

In this part of ISO 14253, the uncertainty of the result of a process of calibration and a process of measurement is handled in the same way:

— calibration is treated as a “measurement of the metrological characteristics of a measuring equipment or a measurement standard”;

— measurement is treated as a “measurement of the geometrical characteristics of a workpiece”.

Therefore, in most cases, no distinction is made in the text between measurement and calibration. The term “measurement” is used as a synonym for both.

Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment —

Part 2: Guidance for the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification

1 Scope

This part of ISO 14253 gives guidance on the implementation of the concept of the "Guide to the estimation of uncertainty in measurement" (in short GUM) to be applied in industry for the calibration of (measurement) standards and measuring equipment in the field of GPS and the measurement of workpiece GPS characteristics. The aim is to promote full information on how to achieve uncertainty statements and provide the basis for international comparison of measurement results and their uncertainties (relationship between purchaser and supplier).

This part of ISO 14253 is intended to support ISO 14253-1. Both parts are beneficial to all technical functions in a company in the interpretation of GPS specifications [i.e. tolerances of workpiece characteristics and values of maximum permissible errors (MPEs) for metrological characteristics of measuring equipment].

This part of ISO 14253 introduces the Procedure for Uncertainty MAnagement (PUMA), which is a practical, iterative procedure based on the GUM for estimating uncertainty of measurement without changing the basic concepts of the GUM. It is intended to be used generally for estimating uncertainty of measurement and giving statements of uncertainty for:

— single measurement results;

— the comparison of two or more measurement results;

— the comparison of measurement results — from one or more workpieces or pieces of measurement equipment — with given specifications [i.e. maximum permissible errors (MPEs) for a metrological characteristic of a measurement instrument or measurement standard, and tolerance limits for a workpiece characteristic, etc.], for proving conformance or non-conformance with the specification.

The iterative method is based basically on an upper bound strategy, i.e. overestimation of the uncertainty at all levels, but the iterations control the amount of overestimation. Intentional overestimation — and not under-estimation — is necessary to prevent wrong decisions based on measurement results. The amount of overestimation is controlled by economical evaluation of the situation.

The iterative method is a tool to maximize profit and minimize cost in the metrological activities of a company. The iterative method/procedure is economically self-adjusting and is also a tool to change/reduce existing uncertainty in measurement with the aim of reducing cost in metrology (manufacture). The iterative method makes it possible to compromise between risk, effort and cost in uncertainty estimation and budgeting.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 14253-1:1998, *Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 1: Decision rules for proving conformance or non-conformance with specifications*

ISO 14660-1:1999, *Geometrical Product Specifications (GPS) — Geometrical features — Part 1: General terms and definitions*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 14253-1, ISO 14660-1, ISO/IEC Guide 98-3 and ISO/IEC Guide 99 and the following apply.

3.1
black box model for uncertainty estimation
model for uncertainty estimation in which the uncertainties associated with the relevant input quantities are directly represented by their influence on the quantity value being attributed to a measurand (in the units of the measurand)

NOTE 1 The “quantity value being attributed to a measurand” is typically a measured value.

NOTE 2 In many cases, a complex method of measurement may be looked upon as one simple black box with stimulus in and result out from the black box. When a black box is opened, it may turn out to contain several “smaller” black boxes or several transparent boxes, or both.

NOTE 3 The method of uncertainty estimation remains a black box method even if it is necessary to make supplementary measurements to determine the values of influence quantities in order to make corresponding corrections.

3.2
transparent box model for uncertainty estimation
model for uncertainty estimation in which the relationship between the input quantities and the quantity value being attributed to a measurand is explicitly expressed with equations or algorithms

3.3
measuring task
quantification of a measurand according to its definition

3.4
overall measurement task
measurement task that quantifies the final measurand

3.5
intermediate measurement task
measurement task obtained by subdividing the overall measurement task into simpler parts

NOTE 1 The subdivision of the overall measuring task serves the goal of simplification of the evaluation of uncertainty.

NOTE 2 The specific subdivisions are arbitrary, as is whether to subdivide at all.

3.6
target uncertainty
U_T
⟨for a measurement or calibration⟩ uncertainty determined as the optimum for the measuring task

NOTE 1 Target uncertainty is the result of a management decision involving e.g. design, manufacturing, quality assurance, service, marketing, sales and distribution.

NOTE 2 Target uncertainty is determined (optimized) taking into account the specification [tolerance or maximum permissible error (MPE)], the process capability, cost, criticality and the requirements of ISO 9001, ISO 9004 and ISO 14253-1.

NOTE 3 See also 8.8.

3.7
required uncertainty of measurement
U_R
uncertainty required for a given measurement process and task

NOTE See also 6.2. The required uncertainty may be specified by, for example, a customer.

3.8
uncertainty management
process of deriving an adequate measurement procedure from the measuring task and the target uncertainty by using uncertainty budgeting techniques

3.9
uncertainty budget
⟨for a measurement or calibration⟩ statement summarizing the estimation of the uncertainty components that contributes to the uncertainty of a result of a measurement

NOTE 1 The uncertainty of the result of the measurement is unambiguous only when the measurement procedure (including the measurement object, measurand, measurement method and conditions) is defined.

NOTE 2 The term “budget” is used for the assignment of numerical values to the uncertainty components and their combination and expansion, based on the measurement procedure, measurement conditions and assumptions.

3.10
uncertainty component
xx
source of uncertainty of measurement for a measuring process

3.11
limit value (variation limit) for an uncertainty component
a_{xx}
absolute value of the extreme value(s) of the uncertainty component, xx

3.12
uncertainty component
u_{xx}
standard uncertainty of the uncertainty component, xx

NOTE The iteration method uses the designation u_{xx} for all uncertainty components.

3.13
influence quantity of a measurement instrument
characteristic of a measuring instrument that affects the result of a measurement performed by the instrument

3.14
influence quantity of a workpiece
characteristic of a workpiece that affects the result of a measurement performed on that workpiece

4 Symbols

For the purposes of this document, the generic symbols given in Table 1 apply.

Table 1 — Generic symbols

Symbol/ abbreviated term	Description
a	limit value for a distribution
a_{xx}	limit value for an error or uncertainty component (in the unit of the measurement result, of the measurand)
a^*_{xx}	limit value for an error or uncertainty component (in the unit of the influence quantity)
α	linear coefficient of thermal expansion
b	coefficient for transformation of a_{xx} to u_{xx}
C	correction (value)
d	resolution of a measurement equipment
E	Young's modulus
ER	error (value of a measurement)
G	function of several measurement values [$G(X_1, X_2, ... X_i, ...)$]
h	hysteresis value
k	coverage factor
m	number of standard deviations in the half of a confidence interval
MR	measurement result (value)
n	number of ...
N	number of iterations
ν	Poisson's number
p	number of total uncorrelated uncertainty components
r	number of total correlated uncertainty components
ρ	correlation coefficient
t	safety factor calculated based on the Student t distribution
TV	true value of a measurement
u, u_i	standard uncertainty (standard deviation)
s_x	standard deviation of a sample
$s_{\bar{x}}$	standard deviation of a mean value of a sample
u_{c}	combined standard uncertainty
u_{xx}	standard deviation of uncertainty component xx — uncertainty component
U	expanded uncertainty of measurement
U_{A}	true uncertainty of measurement
U_{C}	conventional true uncertainty of measurement
U_{E}	approximated uncertainty of measurement (number of iteration not stated)
$U_{\text{E}N}$	approximated uncertainty of measurement of iteration number N
U_{R}	required uncertainty
U_{T}	target uncertainty
U_{V}	uncertainty value (not estimated according to GUM or this part of ISO 14253)
X	measurement result (uncorrected)
X_i	measurement result (in the transparent box model of uncertainty estimation)
Y	measurement result (corrected)

5 Concept of the iterative GUM method for estimation of uncertainty of measurement

By applying the GUM method completely, a conventional true uncertainty of measurement, U_C, can be found.

The simplified, iterative method described in this part of ISO 14253 sets out to achieve estimated uncertainties of measurements, U_E, by overestimating the influencing uncertainty components ($U_E \geqslant U_C$). The process of overestimating provides "worst-case contributions" at the upper bound from each known or predictable uncertainty component, thus ensuring results of estimations "on the safe side", i.e. not underestimating the uncertainty of measurement. The method is based on the following:

— all uncertainty components are identified;

— it is decided which of the possible corrections shall be made (see 8.4.6);

— the influence on the uncertainty of the measurement result from each component is evaluated as a standard uncertainty u_{xx}, called the uncertainty component;

— an iteration process, PUMA (see Clause 6) is undertaken;

— the evaluation of each of the uncertainty components (standard uncertainties) u_{xx} can take place either by a Type A evaluation or by a Type B evaluation;

— Type B evaluation is preferred — if possible — in the first iteration in order to get a rough uncertainty estimate to establish an overview and to save cost;

— the total effect of all components (called the combined standard uncertainty) is calculated by Equation (1):

$$u_C = \sqrt{u_{x1}^2 + u_{x2}^2 + u_{x3}^2 + \ldots + u_{xn}^2} \tag{1}$$

— Equation (1) is only valid for a black box model of the uncertainty estimation and when the components u_{xx} are all uncorrelated (for more details and other equations, see 8.6 and 8.7);

— for simplification, the only correlation coefficients between components considered are

$$\rho = 1, -1, 0 \tag{2}$$

If the uncertainty components are not known to be uncorrelated, full correlation is assumed, either $\rho = 1$ or $\rho = -1$. Correlated components are added arithmetically before put into the formula above (see 8.5 and 8.6);

— the expanded uncertainty U is calculated by Equation (3):

$$U = k \times u_C \tag{3}$$

where $k = 2$; k is the coverage factor (see also 8.8).

The simplified, iterative method normally will consist of at least two iterations of estimating the components of uncertainty:

a) the first very rough, quick and cheap iteration has the purpose of identifying the largest components of uncertainty (see Figure 1);

b) the following iterations — if any — only deal with making more accurate "upper bound" estimates of the largest components to lower the estimate of the uncertainty (u_C and U) to a possible acceptable magnitude.

The simplified and iterative method may be used for two purposes:

1) management of the uncertainty of measurement for a result of a given measurement process (can be used for the results from a known measuring process or for comparison of two or more of such results) — see 6.2;

2) uncertainty management for a measuring process. For the development of an adequate measuring process, i.e. $U_E \leqslant U_T$, see 6.3.

6 Procedure for Uncertainty MAnagement — PUMA

6.1 General

The prerequisite for uncertainty budgeting and management is a clearly identified and defined measuring task, i.e. the measurand to be quantified (a GPS characteristic of a workpiece or a metrological characteristic of a GPS measuring equipment). The uncertainty of measurement is a measure of the quality of the measured value according to the definitions of a GPS characteristic of the workpiece or a metrological characteristic of the GPS measuring equipment given in GPS standards.

GPS standards define the "conventional true values" of the characteristics to be measured by chains of standards and global standards (see ISO/TR 14638). GPS standards in many cases also define the ideal — or conventional true — principle of measurement (see ISO/IEC Guide 99:2007, 2.4), method of measurement (see ISO/IEC Guide 99:2007, 2.5), measurement procedure (see ISO/IEC Guide 99:2007, 2.6) and standard "reference conditions" (see ISO/IEC Guide 99:2007, 4.11).

Deviations from the standardized conventional true values of the characteristics, etc. (the ideal operator) are contributing to the uncertainty of measurement.

6.2 Uncertainty management for a given measurement process

Management of the uncertainty of measurement for a given measuring task (box 1 of Figure 1) and for an existing measurement process is illustrated in Figure 1. The principle of measurement (box 3), measurement method (box 4), measurement procedure (box 5) and measurement conditions (box 6) are fixed and given or decided in this case, and cannot be changed. The only task is to evaluate the consequence on the uncertainty of measurement. A required U_R may be given or decided.

Using the iterative GUM method, the first iteration is only for orientation, and to look for the dominant uncertainty components. The only thing to do — in the management process in this case — is to refine the estimation of the dominant components to come closer to a true estimate of the uncertainty components thus avoiding an excessive overestimate — if necessary.

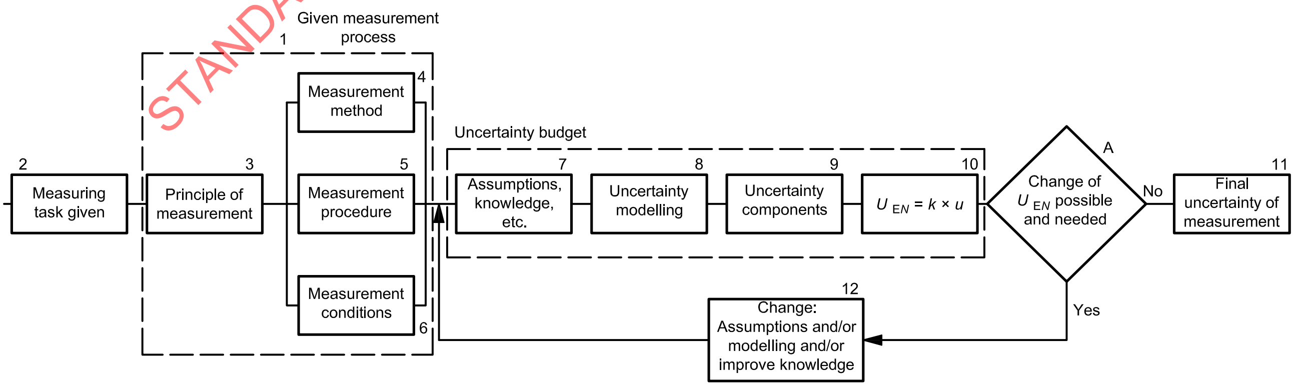


Figure 1 — Uncertainty management for a measurement result from a given measurement process

The procedure is as follows.

a) Make a first iteration based preferably on a black box model of the uncertainty estimation process and set up a preliminary uncertainty budget (boxes 7 to 9) leading to the first rough estimate of the expanded uncertainty, U_{E1} (box 10). For details about uncertainty estimation, see Clause 9. All estimates of uncertainties U_{EN} are performed as upper bound estimates.

b) Compare the first estimated uncertainty, U_{E1}, with the required uncertainty U_R (box A) for the actual measuring task.

   1) If U_{E1} is acceptable (i.e. if $U_{E1} \leqslant U_R$), then the uncertainty budget of the first iteration has proven that the given measurement procedure is adequate for the measuring task (box 11).

   2) If U_{E1} is not acceptable (i.e. if $U_{E1} > U_R$) or if there is no required uncertainty, but a lower and more true value is desired, the iteration process continues.

c) Before the new iteration, analyse the relative magnitude of the uncertainty components. In many cases, a few uncertainty components dominate the combined standard uncertainty and expanded uncertainty.

d) Change the assumptions or improve the knowledge about the uncertainty components to make a more accurate (see ISO/IEC Guide 99:2007, 2.13) upper bound estimation of the largest (dominant) uncertainty components (box 12).

   Change to a more detailed model of the uncertainty estimation process or a higher resolution of the measuring process (box 12).

e) Make the second iteration of the uncertainty budget (boxes 7 to 9) leading to the second, lower and more accurate (see ISO/IEC Guide 99:2007, 2.13) upper bound estimate of the uncertainty of measurement, U_{E2} (box 10).

f) Compare the second estimated uncertainty U_{E2} (box A) with uncertainty required U_R for the actual measuring task.

   1) If U_{E2} is acceptable (i.e. if $U_{E2} \leqslant U_R$), then the uncertainty budget of the second iteration has proven that the given measurement procedure is adequate to the measuring task (box 11).

   2) If U_{E2} is not acceptable (i.e. if $U_{E2} > U_R$), or if there is no required uncertainty, but a lower and more true value is desired, then a third (and possibly more) iteration(s) is (are) needed. Repeat the analysis of the uncertainty components [additional changes of assumptions, improvements in knowledge, changes in modelling, etc. (box 12)] and concentrate on the currently largest uncertainty components.

g) When all possibilities have been used for making more accurate (lower) upper bound estimates of the measuring uncertainties without coming to an acceptable measuring uncertainty $U_{EN} \leqslant U_R$, then it is proven that it is not possible to fulfil the given requirement U_R.

6.3 Uncertainty management for design and development of a measurement process/procedure

Uncertainty management in this case is performed to develop an adequate measurement procedure [measurement of the geometrical characteristics of a workpiece or the metrological characteristics of a measuring equipment (calibration)]. Uncertainty management is performed on the basis of a defined measuring task (box 1 in Figure 2) and a given target uncertainty, U_T (box 2). Definitions of the measuring task and target uncertainty are company policy decisions to be made at a sufficiently high management level. An adequate measurement procedure is a procedure which results in an estimated uncertainty of measurement less than or equal to the target uncertainty. If the estimated uncertainty of measurement is much less than the target uncertainty, the measurement procedure may not be (economically) optimal for performing the measuring task (i.e. the measurement process is too costly).

The PUMA, based on a given measuring task (box 1) and a given target uncertainty U_T (box 2), includes the following (see Figure 2).

a) Choose the principle of measurement (box 3) on the basis of experience and possible measurement instruments present in the company.

b) Set up and document a preliminary method of measurement (box 4), measurement procedure (box 5) and measurement conditions (box 6) on the basis of experience and known possibilities in the company.

c) Make a first iteration based preferably on a black box model of the uncertainty estimation process and set up a preliminary uncertainty budget (boxes 7 to 9) leading to the first rough estimate of the expanded uncertainty, U_{E1} (box 10). For details about uncertainty estimation, see Clause 9. All estimates of uncertainties U_{EN} are performed as upper bound estimates.

d) Compare the first estimated uncertainty, U_{E1}, with the given target uncertainty, U_T (box A).

   1) If U_{E1} is acceptable (i.e. if $U_{E1} \leqslant U_T$), then the uncertainty budget of the first iteration has proven that the measurement procedure is adequate for the measuring task (box 11).

   2) If $U_{E1} << U_T$, then the measurement procedure is technically acceptable, but a possibility may exist to change the method or the procedure (box 13), or both, in order to make the measuring process more cost effective while increasing the uncertainty. A new iteration is then needed to estimate the resulting measurement uncertainty, U_{E2} (box 10).

   3) If U_{E1} is not acceptable (i.e. if $U_{E1} > U_T$), the iteration process continues, or it is concluded that no adequate measurement procedure is possible.

e) Before the new iteration, analyse the relative magnitude of the uncertainty components. In many cases, a few uncertainty components predominate the combined standard uncertainty and expanded uncertainty.

f) If $U_{E1} > U_T$, then change the assumptions or the modelling or increase the knowledge about the uncertainty components (box 12) to make a more accurate (see ISO/IEC Guide 99:2007, 2.13) upper bound estimation of the largest (dominant) uncertainty components.

g) Make the second iteration of the uncertainty budget (boxes 7 to 9) leading to the second, lower and more accurate (see ISO/IEC Guide 99:2007, 2.13) upper bound estimate of the uncertainty of measurement, U_{E2} (box 10).

h) Compare the second estimated uncertainty U_{E2} with the given target uncertainty, U_T (box A).

   1) If U_{E2} is acceptable (i.e. if $U_{E2} \leqslant U_T$), then the uncertainty budget of the second iteration has proven that the measurement procedure is adequate for the measuring task (box 11).

   2) If U_{E2} is not acceptable (i.e. if $U_{E2} > U_T$), then a third (and possibly more) iteration(s) is (are) needed. Repeat the analysis of the uncertainty components [additional changes of assumptions, modelling and increase in knowledge (box 12)] and concentrate on the currently largest uncertainty components.

i) When all possibilities have been used for making more accurate (lower) upper bound estimates of the measuring uncertainties without coming to an acceptable measuring uncertainty $U_{EN} \leqslant U_T$, then it is necessary to change the measurement method or the measurement procedure or the conditions of measurement (box 13) to (possibly) bring down the magnitude of the estimated uncertainty, U_{EN}. The iteration procedure starts again with a first iteration.

j) If changes in the measurement method or the measurement procedure or conditions (box 13) do not lead to an acceptable uncertainty of measurement, it is possible to change the principle of measurement (box 14) and start the above-mentioned procedure again.

k) If changing the measuring principle and the related iterations described above still does not lead to an acceptable uncertainty of measurement, the ultimate possibility is to change the measuring task or target uncertainty (box 15), or both, and to start the above-mentioned procedure again.

l) If changing the measuring task or target uncertainty is not possible, it has been demonstrated that no adequate measurement procedure exists (box 16).

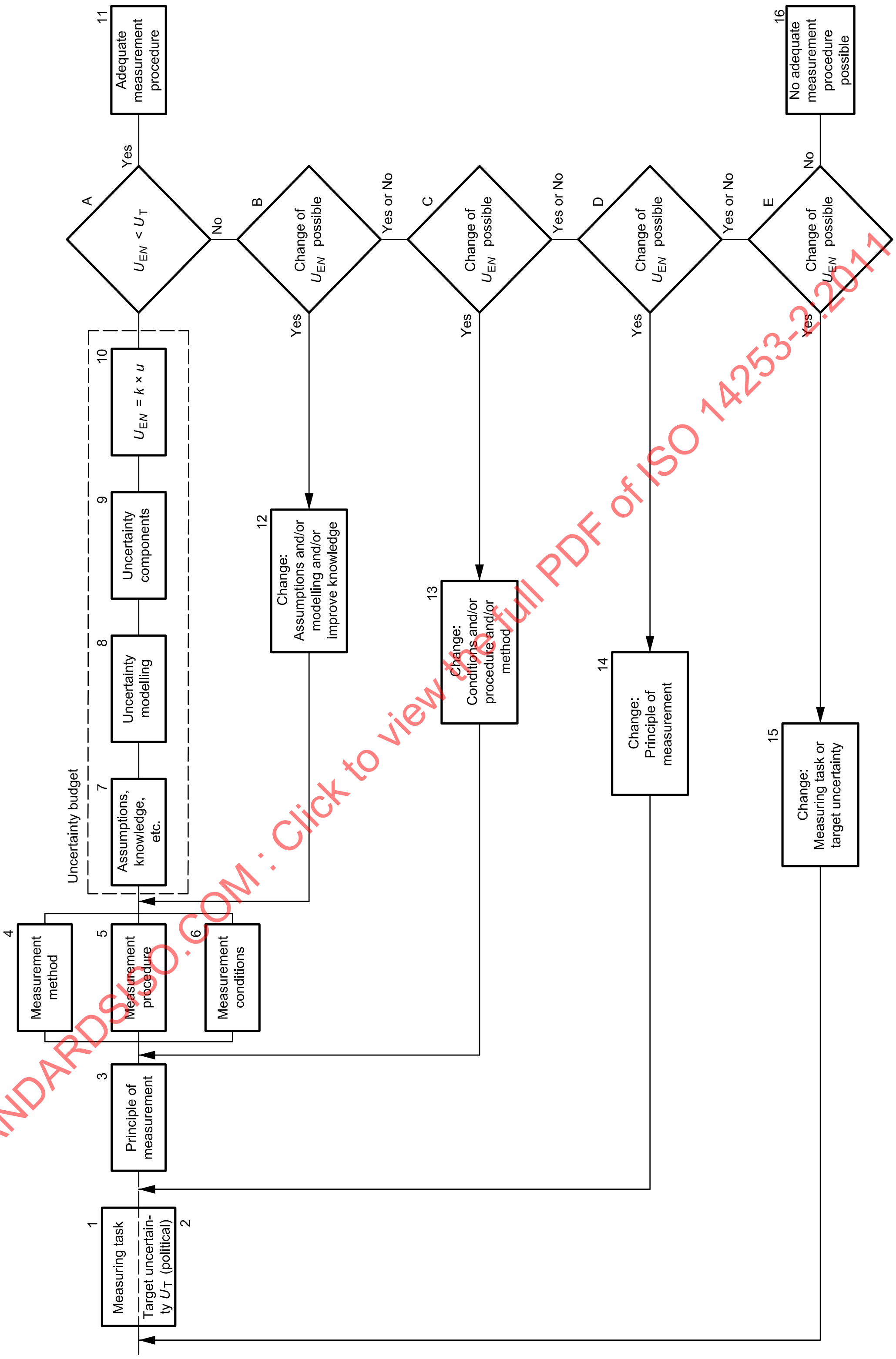


Figure 2 — Procedure for Uncertainty of Measurement MAnagement (PUMA) for a measurement process/procedure

7 Sources of errors and uncertainty of measurement

7.1 Types of errors

Different types of errors regularly show up in measurement results:

- systematic errors;
- random errors;
- drift;
- outliers.

All errors are by nature systematic. When errors are perceived as non-systematic, it is either because the reason for the error is not looked for or because the level of resolution is not sufficient. Systematic errors may be characterized by size and sign (+ or –).

ER = MR – TV

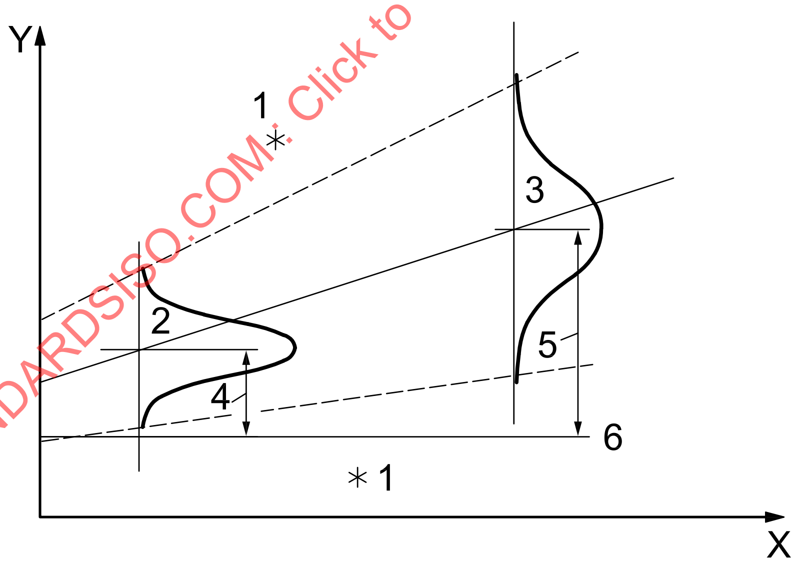
where

ER is the error;

MR is the measurement result;

TV is the true value.

Random errors are systematic errors caused by non-controlled random influence quantities. Random errors may be characterized by the standard deviation and the type of distribution. The mean value of the random errors is often considered as a basis for the evaluation of the systematic error (see Figure 3).



Key

X measured value

Y time

1 outlier

2 dispersion 1

3 dispersion 2

4 systematic error 1

5 systematic error 2

6 true value

Figure 3 — Types of errors in measurement results

Drift is caused by a systematic influence of non-controlled influence quantities. Drift is often a time effect or a wear effect. Drift may be characterized by change per unit time or per amount of use.

Outliers are caused by non-repeatable incidents in the measurement. Noise — electrical or mechanical — may result in outliers. A frequent reason for outliers is human error, i.e. mistakes as reading and writing or wrong handling of measuring equipment. Outliers are impossible to characterize in advance.

Errors or uncertainties in a measuring process will be a mix of known and unknown errors from a number of sources or error components.

The sources or components are not the same in each case, and the sum of the components is not the same.

It is still possible to take a systematic approach. There are always several sources or a combined effect of the ten different ones indicated in Figure 4.

In the following subclauses, examples and further details about each of the ten components are given.

What is often difficult is that each of the components may act individually on the measurement result. But in many cases, they even interfere with each other and cause additional errors and uncertainty.

Figure 4 and the following non-exhaustive lists (see 7.2 to 7.11) shall be used for getting ideas in a systematic way when making uncertainty budgets. In each case, in order to evaluate the actual error/uncertainty component, it is necessary to have knowledge about physics or experience in metrology, or both.

In uncertainty budgets, the uncertainty components may be grouped for convenience.

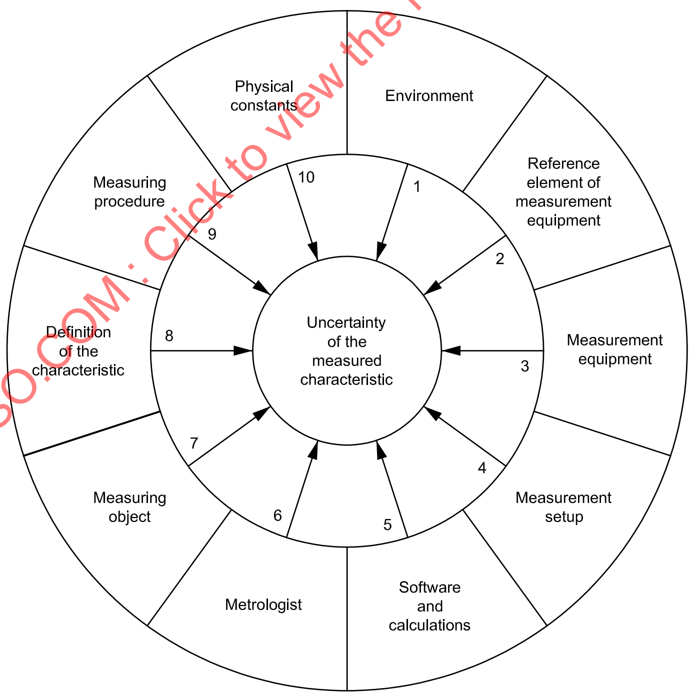


Figure 4 — Uncertainty components in measurement

7.2 Environment for the measurement

In most cases — especially in GPS measurements — the temperature is the main uncertainty component of the environment. Other uncertainty components may be:

- Temperature: absolute temperature, time variance, spatial gradient
- Vibration/noise
- Humidity
- Contamination
- Illumination
- Ambient pressure
- Air composition
- Air flow
- Gravity
- Electromagnetic interference
- Transients in the power supply
- Pressured air (e.g. air bearings)
- Heat radiation
- Workpiece
- Scale
- Instrument thermal equilibrium

7.3 Reference element of measurement equipment

The measuring equipment is divided into “reference element” and the “rest of the equipment”, and it often pays to look at the equipment that way. The “reference element” includes the following items.

- Stability
- Scale mark quality
- Temperature expansion coefficient
- Physical principle: line scale, optical digital scale, magnetic digital scale, spindle, rack & pinion, interferometer
- CCD techniques
- Uncertainty of the calibration
- Resolution of the main scale (analogue or digital)
- Time since last calibration
- Wavelength error

7.4 Measurement equipment

The “rest of the equipment” includes the following items.

- Interpretation system
- Magnification, electrical or mechanical
- Error wavelength
- Zero-point stability
- Force stability/absolute force
- Hysteresis
- Guides/slideways
- Probe system
- Geometrical imperfections
- Stiffness/rigidity
- Reading system
- Linear coefficient for thermal expansion
- Temperature stability/sensitivity
- Parallaxes
- Time since last calibration
- Response characteristic
- Interpolation system, error wavelength
- Interpolation resolution
- Digitization

7.5 Measurement set-up (excluding the placement and clamping of the workpiece)

In many cases, there is no set-up; the measurement equipment can measure "alone".

- Cosine errors and sine errors
- Abbe principle
- Temperature sensitivity
- Stiffness/Rigidity
- Tip radius
- Form deviation of tip
- Stiffness of the probe system
- Optical aperture
- Interaction between workpiece and set-up
- Warming up

7.6 Software and calculations

Observe that even the number of digits or decimals can have an influence.

- Rounding/Quantification
- Algorithms
- Implementation of algorithms
- Number of significant digits in the computation
- Sampling
- Filtering
- Correction of algorithm/Certification of algorithm
- Interpolation/Extrapolation
- Outlier handling

7.7 Metrologist

The human being is not stable; there is a difference from day to day and often a rather large change during the day.

- Education
- Experience
- Training
- Physical disadvantages/Ability
- Knowledge (precision, appreciation)
- Honesty
- Dedication

7.8 Measurement object, workpiece or measuring instrument characteristic

The following factors can affect the outcome.

- Surface roughness
- Form deviations
- E-modulus (Young's modulus)
- Stiffness beyond E-modulus
- Temperature expansion coefficient
- Conductivity
- Weight
- Size
- Shape
- Magnetism
- Hygroscopic characteristic of the material
- Ageing
- Cleanliness
- Temperature
- Internal stress
- Creep characteristics
- Workpiece distortion due to clamping
- Orientation

7.9 Definition of the GPS characteristic, workpiece or measuring instrument characteristic

The following are used in the definition.

- Datum
- Reference system
- Degrees of freedom
- Toleranced feature
- ISO 4288
- Chain link 3 and 4 deviations (ISO/TR 14638)
- Distance
- Angle

7.10 Measuring procedure

The procedure is affected by the following.

- Conditioning
- Number of measurements
- Order of measurements
- Duration of measurements
- Choice of principle of measurement
- Alignment
- Choice of reference — reference item (standard) and value — relative to the measured value
- Choice of apparatus
- Choice of metrologist
- Number of operators
- Strategy
- Clamping
- Fixturing
- Number of points
- Probing principle and strategy
- Alignment of probing system
- Drift check
- Reversal measurements
- Multiple redundancy, error separation

7.11 Physical constants and conversion factors

Knowledge of the correct physical values of, for example, material properties (workpiece, measuring instrument, ambient air, etc.) is important.

8 Tools for the estimation of uncertainty components, standard uncertainty and expanded uncertainty

8.1 Estimation of uncertainty components

Estimation of uncertainty components can be done in two different ways: Type A evaluation and Type B evaluation.

Type A evaluation is evaluation of uncertainty components, u_{xx}, using statistical means. Type B evaluation is evaluation of uncertainty components, u_{xx}, by any other means than statistical.

Type A evaluation will in most cases result in more accurate estimates of uncertainty components than Type B evaluation. In many cases, Type B evaluation will result in sufficiently accurate estimations of uncertainty components.

Therefore, a Type B evaluation shall be chosen in the iterative method, when it is not absolutely necessary to evaluate uncertainty by using Type A evaluation. In a number of cases, no other possibilities exist than to use a Type A evaluation. See “standard cases” for evaluation of uncertainty components in 8.4.

NOTE The iteration method uses the designation u_{xx} for all uncertainty components.

8.2 Type A evaluation for uncertainty components

8.2.1 General

Type A evaluation of the uncertainty component, u_{xx}, needs data from repeated measurements. The standard deviation of the distribution or the standard deviation of the mean value may be calculated using the formulas in 8.2.2.

8.2.2 Statistical tools

Regardless of the type of statistical distribution, the following statistical parameters are defined by the equations:

$$\bar{x} = \frac{1}{n} \times \sum_{1}^{n} X_i$$

The mean value of a number, n, of measurement results X_i. $\bar{x}$ is an estimate of the true value of the mean μ of the distribution.

$$s_x = \sqrt{\frac{\sum_{1}^{n} \left(\bar{x} - X_i\right)^2}{(n-1)}}$$

The standard deviation of the distribution of the sample based on n measurement values. s_x is an estimate of the standard deviation of the distribution σ.

$$s_{\bar{x}} = \sqrt{\frac{\sum_{1}^{n} \left(\bar{x} - X_i\right)^2}{n \times (n-1)}} = \frac{s_x}{\sqrt{n}}$$

The standard deviation of the mean value $s_{\bar{x}}$ of the sample is equal to the standard deviation of the sample divided by the square root of the number of measurements n.

When the mean value or the standard deviation is based on very few repeated measurements the estimated standard deviation values may be wrong, and possibly too small. For this reason, a “safety” factor t is used.

The safety factor t is calculated based on the Student t distribution. The standard deviation of the sample s_x (multiplied by the safety factor t as appropriate) is used in the uncertainty budget as the value for u_{xx}, when the measurement result is obtained using single readings of the component concerned.

$$u_{xx} = s_{x,n} \times t \qquad (4)$$

The standard deviation of the mean value $s_{\bar{x}}$ is the value used for the standard uncertainty u_{xx} in the uncertainty budget when the measurement result is obtained using the mean of several readings of the component concerned.

$$u_{xx} = s_{\bar{x}} \times t \quad \left(s_{\bar{x},n} = \frac{s_{x,n}}{\sqrt{n}} \right) \qquad (5)$$

8.3 Type B evaluation for uncertainty components

8.3.1 General

The evaluation of standard deviations by any means other than statistical is most often limited to previous experiences or by simply “guessing” what might be the standard deviation.

Experience shows that human beings do not “understand” or are not able to estimate standard deviations directly.

Experience shows that human beings remember limit values for variation (error limit values) or are able to develop such by using logical arguments and physical laws. In many cases, specifications are known as limit values. This can be developed into a systematic method to derive standard deviations from limit values.

8.3.2 Transformation tools for error limits

Given a limit of variation, a. For all (limited) distributions, there is a certain ratio between the standard deviation (defined by the same formula valid for all distributions, see 8.2.2) and the limit value, a. Then, if the limit value, a, is known and the type of distribution is known, it is possible to calculate the standard deviation. The limit value designation is chosen as $-a$ and $+a$ (only symmetrical distributions):

$$u_{xx} = a \times b \tag{6}$$

Experience shows that in most cases it is sufficient to use only three types of distribution for transforming limits of variation into standard deviation.

In Figure 5, these three types of distribution are given with the formula for transforming from limit value to uncertainty component u_{xx} (standard uncertainty). The Gaussian distribution is not limited. Two times the standard deviation ($2s$) is used as the limit value for the Gaussian distribution. By experience, it is known that a human being remembers the $2s$ value as the limit value for Gaussian distributed data. The b value for the three types of distribution shown in Figure 5 is:

— Gaussian distribution: $b = 0{,}5$

— rectangular distribution: $b = 0{,}6$

— U-distribution: $b = 0{,}7$

Gaussian distribution: $b = 0{,}5$

$$u_{xx} = \frac{a}{2} \approx 0{,}5 \times a$$

Rectangular distribution: $b = 0{,}6$

$$u_{xx} = \frac{a}{\sqrt{3}} \approx 0{,}58 \times a \approx 0{,}6 \times a$$

U-distribution: $b = 0{,}7$

$$u_{xx} = \frac{a}{\sqrt{2}} \approx 0{,}71 \times a \approx 0{,}7 \times a$$

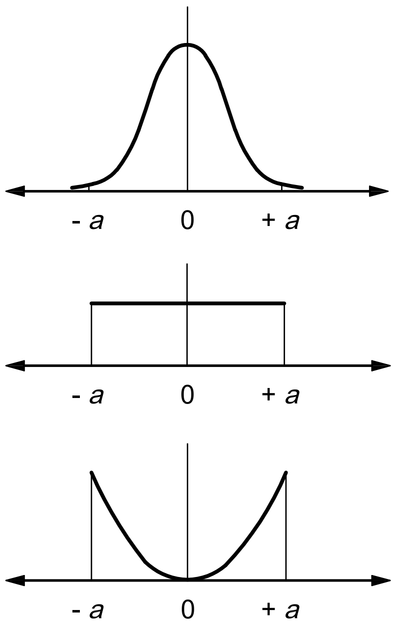


Figure 5 — The three types of distribution used for transforming limits of variation, a, into uncertainty components, u_{xx} (standard uncertainties)

Type B evaluation of the uncertainty component needs a reasonable “guess” or knowledge about the limit value, a. To be sure it is an overestimation make a high, but not too high, guess of the limit value to determine the a value. The next step is to make an assumption about the distribution. In many cases, the type of distribution is known or is obvious. If not, make a conservative assumption. If the distribution is not known to be Gaussian, then choose a rectangular or U-distribution. If the type of distribution is not known to be rectangular, then choose a U-distribution. The U-distribution is the most conservative assumption.

One way to make reasonable estimates of standard uncertainties — for influence quantities — without using statistical methods is by experience or by using physical laws to set up variation limits for a component and then transform these limit values to standard uncertainties by an assumed distribution type for the actual error/uncertainty component.

8.4 Common Type A and B evaluation examples

8.4.1 General

In this clause, some examples of common uncertainty components will be discussed. Examples will be given on how to derive the uncertainty component u_{xx}. The examples are not exhaustive for the problems arising in GPS measurement and calibration.

8.4.2 Experiment or limit value as basis for evaluation of the same uncertainty component

Data from repeated measurements give the possibility of using a Type A evaluation as well as a Type B evaluation of the resulting uncertainty component.

Data can be used to calculate the standard deviation (uncertainty component) using the formulas given in 8.2.2 (Type A evaluation).

The same measured data may also be used in a Type B evaluation of the same uncertainty component only using the extreme values in the data-set as limit values (a values) around a mean. The uncertainty component is then calculated using the formulas in Figure 5.

8.4.3 Repeatability

In every uncertainty budget, repeatability is involved at least one time. In most cases, repeatability can only be evaluated by an experiment (Type A evaluation). The uncertainty component is derived using the formulas for s_x and $s_{\bar{x}}$ given in 8.2.2.

The repeatability-based uncertainty component may be less than the uncertainty component derived from the resolution of the measurement equipment reading. In this case, the latter shall be used instead of the repeatability (see 8.4.4).

8.4.4 Resolution and rounding

The resolution of a measuring equipment (analogue or digital) or the step in last digit/decimal of a measured value or rounded measured value, whichever is the largest, is causing an uncertainty component:

$$u_{xx} = \frac{d}{2 \times \sqrt{3}} \approx \frac{d}{2} \times 0{,}6 \approx 0{,}3 \times d \qquad (7)$$

where d is the resolution or the step in the last digit or decimal. The uncertainty component is equal to the component from a rectangular distribution with limit value $a = 0{,}5 \times d$.

When the repeatability uncertainty component is derived from experimental data, the effect from resolution, etc., is included if the repeatability uncertainty component is greater than the component based on resolution, etc.

An example is given in Annex C.

8.4.5 Maximum permissible error (MPE) of a measuring equipment

When a measuring equipment or measuring standard is known to conform to stated MPE values for each of the metrological characteristics, these MPE values can be used to derive the related uncertainty components:

$$u_{xx} = \text{MPE} \times b \qquad (8)$$

where b is chosen according to the rules given in 8.3.2 and the distribution assumed. When calibration data exist for one measuring equipment or for a larger number of identical pieces of equipment, it is often possible to use these data to find the type of distribution or even in rare cases to evaluate the uncertainty component directly — as a Type A evaluation — by the equations shown in 8.2.2.

8.4.6 Corrections

Errors (ER), for which a magnitude and sign (+ or −) are known, may be compensated for by a correction, C, added to the measurement result:

$$C = -\text{ER} \qquad (9)$$

Even when a correction is made, an uncertainty component (uncertainty of the correction) remains. This uncertainty component shall be less than the error/correction for the correction to have a positive effect on uncertainty of measurement.

It is the responsibility of the person making the uncertainty budget to decide if a known error shall be corrected for. The criteria to correct for a known error or not are based on economy.

Drift may be treated and dealt with as a known error, which may be corrected for.

8.4.7 Hysteresis

Hysteresis, h, in the indication of a measuring equipment may be treated as a symmetrical error/uncertainty around the mean of the two indications forming the hysteresis. The uncertainty component may be derived as a Type A evaluation if sufficient data are present or as a Type B evaluation where the uncertainty component is:

$$u_{xx} = \frac{h}{2} \times b \qquad (10)$$

where b is chosen according to the rules given in 8.3.2 and the distribution assumed.

8.4.8 Influence quantities (temperature, measuring force, direction of measurement, etc.)

Measurements are influenced by a number of influence quantities (see ISO/IEC Guide 99:2007, 2.52), which affect the measuring equipment or the object (e.g. component, measuring instrument, etc.) being measured, or both. Common influence quantities in GPS measurements are temperature, measuring force and direction of measurement. The influence is expressed in a unit other than length [e.g. °C, N and ° (angle)] and shall be transformed by physical laws (equations) into length.

Influence quantities are often known as a value or a range and the uncertainty of the before-mentioned value or range is known as a limit value.

8.4.8.1 Temperature

Standard reference temperature for GPS and GPS measurements is 20 °C (see ISO 1). Influences from temperature, which may be caused by absolute temperature as well as time and spatial temperature gradients, result in linear expansion, bending, etc., of the measurement equipment, the measurement set-up and the object being measured. The transformation from temperature to length is given by the linear expansion equation:

$$\Delta L = \Delta T \times \alpha \times L \quad (11)$$

where ΔT is the relevant temperature difference, α is the temperature expansion coefficient of the material and L is the effective length under consideration.

When temperature has an influence, several transformation equations from temperature to length may be used together with other geometrical or physical equations to form the full description of the influence on the GPS measurement result (length, form, etc.).

8.4.8.2 Measuring force

Standard reference condition for GPS is zero measurement force. The effect on errors and uncertainty of length measurement by non-zero force is caused by elastic and in some cases also plastic deformation of the measurement equipment, the measurement set-up and the measuring object. Especially the effect on the contact geometry between measuring equipment and measurement object shall be investigated.

The effect of measuring force may be quantified by experiments or by physical equations (Hertz formulas and others). The effect depends on the force, the direction of the force, geometry and material constants such as E (Young's modulus), ν (Poisson's number), etc.

8.4.8.3 Direction of measurement

The direction of measurements shall be according to the definition of the geometrical characteristic of the measurement object (see ISO/TR 14638).

The effect of deviation from the defined directions of measurement can be calculated from basic trigonometric equations and be subject to the directional effects of the other influencing quantities.

8.4.9 Definition of the measurand

Measurands in GPS measurements are GPS characteristics of workpieces (often given as requirements on technical drawings) and metrological characteristics for measurement equipment and measurement standards.

These measurands are defined in GPS standards (see ISO/TR 14638 for an overview). In many cases, the measurement procedure is intentionally or by accident not in conformance with the definition of the characteristic. In such cases, these deviations in measurement procedure will result in errors and uncertainties in the measurement result. If the errors are known, correction is possible (see 8.4.6). In practice, a measurement procedure will always result in an uncertainty relative to the definition of the measurand (see also 8.4.11).

8.4.10 Calibration certificates

Calibration certificates give measured values for metrological characteristics and the related uncertainty of measurement. When the given calibrated value is used, the uncertainty component u_{xx} is derived as follows.

— The uncertainty is expressed as "expanded uncertainty", U, with a stated "coverage factor", k, according to GUM:

$$u_{xx} = \frac{U}{k} \quad (12)$$

Some calibration organizations have standardized a default value of k. In these cases, the “coverage factor” is not stated on the certificate.

— The uncertainty is expressed as a value U_V and a stated “level of confidence”, e.g. 95 % or 99 %:

$$u_{xx} = \frac{U_V}{m} \quad (13)$$

where m is the number of standard deviations in the confidence interval corresponding to the stated level of confidence.

Calibration certificates sometimes only — or in addition — certify that the equipment fulfils a defined specification (a set of MPEs) given e.g. in a standard, manufacturer's data sheet, etc. In this case, the nominal MPE value of the metrological characteristic shall be used and the uncertainty component derived from this MPE value given in the specification according to 8.4.5.

8.4.11 Surface texture, form and other geometrical deviations of a measuring object

The surfaces of a measuring object are in contact with the measuring equipment during measurement. Depending on the surface texture, form deviations and other geometrical deviations from nominal geometry, the contact geometry (stylus tip) of the measuring equipment will interact with the surface and cause uncertainty components.

These components may be evaluated by experiments (Type A evaluation) or a Type B evaluation or partly by experiments and partly by a Type B evaluation.

8.4.12 Physical constants

Physical constants (e.g. temperature expansion coefficients, Young's modulus, Poisson's number, etc.) which are part of corrections for or transformation from the influence quantity error or evaluated uncertainties are often not known accurately, but are estimated.

They therefore introduce additional uncertainty components using the same transformation formulas as used for influence quantities above. This evaluation can only be done as a Type B evaluation.

8.5 Black and transparent box model of uncertainty estimation

The uncertainty of a measurement process can be evaluated using different models or different levels of detail, or both. The two extreme cases are the black box method and the transparent box method.

In the black box method, the total measurement process is modelled as a black box with unknown content. The uncertainty budget and the uncertainty components are only describing the total effect on the measurement process. In this choice of model, it may be very difficult to determine the functional relationship between uncertainty components and individual error components.

To have the full benefit of uncertainty budgeting it may be necessary to open the black box and make a more detailed uncertainty budget. This could either be based on several smaller black boxes or the behaviour of all the details in the measurement process, the transparent box model of uncertainty estimation. The black box may also be characterized as a low resolution method and the transparent box method as a high resolution method/model.

In the black box model for uncertainty estimation, the input and output units are the same and the uncertainty components are assumed to be additive, and the sum of the uncertainty components have the expectation value zero. For the purposes of the black box model in this part of ISO 14253 and the PUMA method, all influence quantities are transformed to the unit of the measurand. Therefore, in the black box model, the sensitivity coefficients of the individual uncertainty component are equal to 1 (one).

In the transparent box model for uncertainty estimation, these restrictions of the uncertainty components (additive uncertainty components, input unit the same as output unit and sensitivity coefficient equal to 1) are not valid.

8.6 Black box method of uncertainty estimation — Summing of uncertainty components into combined standard uncertainty, u_c

In the black box method of uncertainty estimation, the result of the measurement is the reading corrected by an eventually known correction:

$$Y = X + C \tag{14}$$

where X is the reading of the measuring instrument and $C = \Sigma C_i$ is the sum of the corresponding additive corrections known from e.g. calibration, temperature correction, deformation correction, etc.

The combined standard uncertainty of measurement is given by Equation (15):

$$u_c = \sqrt{u_r^2 + \sum_1^p u_i^2} \tag{15}$$

where

p is the number of uncorrelated uncertainty components;

u_r is “the sum” of the strongly correlated ($\rho = 1$ and -1) uncertainty components, calculated by the equation:

$$u_r = \sum_1^r u_i \tag{16}$$

where r is the number of strongly correlated uncertainty components.

In total, there are $p + r$ uncertainty components in measurement of Y.

The uncorrelated ($\rho = 0$) uncertainty components are to be added geometrically (the square root of the sum of squares).

The strongly correlated uncertainty components are to be added arithmetically.

A conservative estimate is to consider all uncertainty components which are known not to be fully uncorrelated as strongly correlated.

8.7 Transparent box method of uncertainty estimation — Summing of uncertainty components into combined standard uncertainty, u_c

In the transparent box method of uncertainty estimation, the value of the measurand is modelled as a function of several measured values X_i, which themselves could be functions (transparent box models) or black box models, or both:

$$Y = G\left(X_1, X_2, ..., X_i, ..., X_{p+r}\right) \tag{17}$$

The combined standard uncertainty of measurement is given by the equation:

$$u_{\mathrm{c}} = \sqrt{u_r^2 + \sum_{i=1}^{p} \left(\frac{\partial Y}{\partial X_1} \times u_{Xi} \right)^2} \tag{18}$$

where u_r is the "sum" of the strongly correlated components of measuring uncertainty:

$$u_r = \sum_{i=1}^{r} \frac{\partial Y}{\partial X_i} \times u_{Xi} \tag{19}$$

where

$\frac{\partial Y}{\partial X_i}$ is the partial differential coefficient of the function Y with respect to X_i;

u_{Xi} is the combined standard uncertainty of measurement of the number i measured value (function), which is part of the transparent box method of uncertainty estimation for the measurement of Y.

u_{Xi} may be the result (u_{c} — combined standard uncertainty) of either a black box (see 8.6) or another transparent box method of uncertainty estimation.

The uncorrelated ($\rho = 0$) components of measuring uncertainty shall be added geometrically (the square root of the sum of squares).

The strongly correlated components of uncertainty shall be added arithmetically (the number of strongly correlated components of uncertainty is r).

A conservative estimate is to take as strongly correlated all components which are not known to be fully uncorrelated.

The number of uncorrelated components of uncertainty is p.

In total, there have been $p + r$ components of uncertainty in this transparent box method of uncertainty estimation of Y, which again — each of them — could be a combination of a number of components of uncertainty of measurement.

8.8 Evaluation of expanded uncertainty, U, from combined standard uncertainty, u_{c}

The expanded uncertainty of measurement, U, in GPS measurements is calculated as:

$$U = u_{\mathrm{c}} \times k = u_{\mathrm{c}} \times 2 \tag{20}$$

Unless otherwise specified, the coverage factor $k = 2$ in GPS measurements (see ISO 14253-1).

8.9 Nature of the uncertainty of measurement parameters u_{c} and U

The uncertainty components and the combined uncertainty of measurement are, as shown, estimated as a standard uncertainty u_{xx} and u_{c} respectively. In practical industrial GPS measurements, the uncertainty components are a mix of constant and varying components with time constants covering several orders of magnitude. The uncertainty of measurement includes all systematic errors, which are not corrected for, regardless of the reason. It is impossible to correct for all systematic errors.

Therefore, in most cases, u_{c} and U are not stochastic variables. They represent quasi-constant, but not known errors. U and u_{c} shall, therefore, not be treated as standard deviations, but as constant (unknown) errors.

9 Practical estimation of uncertainty — Uncertainty budgeting with PUMA

9.1 General

The use of the PUMA method and how to make uncertainty budgets and related documentation are given as examples in Annex A.

This clause only gives the sequence in the documentation and procedure of estimating each of the components of uncertainty to be put in an uncertainty budget.

9.2 Preconditions for an uncertainty budget

Setting up an uncertainty budget is only possible when:

— the measuring task is properly defined. The characteristic of the feature of the workpiece or the characteristic of the measurement equipment shall be defined and pointed out as the task (box 1 in Figure 2);

— the measurement principle is properly defined and known, or at least known initially as a draft (box 3 in Figure 2);

— the measurement method is properly defined and known, or at least known initially as a draft (box 4 in Figure 2);

— the measurement procedure is properly documented and known, or at least known initially as a draft (box 5 in Figure 2).

  — The measurement procedure includes the choice of measurement equipment.

  — The measurement procedure gives all the details of how the measuring equipment and the workpiece are handled during measurement. The uncertainty budget is mirroring the activities and steps in the procedure;

— the measurement conditions are defined and known, or at least known initially as a draft (box 6 in Figure 2).

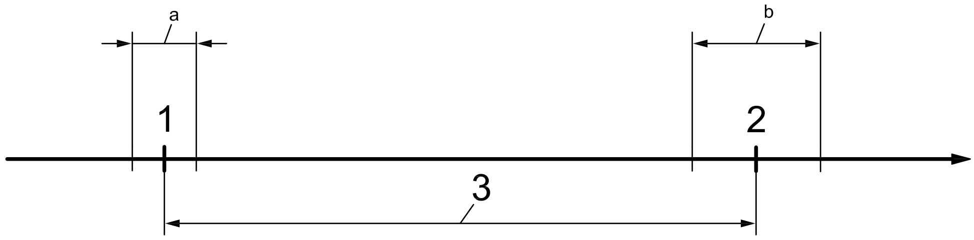
Observe that every measurement will include the three elements (1, 2 and 3) illustrated in Figure 6. The uncertainty budget shall reflect the three elements:

— determination of a reference point (1 in Figure 6), often a zero point. In many cases, the zero point of the measurement equipment is set as an activity in the calibration procedure. Uncertainty is related to the setting of the reference point or zero point;

— determination of a measuring point (2 in Figure 6), the reading of the measurement equipment when measuring the characteristic of the workpiece or measurement equipment. Uncertainty is related to the reading itself depending on characteristics of the equipment and the measuring object;

— a travel of the measurement equipment (3 in Figure 6) from the reference point to the measurement point. The error or uncertainty, or both, of this travel is known from the calibration of the equipment.

Each of the three elements is again and additionally influenced by the error sources given in Clause 7. The influence from the error/uncertainty sources shall be systematically checked in the uncertainty budget.



Key

1 reference point

2 measuring point

3 travel of measuring equipment

a Uncertainty range of reference point.

b Uncertainty range of measuring point.

Figure 6 — Generic model of the three elements in a measurement

9.3 Standard procedure for uncertainty budgeting

9.3.1 The following procedure may be helpful for setting up and documenting an uncertainty budget, first iteration of the PUMA method.

9.3.2 Define and document the overall measuring task (characteristic to be measured) and the basic measurement value [basic measurement result (see 9.2)] for which the uncertainty budget shall be set up.

9.3.3 Document the

— measurement principle,

— measurement method,

— measurement procedure, and

— measurement conditions.

If not fully known, choose and document the initial or assumed draft principle, draft method, draft procedure and draft conditions in accordance with the principle of overestimation of uncertainty components given in Clause 5.

9.3.4 Make a graphical presentation of the measurement set-up(s). The figure(s) may be of help for understanding the uncertainty components present in the measurement.

9.3.5 Document the mathematical relations between measured values and the characteristics of the overall measuring task.

The mathematical relation is normally not needed when the measuring task can be solved by a black box method (see 8.6).

The mathematical relation is needed when the measuring task shall be solved by a transparent box method (see 8.7).

9.3.6 Make an initial investigation and documentation of all possible uncertainty components. The result and the documentation may be stated in a table as illustrated in Figure 7.

The investigation is made in a systematic sequence using the three elements given in Figure 6, the potential error sources given in Clause 7 and the already documented information of 9.3.2 and 9.3.3.

The subdivision of the uncertainty of measurement into uncertainty components should be done in a way that does not include the same component more than once, but in many practical cases this is not possible. The principle is most important for the dominant components in an uncertainty budget.

Designation (low resolution)	Designation (high resolution)	Name	Comments (initial)
u_{xx}	u_{xa}	Name of xa	Initial observations, information, comments and decisions related to uncertainty component xa
	u_{xb}	Name of xb	Initial observations, information, comments and decisions related to uncertainty component xb
	u_{xc}	Name of xc	Initial observations, information, comments and decisions related to uncertainty component xc
		Name of total xx	Initial observations, information, comments and decisions related to uncertainty component total xx
u_{yy}	u_{ya}	Name of ya	Initial observations, information, comments and decisions related to uncertainty component ya
	u_{yb}	Name of yb	Initial observations, information, comments and decisions related to uncertainty component yb
		Name of total yy	Initial observations, information, comments and decisions related to uncertainty component total yy
u_{zz}		Name of zz	Initial observations, information, comments and decisions related to uncertainty component zz

Figure 7 — Initial overview, designation, naming and commenting on the uncertainty components of an uncertainty budget

The table in Figure 7 has two levels of resolution. These levels are useful in the initial phase and before the first PUMA iteration, where the modelling of the uncertainty is not yet established. Low resolution often means one single black box as the model. High resolution gives the possibility of splitting the single black box into several smaller black boxes.

For each uncertainty component, define and document mathematical designations and names (labels) on the two levels of resolution.

Use the comments column in Figure 7 to sum up information, conditions and even initial decisions related to the actual uncertainty component. The comments column is a note pad.

9.3.7 Based on the information present and documented in Figure 7, investigate and establish for the uncertainty modelling for the actual iteration step.

For each uncertainty component:

— decide on the evaluation method, Type A or B evaluation (see 8.2 and 8.3);

— document and argue for the evaluation of the uncertainty component value, the background, etc.;

— in the case of a Type A evaluation, state the component value and the number of measurements on which it is based;

— in the case of a Type B evaluation, state the limit value a^* (variation limit in the unit of the influence quantity), a, the assumed distribution and the resulting uncertainty component value.

9.3.8 Investigate, search for and document any possible correlation between the documented uncertainty components in accordance with Clause 5.

9.3.9 Choose the correct formulas depending on modelling and correlation and calculate the combined standard deviation, u_c (see 8.6 and 8.7).

9.3.10 Derive the expanded uncertainty, U, where $U = 2 \times u_c$ (see 8.8).

9.3.11 Make a summary table containing all key information in the uncertainty budget (see example in Figure 8). Investigate possible changes which may change the uncertainty estimate — to be ready for the next iteration — if necessary now or later. Especially make an economical evaluation.

Component name	**Evaluation type**	**Distribution type**	**Number of measurements**	**Variation limit** a^* influence units	**Variation limit** a µm	**Correlation coefficient**	**Distribution factor** b	**Uncertainty comp.** u_{xx} µm
u_{Xa} Name of xa	A		10			0		1,60
u_{Xb} Name of xb	B	Gaussian		1,90	1,90	0	0,5	0,95
u_{Xc} Name of xc	B	Rectangular		3,42	3,42	0	0,6	2,05
u_{Ya} Name of ya	A		15			0		1,20
u_{Yb} Name of yb	A		15			0		0,60
u_{Za} Name of za	B	U		10 °C	1,57	0	0,7	1,10
u_{Zb} Name of zb	B	U		15 °C $\alpha_1/\alpha_2 = 1,1$	0,60	0	0,7	0,42
Combined standard uncertainty, u_c								**3,29**
Expanded uncertainty ($k = 2$), U								**6,58**

Figure 8 — Example of a summary table with all key information of an uncertainty budget

10 Applications

10.1 General

A normal uncertainty budgeting for a GPS measurement may result in the following equation. The uncertainty components are grouped depending on their origin:

$$u_c = \sqrt{u_{MPEx}^2 + \ldots + u_{Mx}^2 + \ldots + u_{Bx}^2 + \ldots + u_{Ox}^2 + \ldots + u_{Ex}^2 + \ldots} \quad (21)$$

$$U = u_c \times k \ (k = 2) \quad (22)$$

The groups of uncertainty components originate from, for example:

- measurement equipment (or measurement standard) $u_{MPE1}, u_{MPE2}, u_{MPE3}, \cdots$
- environment $u_{M1}, u_{M2}, u_{M3}, \cdots$
- personnel/staff $u_{B1}, u_{B2}, u_{B3}, \cdots$
- measurement set-up $u_{O1}, u_{O2}, u_{O3}, \cdots$
- measurement object (workpiece or measurement equipment) $u_{E1}, u_{E2}, u_{E3}, \cdots$
- definition of the characteristic of the object $u_{D1}, u_{D2}, u_{D3}, \cdots$
- measurement procedure $u_{P1}, u_{P2}, u_{P3}, \cdots$
- etc. $u_{etc.x}, \cdots$

Experience is that the different groups of uncertainty components in many cases do not influence each other when the changes in one of the other groups are small. This means that the equation can be used to evaluate the influence from one or more of the groups on the uncertainty of measurement, absolute as well as relative.

It is possible to “transform” the uncertainty budget and the changes in one or more of the groups into economical terms and effect, and thus use the uncertainty budget to evaluate the economical influence of the uncertainty components.

In the following subclauses, applications of uncertainty budgets and the PUMA method are given. The list is non-exhaustive.

10.2 Documentation and evaluation of the uncertainty value

As demonstrated in many cases in this part of ISO 14253, the uncertainty budget is able to give an estimate of the uncertainty value for an existing measurement or calibration process.

10.3 Design and documentation of the measurement or calibration procedure

10.3.1 Documentation and optimization of measurement and calibration processes

The PUMA method gives the opportunity to document and optimize a measurement or a calibration process by taking into account technical or economical criteria, or both, when optimizing through a number of iterations.

10.3.2 Development of measurement procedures and instructions

Through parallel development of measurement procedures and uncertainty budgets, the PUMA method provides the opportunity to analyse the effect of every sub-procedure based on the effect on the uncertainty. Thus, develop (and optimize) the total measurement procedure and the related instruction.

10.3.3 Development of calibration procedures and instructions

Through parallel development of calibration procedures and uncertainty budgets, the PUMA method provides the opportunity to analyse the effect of every sub-procedure based on the effect on the uncertainty. Thus, the total calibration procedure and the related instruction are developed and optimized.

10.3.4 Qualification or disqualification of secondary measurement methods and equipment

In many cases, the ideal measuring method and measurement equipment — according to the definition of the characteristic to be measured (GPS characteristic of a workpiece or metrological characteristic of measurement equipment) — is too expensive or too slow, or both. Results of analysis of the measuring object for form and angular deviations and investigation of the influence on the uncertainty budget give the possibility of qualifying or disqualifying secondary measurement methods and equipment and cut costs, e.g. investigate if a three-point measurement (secondary method) in a V-block may be a valid substitute for measurement of roundness by variation in roundness (ideal method in accordance with the definition of roundness).

10.3.5 Qualification of measurement equipment and set-ups

The influence on the uncertainty of measurement from a specific measurement equipment, $u_{\mathrm{MPE}x}$, and measurement set-up, $u_{\mathrm{O}x}$, can be seen from the uncertainty budget. All other uncertainty components are taken as invariable. When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the equipment and the set-up are qualified for the measurement task.

10.3.6 Demonstration of best measuring capability, BMC

The best measuring capability (BMC) is the least possible uncertainty of measurement achievable in a company or a laboratory for a specific measuring task. When all uncertainty components in an uncertainty budget are minimized, $u_{\mathrm{c,min}}$ is the BMC for the task.

10.4 Design, optimization and documentation of the calibration hierarchy

10.4.1 Design of the calibration hierarchy

The uncertainty budget results in an equation which gives a functional relation between two levels in the calibration hierarchy in a company or in a calibration laboratory (see the example in Annex A and Figure 9). Use of the PUMA method — with a stated “target uncertainty” — on representative shop-floor measurements, with the uncertainty components originating from the measurement equipment, $u_{\mathrm{MPE}x}$, as variables — and all other uncertainty components as fixed values — results in minimum requirements (MPEs) for the metrological characteristics of the measurement equipment (see Figure 9).

The same procedure used on the calibration measurements of the measurement equipment will result in minimum requirements for the metrological characteristics of the measurements standards. The procedure can be used at all levels of the calibration hierarchy and thus design the full hierarchy in a company or a laboratory.

10.4.2 Requirements for and qualification of measurement standards

The influence on the uncertainty of measurement in calibration from a specific measurement standard, $u_{\mathrm{MPE}x}$, can be seen from the uncertainty budget. All other uncertainty components are taken as invariable. When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the measurement standard is qualified for the calibration task.

10.4.3 Requirements for and qualification of external calibration certificates

The metrological characteristics of the reference standards in a company or laboratory result in uncertainty components in the uncertainty budgets for calibration of the next lower level of the calibration hierarchy. The reference standards are acting as "measurement equipment"; the equipment at the next lower level is acting as measurement object. Taking all other uncertainty components as invariable and the uncertainty components from the reference standard, $u_{\mathrm{MPE}x}$, as variables, the requirements to the calibration certificates can be derived from the formula:

$$u_{\mathrm{T}} \geqslant u_{\mathrm{c}} = \sqrt{u_{\mathrm{MPE}x}^2 + \ldots + u_{\mathrm{M}x}^2 + \ldots + u_{\mathrm{B}x}^2 + \ldots + u_{\mathrm{O}x}^2 + \ldots + u_{\mathrm{E}x}^2 + \ldots + u_{\mathrm{D}x}^2 + \ldots + u_{\mathrm{P}x}^2 + \ldots +} \quad (23)$$

When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the calibration certificate is qualified.

10.4.4 Evaluation of the use of check standards

Check standards used in the workshop — in addition to calibration — may be a way to decrease the uncertainty of measurement. By substitution of the relevant uncertainty components in the original uncertainty budget, based on the calibrated measurement equipment, and adding possible new uncertainty components, the effect of a check standard on the uncertainty of measurement can be evaluated (see the example in Annex A).

10.5 Design and documentation of new measurement equipment

10.5.1 Specification for new measurement equipment

The uncertainty budget for a specific measuring task can be set up with the uncertainty components from the measurement equipment, $u_{\mathrm{MPE}x}$, as unknown variables and all other uncertainty components as invariable. The requirements for new measurement equipment, which does not exist yet in the company, can be derived from Equation (23).

10.5.2 Design of special measurement equipment

The uncertainty budget for a specific measuring task can be set up with the uncertainty components from the not yet designed measurement equipment as unknown variables and all other uncertainty components as invariable. The design requirements for the new measurement equipment can be derived from Equation (23).

10.6 Requirements for and qualification of the environment

The influence on the uncertainty of measurement from the environment, $u_{\mathrm{M}x}$, can be seen from the uncertainty budget. All other uncertainty components are invariable. The uncertainty components from the environment are taken as variables. It is then possible to derive requirements for the environment from Equation (23).

When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the environment is qualified for the measurement task.

10.7 Requirements for and qualification of measurement personnel

The influence on the uncertainty of measurement from the personnel, $u_{\mathrm{B}x}$, can be seen from the uncertainty budget. All other uncertainty components are invariable. The uncertainty components from the personnel are taken as variables. It is then possible to derive requirements for the personnel from Equation (23).

When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the personnel is qualified for the measurement task.

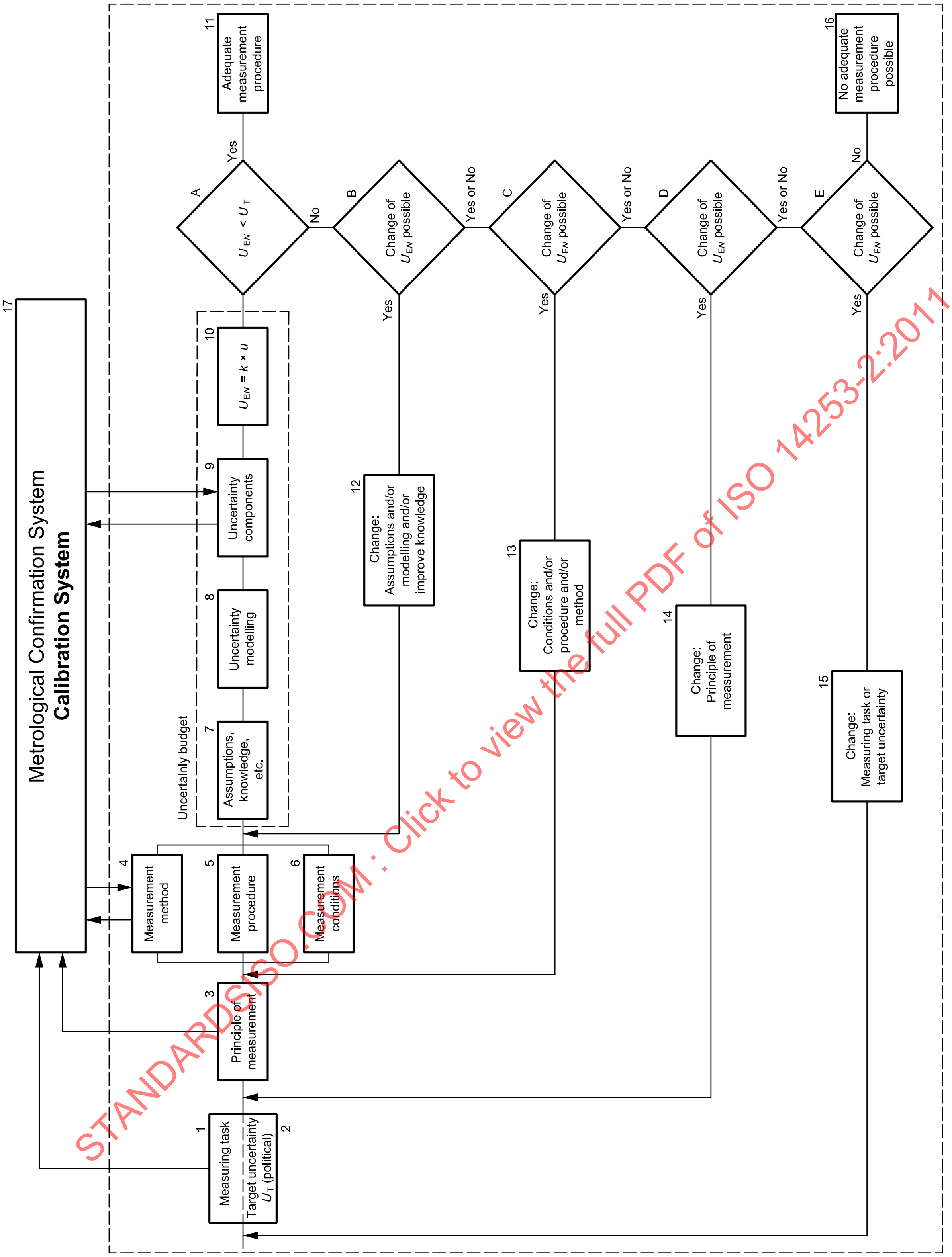


Figure 9 — Relationship between the uncertainty budget and the calibration level for the measurement equipment or measurement standard used in the measurement

Annex A
(informative)

Example of uncertainty budgets — Calibration of a setting ring

WARNING — It shall be recognized that the following example is constructed to illustrate the PUMA only. It only includes uncertainty components significant in the illustrated cases. For different target uncertainties and applications, other uncertainty components may be significant.

A.1 Overview

This example covers the estimation of uncertainty of measurement and qualification of a measurement procedure and measurement conditions for a measurement task using the PUMA method.

A.2 Task and target uncertainty

A.2.1 Measuring task

The measuring task consists of calibrating a ∅ 100 mm × 15 mm setting ring, two-point diameter in one defined direction in the symmetry plane. The roundness in the symmetry plane is 0,2 µm.

A.2.2 Target uncertainty

A target uncertainty (see 3.6) of 1,5 µm was chosen.

A.3 Principle, method, procedure and condition

A.3.1 Measurement principle

Mechanical contact — Comparison with a known length (reference ring).

A.3.2 Measurement method

Differential, comparison of a ∅ 100 mm reference standard and the “unknown” ∅ 100 mm setting ring.

A.3.3 Initial measurement procedure

The following procedure applies.

— The setting ring is measured on a horizontal measuring machine.

— A reference ring (∅ 100 mm) is used.

— The horizontal measuring machine is used as a comparator.

A.3.4 Initial measurement conditions

The following conditions apply.

— The horizontal measuring machine is within manufacturer's specification (see Table A.1).

— The digital step in the readout display is 0,1 µm.

— The temperature in the laboratory is 20 °C ± 1 °C.

— The temperature variation of the measuring machine over time is registered to 0,25 °C.

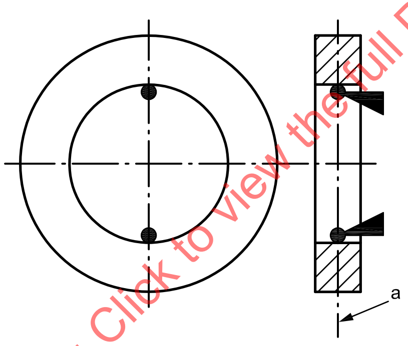
— The temperature difference between setting ring and reference ring is less than 1 °C.

— The measuring machine and the rings are made of steel.

— The operator is trained and familiar with the use of the measuring machine.

A.4 Graphical illustration of measurement set-up

See Figure A.1.



a Symmetry plane.

Figure A.1 — Measurement set-up

A.5 List and discussion of the uncertainty components

See Table A.1.

Table A.1 — Overview and comments table for uncertainty components in diameter measurements

<table>
<tr><th>Designation
Low resolution</th><th>Designation
High resolution</th><th>Name
Uncertainty component</th><th colspan="2">Comments</th></tr>
<tr><td>u_{RS}</td><td></td><td>Reference standard (ring)</td><td colspan="2">The uncertainty is stated for the ∅ 100 mm diameter on the (accredited) calibration certificate as $U = 0,8$ µm.</td></tr>
<tr><td>u_{EC}</td><td></td><td>Error of indication of the measuring machine</td><td colspan="2">The measuring machine is calibrated and is documented within the specifications (MPE values). The scale error is within: 0,6 µm + 4,5 µm/m for a floating zero.</td></tr>
<tr><td>u_{PA}</td><td></td><td>Alignment of measuring anvils</td><td colspan="2">Since the reference ring and the ring to be calibrated are contacted the same way (as long as their diameters are within a reasonable range), the parallelism error can be neglected.</td></tr>
<tr><td>u_{RR}</td><td>u_{RA}</td><td>Resolution</td><td>$u_{RA} = \frac{d}{2 \times \sqrt{3}} = \frac{0,1\,\mu m}{2 \times \sqrt{3}} \approx 0,029 \mu m$</td><td rowspan="2">The largest of the two is equal to u_{RR}.</td></tr>
<tr><td></td><td>u_{RE}</td><td>Repeatability</td><td>A repeatability study has been conducted. The limit of variation is found to be 0,7 µm. (This corresponds to 0,5 µm for measuring the master ring and 0,5 µm for measuring the gauge ring, when squared together.)</td></tr>
<tr><td>u_{TD}</td><td></td><td>Temperature difference between the two rings</td><td colspan="2">The temperature difference between the master ring and the ring being calibrated is assumed to follow a U-shaped distribution. It is assumed that the two measurements are so close together in time that the measuring machine does not change temperature.</td></tr>
<tr><td>u_{TA}</td><td></td><td>Difference in temperature expansion coefficients</td><td colspan="2">The temperature is assumed to follow a U-shaped distribution. It is assumed that the two measurements are so close together in time that the measuring machine does not change temperature.</td></tr>
<tr><td>u_{RO}</td><td></td><td>Roundness of setting ring</td><td colspan="2">The roundness is measured as 0,2 µm. The ring has an elliptical shape error.</td></tr>
</table>

A.6 First iteration

A.6.1 First iteration — Documentation and calculation of the uncertainty components

u_{RS} — **Reference standard (ring)** **Given in calibration certificate**

According to the calibration certificate (Certificate no. XPQ-23315-97), the expanded uncertainty of the certified diameter of the reference ring is 0,8 µm (coverage factor $k = 2$):

$$u_{RS} = \frac{U}{k} = \frac{0,8\,\mu m}{2} = 0,8\,\mu m \times 0,5 = 0,4\,\mu m$$

u_{EC} — **Error of indication of the horizontal measuring machine** **Type B evaluation**

The MPE value of the error of indication curve (based on floating zero) is 0,6 µm + 4,5 µm/m. The measurement distance (difference in diameter) between the reference ring and the ring calibrated is very small (<< 1 mm). Therefore:

$$a_{EC} = 0,6\,\mu m$$

For safety reasons, a rectangular distribution ($b = 0,6$) is assumed. This results in an uncertainty component of:

$$u_{EC} = 0,6\,\mu m \times 0,6 = 0,36\,\mu m$$

u_{PA} — **Alignment of measuring anvils** **Type B evaluation**

Since the reference ring and the setting ring to be calibrated are contacted the same way (as long as their diameters are within a reasonable range), the parallelism error can be neglected.

$$u_{PA} \approx 0\ \mu m$$

u_{RR} — **Repeatability/resolution** **Type A evaluation**

A repeatability study has been conducted on the difference of ring diameters. The limit of variation is found to be 0,7 µm. (This corresponds to 0,5 µm for measuring the master ring and 0,5 µm for measuring the gauge ring, when squared together.)

Assuming the variation corresponds to 6 standard deviations, this gives an uncertainty component of:

$$u_{RR} = \frac{0{,}7\ \mu m}{6} = 0{,}12\ \mu m$$

u_{TD} — **Temperature difference between the two rings** **Type B evaluation**

The temperature difference between the two rings is not seen to be greater than 1 °C. The temperature expansion coefficient for the two rings is assumed equal to $\alpha = 1{,}1\ \mu m/(100\ mm \times °C)$. This means:

$$a_{TD} = 1{,}1 \frac{\mu m}{(100\ mm \times °C)} \times 1\ °C \times 100\ mm = 1{,}1\ \mu m$$

A U-distribution is assumed ($b = 0{,}7$):

$$u_{TD} = 1{,}1\ \mu m \times 0{,}7 = 0{,}77\ \mu m$$

u_{TA} — **Difference in temperature expansion coefficients** **Type B evaluation**

The maximum deviation from 20 °C is 1 °C. The difference in temperature expansion coefficients is assumed to be less than 10 %. Therefore:

$$a_{TD} = \frac{1{,}1\ \mu m}{(100\ mm \times °C)} \times 1\ °C \times 100\ mm \times 10\ \% = 0{,}11\ \mu m$$

A U-distribution is assumed ($b = 0{,}7$):

$$u_{TA} = 0{,}11\ \mu m \times 0{,}7 \approx 0{,}08\ \mu m$$

u_{RO} — **Roundness of the setting ring** **Type B evaluation**

The form error is elliptical and the out of roundness is 0,2 µm. The diameter is defined and measured in one specified direction in the ring. Therefore the roundness has no significant effect.

$$u_{RO} \approx 0\ \mu m$$

A.6.2 First iteration — Correlation between uncertainty components

It is estimated that no correlation occurs between the uncertainty components.

A.6.3 First iteration — Combined and expanded uncertainty

When no correlation exists between the uncertainty components, the combined standard uncertainty is:

$$u_c = \sqrt{u_{RS}^2 + u_{EC}^2 + u_{PA}^2 + u_{RR}^2 + u_{TD}^2 + u_{TA}^2 + u_{RO}^2}$$

The values from A.6.1:

$$u_c = \sqrt{\left(0{,}40^2 + 0{,}36^2 + 0^2 + 0{,}12^2 + 0{,}77^2 + 0{,}08^2 + 0^2\right)\,\mu\text{m}^2}$$

$$u_c = 0{,}95\ \mu\text{m}$$

Expanded uncertainty:

$$U = u_c \times k = 0{,}95\ \mu\text{m} \times k = 1{,}90\ \mu\text{m}$$

A.6.4 Summary of uncertainty budget — First iteration

See Table A.2.

Table A.2 — Summary of uncertainty budget (first iteration)

Component name		Evaluation type	Distribution type	Number of measurements	Variation limit a^* influence units	Variation limit a µm	Correlation coefficient	Distribution factor b	Uncertainty component u_{xx} µm
u_{RS}	Reference standard (ring)	Cert.					0	0,5	0,40
u_{EC}	Error of indication of the measuring machine	B	Rect.		0,6 µm	0,6	0	0,6	0,36
u_{PA}	Alignment of measuring anvils	B	Rect.		0 µm	0	0	0,6	0
u_{RR}	Repeatability/resolution	A		6			0		0,12
u_{TD}	Temperature difference between the two rings	B	U		1 °C	1,1	0	0,7	0,77
u_{TA}	Difference in temperature expansion coefficients	B	U		1 °C	0,11	0	0,7	0,08
u_{RO}	Roundness of setting ring	B			0 µm	0	0		0
Combined standard uncertainty, u_c									**0,95**
Expanded uncertainty ($k = 2$), U									**1,90**

A.6.5 First iteration — Discussion of the uncertainty budget

The criterion $U_{E1} < U_T$ is not met. There is one dominant uncertainty component, u_{TD}, caused by the temperature difference of 1 °C. It is not possible to make a smaller estimate u_{TD} from the existing information. The only solution is to change the measurement conditions. The temperature acclimatization shall be better, which means more time for the acclimatization and probably a more efficient heat protection from body parts of the operator during handling and measurement.

Change (decrease) of other uncertainty components — other than the temperature related uncertainty components — in the uncertainty budget will have nearly no effect on the combined standard deviation and the expanded uncertainty.

A.6.6 Conclusion on the first iteration

The measurement procedure is qualified by the first iteration, but the measurement conditions need improvement.

The maximum temperature difference between the two rings shall not exceed 0,5 °C.

A.7 Second iteration

The temperature conditions are changed from 1 °C to 0,5 °C in the formulas for u_{TD} and u_{TA} (see A.6.1). Documentation and calculation of the uncertainty components shall be changed accordingly.

A.8 Conclusion on the second iteration

In the second iteration, the temperature difference is limited to 0,5 °C. Table A.3 gives the documentation; the target uncertainty criterion is met:

$$u_{\mathrm{E2}} = 1{,}35\ \mu\mathrm{m} \leqslant U_{\mathrm{T}} = 1{,}5\ \mu\mathrm{m}$$

By the second iteration, the measurement conditions are qualified.

A.9 Comments — Summary of example

Through this example, it is demonstrated that it is possible to qualify a measurement procedure and a set of measurement conditions using the PUMA method to fulfil a given target uncertainty criterion:

$$U_{\mathrm{E}N} \leqslant U_{\mathrm{T}}$$

After the first iteration, where the target uncertainty criterion is not met, it is — in this case — obvious what to do. There is only one dominant uncertainty component. The temperature conditions shall be better to meet the target uncertainty criterion. It is demonstrated how the individual uncertainty component influences the combined standard uncertainty and expanded uncertainty after the first iteration. Depending on the relative size of the uncertainty components, a strategy for a decreasing of the uncertainty can be made.

Table A.3 — Summary of uncertainty budget (second iteration)

Component name		Evaluation type	Distri-bution type	Number of measure-ments	Variation limit a^* influence units	Variation limit a µm	Correlation coefficient	Distri-bution factor b	Uncer-tainty compo-nent u_{xx} µm
u_{RS}	Reference standard (ring)	Cert.					0	0,5	0,40
u_{EC}	Error of indication of the measuring machine	B	Rect.		0,6 µm	0,6	0	0,6	0,36
u_{PA}	Alignment of measuring anvils	B	Rect.		0 µm	0	0	0,6	0
u_{RR}	Repeatability/resolution	A		6			0		0,12
u_{TD}	Temperature difference between the two rings	B	U		0,5 °C	0,55	0	0,7	0,39
u_{TA}	Difference in temperature expansion coefficients	B	U		0,5 °C	0,06	0	0,7	0,04
u_{RO}	Roundness of setting ring	B			0 µm	0	0		0
Combined standard uncertainty, u_c									**0,67**
Expanded uncertainty ($k = 2$), U									**1,35**
NOTE The change in uncertainty components is indicated by thick lines.									

Annex B
(informative)

Example of uncertainty budgets — Design of a calibration hierarchy

WARNING — It shall be recognized that the following example has been constructed to illustrate the PUMA only. It only includes uncertainty components significant in the illustrated cases. For different target uncertainties and applications, other uncertainty components may be significant.

B.1 Overview

This example illustrates how the PUMA method may be used in industry to optimize and plan in detail the metrological (calibration) hierarchy. The example includes:

— measurement of local diameter with external micrometer;

— calibration of an external micrometer;

— calibration requirements for measurement standards for calibration of an external micrometer;

— use of check standard as a supplement to calibration.

Furthermore, it includes the estimation of uncertainty of measurement and evaluation of the requirements for metrological characteristics at the lower three levels of the traceability hierarchy shown in Figure B.1. These three levels are:

III measurement of the local (two-point) diameter of a cylinder using an external micrometer. The measurement procedure is evaluated by the PUMA method and a given target uncertainty U_{T} (see B.2);

II calibration of the metrological characteristics (which influence the uncertainty of measurement in sub-example I) of an external micrometer (see B.3, B.4 and B.5);

I calibration requirements (MPE values) for the metrological characteristics of the calibration standards needed for calibration of the external micrometer (see B.6).

Use of a check standard as a supplement to calibration of the external micrometer is evaluated by the uncertainty budget as a variant of the measurement of two-point diameter (see B.7).

At level III, the uncertainty of measurement for the two-point diameter measurement is evaluated. The maximum permissible errors (MPEs) of the metrological characteristics of the external micrometer [$\mathrm{MPE}_{\mathrm{ML}}$ (error of indication), $\mathrm{MPE}_{\mathrm{MF}}$ (flatness of measuring anvils), and $\mathrm{MPE}_{\mathrm{MP}}$ (parallelism of measuring anvils)] are taken as unknown variables. From the function:

$$U_{\mathrm{T}} \geqslant U_{\mathrm{WP}} = f(\mathrm{MPE}_{\mathrm{ML}}, \mathrm{MPE}_{\mathrm{MF}}, \mathrm{MPE}_{\mathrm{MP}}, \text{other uncertainty components})$$

the MPE values for the three metrological characteristics ($\mathrm{MPE}_{\mathrm{ML}}$, $\mathrm{MPE}_{\mathrm{MF}}$, and $\mathrm{MPE}_{\mathrm{MP}}$) of the external micrometer can be derived. At level II, the uncertainty of measurement in calibration of the three metrological characteristics (error of indication, flatness of measuring anvils and parallelism of measuring anvils) is estimated. At level I, the MPE values for the metrological characteristics of the three measurement standards are derived with the same technique used for the MPEs of the micrometer, but now taking the MPE values of the three measurement standards as unknown variables.

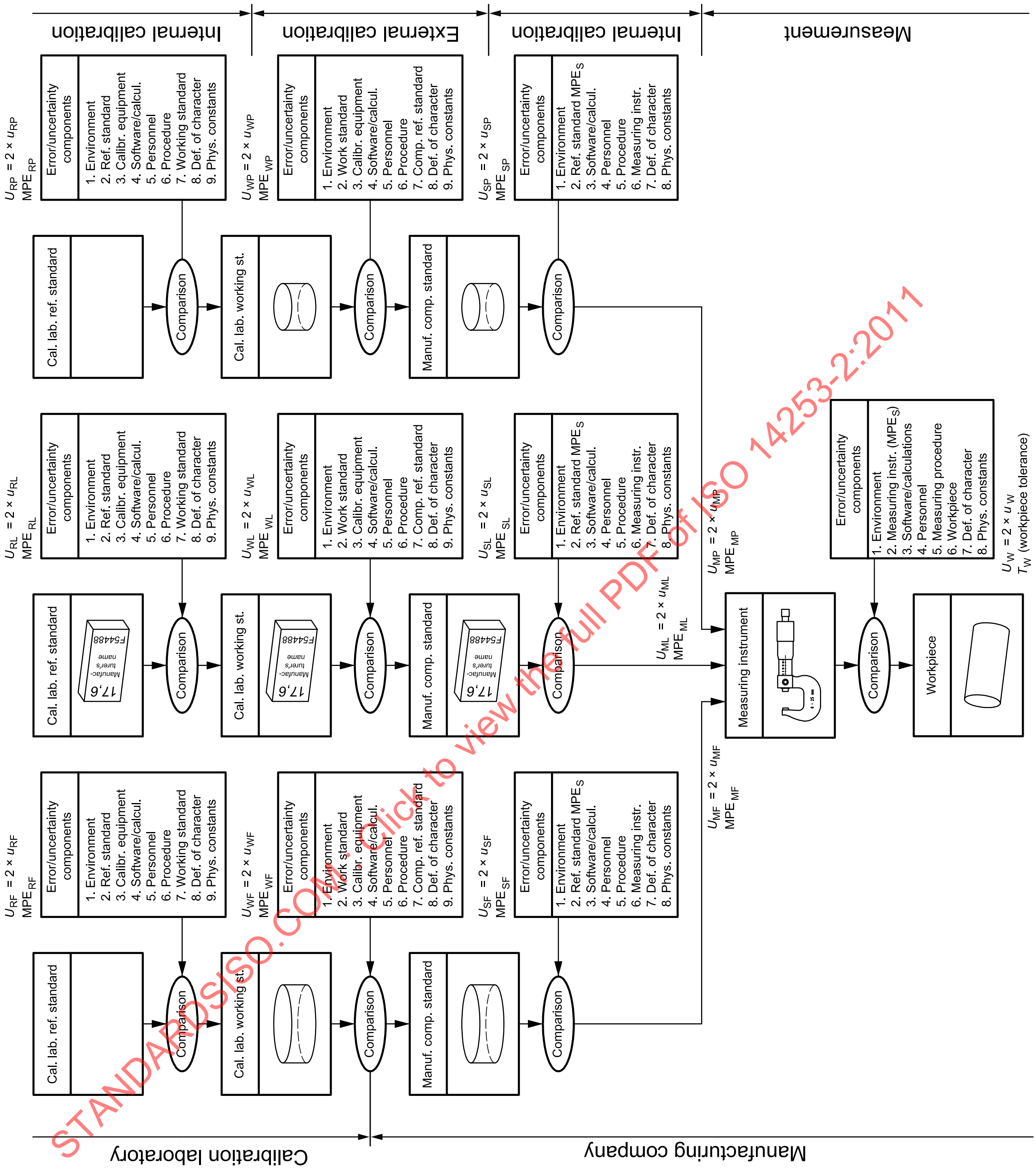


Figure B.1 — Calibration hierarchy for measurement of local diameter and calibration of external micrometers

The result of uncertainty budgeting on the three levels is the following:

— the MPE values for the external micrometer are optimized and directly derived from the need for uncertainty of measurement on the workshop floor;

— the MPE values for the measurement standards (gauge blocks, optical flat and optical parallels) are optimized to calibration of the above external micrometer. These MPE values are the minimum requirements to calibration certificates;

— the improvement of the uncertainty of measurement using a check standard as a supplement to calibration can be quantified.

B.2 Measurement of local diameter

B.2.1 Task and target uncertainty

B.2.1.1 Measuring task

The measuring task consists of measuring the local diameter (two-point diameter) on a series of fine turned steel shafts, with nominal dimensions ∅ 25 mm × 150 mm.

B.2.1.2 Target uncertainty

A target uncertainty (see 3.6) of 8 µm was chosen.

B.2.2 Principle, method and conditions

B.2.2.1 Measurement principle

Measurement of length — Comparison with a known length.

B.2.2.2 Measurement method

The measurements are performed with an analogue external micrometer with flat (∅ 6 mm) measuring anvils, measuring range 0 mm to 25 mm with a vernier scale interval of 1 µm.

B.2.2.3 Initial measurement procedure

The following procedure applies.

— The diameter is measured while the shaft is still clamped in the chuck of the machine tool.

— Only one measurement of the diameter is allowed.

— The shaft is cleaned with a cloth before measurement.

— The friction/ratchet drive shall be used during measurements.

— The spindle clamp shall not be used.

B.2.2.4 Initial measurement conditions

The following conditions apply.

- It is demonstrated that the temperature in the shafts and in the micrometer varies over time. The maximum deviation from standard reference temperature 20 °C is 15 °C.
- Maximum temperature difference between the shafts and the micrometer is 10 °C.
- Three different operators use the machine tool and the micrometer for the production of the shafts.
- The cylindricity of the shafts is found to be better than 1,5 µm. The major part of the cylindricity is out of roundness.

B.2.3 Graphical illustration of the measurement set-up

See Figure B.2.

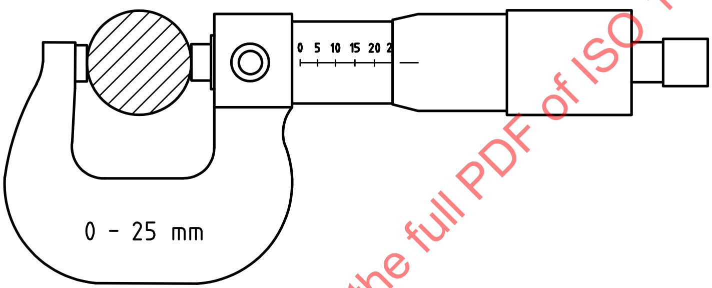


Figure B.2 — Measurement set-up for measurement of local ∅ 25 mm diameter

B.2.4 List and discussion of the uncertainty components

The two-point diameter measurement is modelled as a black box uncertainty estimation process. No corrections are used. All error contributions are included in the uncertainty of measurement.

In Table B.1, all the uncertainty components are mentioned and named, which are assumed to influence the uncertainty of the actual diameter measurements.

Table B.1 — Overview and comments table for uncertainty components in measurement of local diameter (two-point diameter)

<table>
<tr><th>Designation
Low resolution</th><th>Designation
High resolution</th><th>Name
Uncertainty component</th><th colspan="2">Comments</th></tr>
<tr><td>u_{ML}</td><td></td><td>Micrometer — Error of indication</td><td colspan="2">Requirement for error of indication MPE_{ML} of the micrometer is an unknown variable. Initially it is set to 6 µm — and symmetrical positioning of the error of indication curve by zero adjustment after calibration.</td></tr>
<tr><td>u_{MF}</td><td></td><td>Micrometer — Flatness of measuring anvils</td><td colspan="2">Requirement for out of flatness for the two measuring anvils M_{MF} is an unknown variable. Initially it is set to 1 µm.</td></tr>
<tr><td>u_{MP}</td><td></td><td>Micrometer — Parallelism of measuring anvils</td><td colspan="2">Requirement for out of parallelism between the two measuring anvils M_{MP} is an unknown variable. Initially it is set to 2 µm.</td></tr>
<tr><td>u_{MX}</td><td></td><td>Effect of spindle clamping, orientation of the micrometer and time of handling</td><td colspan="2">These effects are in this case not active. The spindle clamp is not used. The orientation and time of handling have no significant effect on a 0 mm to 25 mm micrometer.</td></tr>
<tr><td>u_{RR}</td><td>u_{RA}</td><td>Resolution</td><td>$u_{RA} = \frac{d}{2\times\sqrt{3}} = \frac{1\ \mu m}{2\times\sqrt{3}} = 0,29\ \mu m$</td><td rowspan="2">The largest of the two is equal to u_{RR}.</td></tr>
<tr><td></td><td>u_{RE}</td><td>Repeatability</td><td>It is demonstrated by experiments that the three operators have the same repeatability. The experiment includes more than 15 measurements for each operator on “perfect” Ø 25 mm plug gauges. The effect of the flexibility of the micrometer is included in the repeatability.</td></tr>
<tr><td>u_{NP}</td><td></td><td>Variation of zero point between three operators</td><td colspan="2">The three operators use the micrometer in a different way. The zero point is not the same as set by the calibration “person”. Experiment (more than 15 measurements for each operator on “perfect” Ø 25 mm plug gauges).</td></tr>
<tr><td>u_{TD}</td><td></td><td>Temperature difference</td><td colspan="2">Maximum difference between shafts and micrometer seen during observation period is 10 °C.</td></tr>
<tr><td>u_{TA}</td><td></td><td>Temperature</td><td colspan="2">Maximum deviation from standard reference temperature (20 °C) is 15 °C.</td></tr>
<tr><td>u_{WE}</td><td></td><td>Workpiece form error</td><td colspan="2">Cylindricity measured is 1,5 µm. The major part of the cylindricity is out of roundness. The effect on diameter is two times the cylindricity, 3 µm.</td></tr>
</table>

B.2.5 First iteration

B.2.5.1 First iteration — Documentation and calculation of the uncertainty components

u_{ML} — Micrometer — Error of indication **Type B evaluation**

MPE_{ML} for the metrological characteristic error of indication of an external micrometer is usually defined as the maximum range of the error of indication curve, and not related to the zero error of indication. Position of the error of indication curve to zero error is another (independent) metrological characteristic.

In this case, it is assumed that the error of indication curve is positioned — during the calibration procedure — so that the largest negative and positive errors of indication are of the same absolute value.

The definitive value of MPE_{ML} is not yet fixed. It is one of the tasks of the uncertainty budget. As an initial setting of MPE_{ML}, 6 µm is chosen. Because of the zero setting procedure mentioned the error limit value is:

$$a_{ML} = \frac{6\ \mu m}{2} = 3\ \mu m$$

A rectangular distribution is assumed (overestimation principle, because Gaussian distribution cannot be proved on the given basis) ($b = 0{,}6$):

$$u_{ML} = 3\ \mu m \times 0{,}6 = 1{,}8\ \mu m$$

u_{MF} **— Micrometer — flatness of measuring anvils** **Type B evaluation**

The flatness deviation is active in diameter measurements on shafts, while the calibration of the error of indication curve is performed on gauge blocks with plane and parallel surfaces.

The definitive value of MPE_{MF} is not yet fixed. It is one of the tasks of the uncertainty budget. As an initial setting of MPE_{MF}, 1 µm is chosen.

MPE_{MF} influences the uncertainty budget twice, once for each of the two measuring anvils. A Gaussian distribution is assumed ($b = 0{,}5$):

$$u_{MF} = 1\ \mu m \times 0{,}5 = 0{,}5\ \mu m$$

u_{MP} **— Micrometer — parallelism of measuring anvils** **Type B evaluation**

The parallelism deviation is active in diameter measurements on shafts, while the calibration of the error of indication curve is performed on gauge blocks with plane and parallel surfaces.

The definitive value of MPE_{MP} is not yet fixed. It is one of the tasks of the uncertainty budget. As an initial setting of MPE_{MP}, 2 µm is chosen. A Gaussian distribution is assumed ($b = 0{,}5$):

$$a_{MP} = 2\ \mu m$$

$$u_{MP} = 2\ \mu m \times 0{,}5 = 1\ \mu m$$

u_{RR} **— Repeatability/resolution** **Type A evaluation**

All three operators have the same repeatability. It is tested in an experiment, where ∅ 25 mm plug gauges have been used as “workpieces”. Hence the form error from the real workpieces is not included in the repeatability study. All operators have performed 15 measurements. The common standard deviation is:

$$u_{RR} = 1{,}2\ \mu m$$

The resolution uncertainty component, u_{RA}, is included in u_{RR}, in this case ($u_{RA} < u_{RE}$).

u_{NP} **— Variation of zero point between three operators** **Type A evaluation**

From the same experiments used for repeatability the differences in zero point between the three operators and the calibration personnel are investigated:

$$u_{NP} = 1\ \mu m$$

u_{TD} **— Temperature difference** **Type B evaluation**

The temperature difference between micrometer and workpieces is observed to maximum 10 °C. There is no information about which of them has the highest temperature. Therefore ± 10 °C is assumed. The linear coefficient of thermal expansion, α, is assumed to be 1,1 µm/(100 mm × °C) for the micrometer and the workpieces. The limit value is:

$$a_{TD} = \Delta T \times \alpha \times D = 10\ °C \times 1{,}1 \frac{\mu m}{(100\ mm \times °C)} \times 25\ mm = 2{,}8\ \mu m$$

A U-distribution is assumed ($b = 0{,}7$):

$$u_{TD} = 2{,}8\ \mu m \times 0{,}7 = 1{,}96\ \mu m$$

u_{TA} **— Temperature** **Type B evaluation**

The observed maximum deviation from standard reference temperature (20 °C) is 15 °C. There is no information about the sign of this deviation, therefore ±15 °C is assumed. A 10 % maximum difference between the two linear coefficients of thermal expansion ($\alpha_{micrometer}$ and $\alpha_{workpiece}$) is assumed. The limit value is:

$$a_{TA} = 0{,}1 \times \Delta T_{20} \times \alpha \times D = 0{,}1 \times 15\ °C \times 1{,}1 \frac{\mu m}{(100\ mm \times °C)} \times 25\ mm = 0{,}4\ \mu m$$

A U-distribution is assumed ($b = 0{,}7$):

$$u_{TA} = 0{,}4\ \mu m \times 0{,}7 = 0{,}28\ \mu m$$

u_{WE} **— Workpiece form error** **Type B evaluation**

The cylindricity is measured on a sample of shafts and found to be 1,5 µm. Cylindricity is a measure for the variation of radius. The effect on the diameter is assumed to be two times the cylindricity deviation, while no information exists to make it smaller. The limit value is:

$$a_{WE} = 3\ \mu m$$

A rectangular distribution is assumed ($b = 0{,}6$):

$$u_{WE} = 1{,}8\ \mu m$$

B.2.5.2 First iteration — Correlation between uncertainty components

It is estimated that no correlation occurs between the uncertainty components.

B.2.5.3 First iteration — Combined and expanded uncertainty

When there is not any correlation between the uncertainty components, the combined standard uncertainty is:

$$u_c = \sqrt{u_{ML}{}^2 + u_{MF}{}^2 + u_{MF}{}^2 + u_{MP}{}^2 + u_{RR}{}^2 + u_{NP}{}^2 + u_{TD}{}^2 + u_{TA}{}^2 + u_{WE}{}^2}$$

Inserting the values from B.2.5.1 gives:

$$u_c = \sqrt{\left(1{,}8^2 + 0{,}5^2 + 0{,}5^2 + 1{,}0^2 + 1{,}2^2 + 1{,}0^2 + 1{,}96^2 + 0{,}28^2 + 1{,}8^2\right)\mu m^2}$$

$$u_c = 3{,}79\ \mu m$$

$$U = u_c \times k = 3{,}79\ \mu m \times 2 = 7{,}58\ \mu m$$

B.2.5.4 Summary of uncertainty budget — First iteration

See Table B.2.

Table B.2 — Summary of uncertainty budget (first iteration) — Measurement of two-point diameter

Component name		Evaluation type	Distribution type	Number of measurements	Variation limit a^* influence units	Variation limit a µm	Correlation coefficient	Distribution factor b	Uncertainty component u_{xx} µm
u_{ML}	Micrometer — error indication	B	Rect.		3,0 µm	3,0	0	0,6	1,80[(1)]
u_{MF}	Micrometer — flatness 1	B	Gaussian		1,0 µm	1,0	0	0,5	0,50[(3)]
u_{MF}	Micrometer — flatness 2	B	Gaussian		1,0 µm	1,0	0	0,5	0,50[(3)]
u_{MP}	Micrometer — parallelism	B	Gaussian		2,0 µm	2,0	0	0,5	1,00[(2)]
u_{RR}	Repeatability	A		15			0		1,20[(2)]
u_{NP}	Variation of 0 point	A		15			0		1,00[(2)]
u_{TD}	Temperature difference	B	U		10 °C	2,8	0	0,7	1,96[(1)]
u_{TA}	Temperature	B	U		15 °C $\alpha_1/\alpha_2 = 1{,}1$	0,4	0	0,7	0,28[(3)]
u_{WE}	Workpiece form error	B	Rect.		3,0 µm	3,0	0	0,6	1,80[(1)]
Combined standard uncertainty, u_c									**3,79**
Expanded uncertainty ($k = 2$), U									**7,58**
NOTE For an explanation of the indications (1), (2) and (3) concerning the uncertainty components, see B.2.5.5.									

B.2.5.5 First iteration — Discussion of the uncertainty budget

It has been documented that $U_{\text{first iteration}} = 7{,}6$ µm. This value is less than the target uncertainty ($U_T = 8$ µm).

In Table B.2, there are three large [marked (1)], three mid-size [marked (2)] and three small [marked (3)] uncertainty components in the uncertainty of measurement.

The uncertainty components are squared in the formula for combined standard uncertainty. It is therefore difficult to see and understand their influence on u_c. If the variances (u^2) are used instead, another, and sometimes more understandable, picture of the influence of the individual uncertainty components (see Table B.3) emerges.

Table B.3 — Influence of the individual uncertainty components on u_c and u_c^2 (25 mm two-point diameter measurement)

Component name		Uncertainty source	Uncertainty component u_{xx} µm	Variance u_{xx}^2 µm^2	Percentage of u_c^2 %	Percentage of u_c^2 %	Uncertainty source
u_{ML}	Micrometer — error indication	Measuring equipment	1,80	3,24	23	33	Measuring equipment
u_{MF}	Micrometer — flatness 1		0,50	0,25	2		
u_{ML}	Micrometer — flatness 2		0,50	0,25	2		
u_{MP}	Micrometer — parallelism		1,00	1,00	7		
u_{RR}	Repeatability	Operator	1,20	1,44	10	17	Operator
u_{NP}	Variation of 0 point		1,00	1,00	7		
u_{TD}	Temperature difference	Environment	1,96	3,84	27	27	Environment
u_{TA}	Temperature		0,28	0,08	0		
u_{WE}	Workpiece form error	Workpiece	1,80	3,24	23	23	Workpiece
Combined standard uncertainty u_c			**3,79**	**14,34**	**100**	**100**	**Total**

From Table B.3, the following can be seen:

— if the external micrometer did not have any errors, U would be reduced from 7,6 µm to 6,2 µm;

— if the operator, environment and workpiece were perfect, then U would be reduced from 7,6 µm to 4,4 µm.

It is obvious in this case that the uncertainty components linked to the measuring process are the dominant components — not the measuring equipment.

The result is $U = 7{,}6$ µm, and if the rules of ISO 14253-1 apply, then the diameter tolerance of the workpiece is reduced to $2 \times 7{,}6$ µm $= 15{,}2$ µm during the production of shafts. This reduction at ∅ 25 mm is equal to the full size of the tolerance IT6 (13 µm).

If U is only 10 % of the workpiece tolerance, then the workpiece tolerance is IT10 (84 µm). At smaller tolerances, U will be more than 10 % of the tolerance. At tolerance IT8 (33 µm), U will be 45 % of the tolerance, and there will be only 10 % of the tolerance left for the production of shafts.

If the target uncertainty is taken to be 6 µm instead of 8 µm, then the uncertainty of measurement from the first iteration is too large ($U_{E1} = 7{,}6$ µm). The needed reduction is at least 1,6 µm. This is equal to a reduction of 38 % for u^2.

It is necessary to look at the most dominant uncertainty component, i.e. the temperature difference between workpiece and measuring equipment. It is possible to reduce this 27 % component (27 % of u_c^2) to nearly 0 by changing the procedure or measuring the temperature during production, or both.

Intensive training of the three operators will result in a reduction of the repeatability u_{RR} and the variation between their 0-points (u_{NP}). This will give up to 15 % of the necessary 38 % reduction.

The uncertainty component originating from the form errors of the workpiece is impossible to reduce when doing only one single measurement of the workpiece. If the number of measurements were increased, then this component could be reduced. Doing four measurements and using the mean value will result in a reduction of 20 % of the necessary 38 %. But the effect will be an increase in measuring time. And time is often money.

In this case, there are many ways of reducing the uncertainty of measurement. The choice amongst these can only be made on the basis of a cost analysis. The costs shall always be the guide of how to reduce the uncertainty of measurement.

In this case, a reduction of the components from the micrometer will not be a realistic possibility. The only "equipment solution" is to choose other equipment with smaller (possible) MPE values. This might be an economically sound solution if the measurement time is also reduced, and if it is possible to measure several diameters without influence from the operator.

This could bring down the expanded uncertainty from $U = 7{,}6$ µm to $U = 2{,}6$ µm.

B.2.5.6 Conclusion on the first iteration

As illustrated in the example above, the initial setting of the three micrometer MPE values is sufficient for the given target uncertainty and the actual measuring task. The requirements for the micrometer should then be confirmed as:

— Error curve (max. – min.): $\text{MPE}_{\text{ML}} = 6$ µm (bilateral specification)

— Flatness of measuring anvils: $\text{MPE}_{\text{MF}} = 1$ µm (unilateral specification)

— Parallelism between anvils: $\text{MPE}_{\text{MP}} = 2$ µm (unilateral specification)

The micrometer shall comply with these requirements, but with the uncertainties present during the calibration measurements, i.e. U_{SL}, U_{SF} and U_{SP}, reduced according to ISO 14253-1 (see B.3, B.4, B.5 and Figure B.1). It is necessary to know these three uncertainties when calibrating the micrometer.

B.2.6 Second iteration

No second iteration is needed in this case. A small decrease of the U value from the first iteration would be possible but no big reduction is possible — as demonstrated — without major changes of the measurement method and procedure.

B.3 Calibration of error of indication of an external micrometer

B.3.1 Requirements

The requirements (MPEs) for the measurement standards (gauge blocks) have not yet been established. These requirements shall be fixed as one of the tasks of the uncertainty budget.

B.3.2 Task and target uncertainty

B.3.2.1 Overall task

The overall task is to measure the range of the error of indication curve. In the error of indication curve, there are 11 basic measurements — 11 measurements with a different uncertainty of measurement in the range from 0 mm to 25 mm. To avoid unnecessary uncertainty budgeting work, look for the largest of the 11 uncertainties (25 mm) and see if it is possible to "live" with this uncertainty in the 10 other cases. Also try the smallest (0 mm) uncertainty as a check.

B.3.2.2 Basic measuring task

The basic task is to measure the error of indication in 11 positions within the measuring range (0 mm to 25 mm), zero; 2,5 mm; 5 mm; .... 22,5 mm and 25 mm.

B.3.2.3 Target uncertainty for the basic measurements

A target uncertainty (see 3.6) of 1 µm was chosen.

B.3.3 Principle, method, procedure and conditions

B.3.3.1 Measurement principle

Measurement of length — Comparison with a known length.

B.3.3.2 Measurement method

The calibration is performed using 10 special gauge blocks with a 2,5 mm module (L = 2,5; 5; ....; 22,5; 25 mm).

B.3.3.3 Initial measurement procedure

The following procedure applies.

— The reading of the external micrometer is compared with the length of a gauge block positioned between the measuring anvils.

— One (calibration) measurement per gauge block. Error of indication:

Error = Micrometer reading − Gauge block length

B.3.3.4 Initial measurement conditions

The following conditions apply.

— The calibration personnel is experienced.

— The room temperature is not controlled.

— A variation over the year in the room is observed to 20 °C ± 8 °C.

— The temperature variation over one hour is less than 0,5 °C.

B.3.4 Graphical illustration of measurement set-up

See Figure B.3.

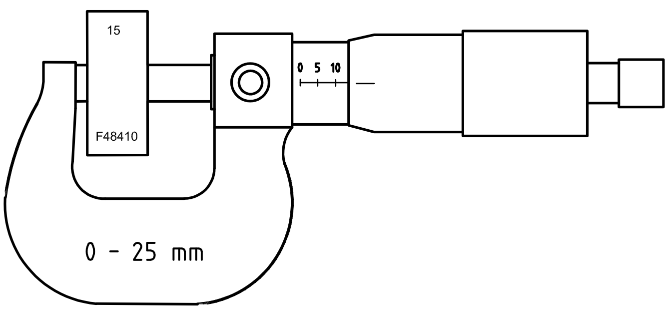


Figure B.3 — Measurement set-up

B.3.5 List and discussion of the uncertainty components

See Table B.4.

Table B.4 — Overview and comments table for uncertainty components — Calibration of error of indication of a micrometer in the 25 mm measuring point

<table>
<tr><th>Designation
Low resolution</th><th>Detailed designation</th><th>Name
Uncertainty component</th><th colspan="2">Comments</th></tr>
<tr><td>u_{SL}</td><td></td><td>Gauge block length — MPE_{SL}</td><td colspan="2">Requirements for gauge block MPE_{SL} is an unknown variable. Initially gauge block grade 2 (ISO 3650) is chosen.</td></tr>
<tr><td>u_{RR}</td><td>u_{RA}</td><td>Resolution</td><td>$u_{RA} = \frac{d}{2 \times \sqrt{3}} = \frac{1\ \mu m}{2 \times \sqrt{3}} = 0{,}29\ \mu m$</td><td rowspan="2">The largest of the two is equal to u_{RR}.</td></tr>
<tr><td></td><td>u_{RE}</td><td>Repeatability</td><td>An experiment with at least 15 measurements on the same 25 mm gauge block is performed.</td></tr>
<tr><td>u_{TD}</td><td></td><td>Temperature difference</td><td colspan="2">Maximum difference observed between the gauge blocks and the micrometer is 1 °C.</td></tr>
<tr><td>u_{TA}</td><td></td><td>Temperature</td><td colspan="2">Maximum deviation from standard reference temperature 20 °C is 8 °C.</td></tr>
</table>

B.3.6 First iteration

B.3.6.1 First iteration — Documentation and calculation of the uncertainty components

u_{SL} **— Gauge block length** **Type B evaluation**

The definitive value of MPE_{SL} has not yet been fixed. It is one of the tasks of the uncertainty budget. Initially, gauge blocks of grade 2 are chosen and, as MPE_{SL}, the tolerance limit values are taken from ISO 3650. The limit value for a 25 mm gauge block is:

$$a_{SL} = 0{,}6\ \mu m$$

Based on experience from calibration certificates for gauge blocks of the actual make, a rectangular distribution is assumed (b = 0,6):

$$u_{SL} = 0{,}6 \times 0{,}6\ \mu m = 0{,}36\ \mu m$$

u_{RR} **— Repeatability/resolution** **Type B evaluation**

A repeatability experiment has been made taking 15 measurements on a 25 mm gauge block with the actual micrometer. The standard deviation of the experiment is $u_{RE} = 0{,}19\ \mu m$. Therefore, the resolution uncertainty component, u_{RA}, is chosen as u_{RR} ($u_{RA} > u_{RE}$):

$$u_{RR} = 0{,}29\ \mu m$$

u_{TD} **— Temperature difference** **Type B evaluation**

The temperature difference between micrometer and gauge blocks is observed to maximum 1 °C. There is no information about which have the highest temperature. Therefore, ±1 °C is assumed. The linear coefficient of thermal expansion, α, is assumed to be 1,1 µm/(100 mm × °C) for the micrometer and the gauge block. The limit value is:

$$a_{\text{TD}} = \Delta T \times \alpha \times D = 1\,°\text{C} \times 1{,}1 \frac{\mu\text{m}}{100\ \text{mm} \times °\text{C}} \times 25\ \text{mm} = 0{,}28\ \mu\text{m}$$

A U-distribution is assumed ($b = 0{,}7$):

$$u_{\text{TD}} = 0{,}28\ \mu\text{m} \times 0{,}7 = 0{,}20\ \mu\text{m}$$

u_{TA} **— Temperature** **Type B evaluation**

The observed maximum difference from standard reference temperature (20 °C) is 8 °C. There is no information about the sign of this deviation, therefore ± 8 °C is assumed. A 10 % maximum difference between the two linear temperature expansion coefficients ($\alpha_{\text{micrometer}}$ and $\alpha_{\text{gauge block}}$) is assumed. The limit value is:

$$a_{\text{TA}} = 0{,}1 \times \Delta T_{20} \times \alpha \times D = 0{,}1 \times 8\ °\text{C} \times 1{,}1 \frac{\mu\text{m}}{100\ \text{mm} \times °\text{C}} \times 25\ \text{mm} = 0{,}2\ \mu\text{m}$$

A U-distribution is assumed ($b = 0{,}7$):

$$u_{\text{TA}} = 0{,}2\ \mu\text{m} \times 0{,}7 = 0{,}14\ \mu\text{m}$$

B.3.6.2 First iteration — Correlation between uncertainty components

It is estimated that no correlation occurs between the uncertainty components.

B.3.6.3 First iteration — Combined and expanded uncertainty

No uncertainty components are correlated. The combined standard deviation is:

$$u_{\text{c}} = \sqrt{u_{\text{SL}}^2 + u_{\text{RR}}^2 + u_{\text{TD}}^2 + u_{\text{TA}}^2}$$

The values from B.3.6.1:

$$u_{\text{c}} = \sqrt{\left(0{,}36^2 + 0{,}29^2 + 0{,}20^2 + 0{,}14^2\right)\mu\text{m}^2} = 0{,}5\ \mu\text{m}$$

The expanded uncertainty for the 25 mm measuring point is (coverage factor $k = 2$):

$$U_{25\ \text{mm}} = 0{,}5\ \mu\text{m} \times 2 = 1{,}0\ \mu\text{m}$$

The expanded uncertainty for the zero-measuring point is:

$$U_{0\ \text{mm}} = 0{,}4\ \mu\text{m} \times 2 = 0{,}8\ \mu\text{m}$$

B.3.6.4 Summary of uncertainty budget — First iteration

See Table B.5.

Table B.5 — Summary of uncertainty budget (first iteration) — Measurement of error of indication (25 mm measuring point)

Component name	Evaluation type	Distri-bution type	Number of measure-ments	Variation limit a^* influence units	Variation limit a µm	Correlation coefficient	Distribution factor b	Uncer-tainty compo-nent u_{xx} µm
u_{SL} Gauge block — MPE_{SL}	B	Rect.		0,6 µm	0,6	0	0,6	0,36
u_{RR} Resolution	B	Rect.		0,5 µm	0,5	0	0,6	0,29
u_{TD} Temperature difference	B	U		1 °C	0,20	0	0,7	0,20
u_{TA} Temperature	B	U		8 °C	0,14	0	0,7	0,14
Combined standard uncertainty, u_c								**0,50**
Expanded uncertainty ($k = 2$), U								**1,00**

B.3.6.5 First iteration — Discussion of the uncertainty budget

The dominant uncertainty components are gauge blocks and resolution. There is no need to reduce the uncertainty of measurement u_c and U in a second iteration. $U < 1$ µm cannot be used because of the resolution 1 µm. Observe that the temperature requirement during calibration is 20 °C ± 8 °C. This temperature range has no significant effect on the uncertainty in this case — short distances. For larger micrometers, this temperature range will result in dominant uncertainty components.

A conservative estimate is to use $U = 1{,}0$ µm for all measuring points between 0 mm and 25 mm. The maximum allowed difference in error of indication during calibration is therefore (see ISO 14253-1):

$$4\ \mu\text{m i.e.} \left[\text{MPE}_{\text{ML}} - (2 \times U) = 6\ \mu\text{m} - (2 \times 1{,}0\ \mu\text{m}) = 4\ \mu\text{m}\right]$$

B.3.6.6 Conclusion on the first iteration

The target uncertainty criterion is met by the initial assumptions and settings. This fact qualifies grade 2 gauge blocks as measurement standards and qualifies the temperature condition of the room: 20 °C ± 8 °C.

B.3.7 Second iteration

No second iteration is needed.

B.4 Calibration of flatness of the measuring anvils

B.4.1 Task and target uncertainty

B.4.1.1 Measuring task

The measuring task consists of measuring the flatness on two ∅ 6 mm measuring anvils of an external micrometer.

B.4.1.2 Target uncertainty

A target uncertainty (see 3.6) of 0,15 µm was chosen.

B.4.2 Principle, method, procedure and condition

B.4.2.1 Measurement principle

Light interference — Comparison with a flat surface.

B.4.2.2 Measurement method

An optical flat is placed on top of the measuring anvil surface parallel to the general direction of the surface. The number of interference lines is evaluated.

B.4.2.3 Measurement procedure

The following procedure applies.

- An optical flat is wrung to the surface of the measuring anvil.
- The number of interference lines is observed on the nearly symmetrical image [see Figure B.4 b)].
- The deviation from flatness is taken as number of lines times half the wavelength of the monochromatic light used.

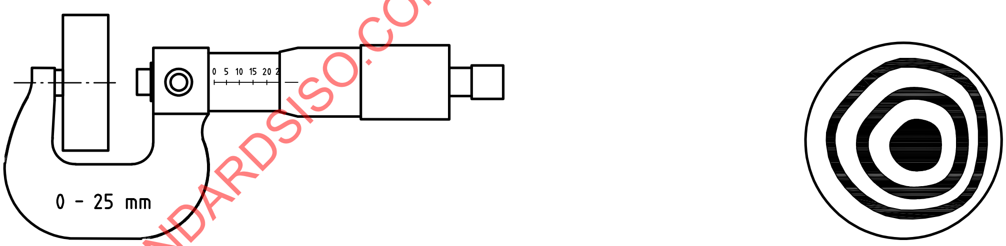
B.4.2.4 Measurement conditions

The following conditions apply.

- There are not any temperature conditions.
- The optical flat shall be acclimatized for at least 1 h.

B.4.3 Graphical illustration of measurement set-up

See Figure B.4.



a) Measurement of flatness on the measuring anvils

b) Image to be evaluated

Figure B.4 — Measurement set-up

B.4.4 List and discussion of the uncertainty components

See Table B.6.

The calibration of flatness of the measuring anvils has only two significant uncertainty components. Flatness of the optical flat and the resolution of reading the interference image pattern. The optical flat is used in a way such that the pattern is symmetrical [see Figure B.4 b)].

Table B.6 — Overview and comments table for uncertainty components for calibration of flatness of measuring anvils

Designation Low resolution	Designation High resolution	Name Uncertainty component	Comments
u_{SF}		Flatness — MPE_{SF}	The optical flat is ∅ 31 mm — the flatness is given for this whole area. The area used is only ∅ 6 mm to ∅ 8 mm.
u_{RR}		Resolution	The resolution is estimated 0,5 × line distance: $d = 0{,}15$ µm

B.4.5 First iteration

B.4.5.1 First iteration — Documentation and calculation of the uncertainty components

u_{SF} — Flatness of optical flat **Type B evaluation**

The definitive value of MPE_{SF} is not yet fixed. It is one of the tasks of the uncertainty budget. Initially MPE_{SF} is set to 0,05 µm for a ∅ 8 mm area in the middle of the surface. The limit value:

$$a_{SF} = 0{,}05\ \mu\text{m}$$

A rectangular distribution is assumed ($b = 0{,}6$):

$$u_{SF} = 0{,}05\ \mu\text{m} \times 0{,}6 = 0{,}03\ \mu\text{m}$$

u_{RR} — Resolution **Type B evaluation**

The wavelength of the light used is assumed to be 0,6 µm. The height difference between the lines of Figure B.4 b) is half a wavelength, i.e. 0,3 µm. The resolution is assumed to be:

$$d = 0{,}5 \times \text{line distance} = 0{,}15\ \mu\text{m}$$

The uncertainty component u_{RR} (see 8.4.4) is:

$$u_{RR} = \frac{d}{2} \times 0{,}6 = \frac{0{,}15\ \mu\text{m}}{2} \times 0{,}6 = 0{,}05\ \mu\text{m}$$

B.4.5.2 First iteration — Correlation between uncertainty components

It is estimated that no correlation occurs between the uncertainty components.

B.4.5.3 First iteration — Combined and expanded uncertainty

$$u_c = \sqrt{u_{SF}^2 + u_{RR}^2}$$

The values from B.4.5.1:

$$u_c = \sqrt{\left(0{,}03^2 + 0{,}05^2\right)\ \mu\text{m}^2} = 0{,}06\ \mu\text{m}$$

The expanded uncertainty (coverage factor $k = 2$) is:

$$U = 0{,}06\ \mu\text{m} \times 2 = 0{,}12\ \mu\text{m}$$

B.4.5.4 Summary of uncertainty budget — First iteration

See Table B.7.

Table B.7 — Summary of uncertainty budget (first iteration) — Calibration of flatness of measuring anvils

Component name	Eval-uation type	Distri-bution type	Number of measure-ments	Variation limit a^* influence units	Variation limit a µm	Correlation coefficient	Distribution factor b	Uncer-tainty com-ponent u_{xx} µm
u_{SF} Flatness of optical flat	B	Rect.		0,05 µm	0,05	0	0,6	0,03
u_{RR} Resolution of interference image	B	Rect.		0,075 µm	0,075	0	0,6	0,05
Combined standard uncertainty, u_c								**0,06**
Expanded uncertainty ($k = 2$), U								**0,12**

B.4.5.5 First iteration — Discussion of the uncertainty budget

It is obvious that the dominant uncertainty component is the resolution or the reading of the pattern. The flatness deviation of the optical flat is not very important compared with the influence of the resolution. U is in the order of 12 % of the flatness requirement for the measuring anvils of the micrometer $MPE_{MF} = 1$ µm.

B.4.5.6 Conclusion on the first iteration

The target uncertainty requirement is met. The maximum permissible measured deviation from perfect flatness during calibration is:

$MPE_{MF} - U = 1{,}00$ µm $- 0{,}12$ µm $= 0{,}88$ µm (rule from ISO 14253-1 as it applies to a unilateral tolerance)

For transformation of the MPE_{SF} ∅ 8 mm requirement to ∅ 30 mm, see B.6.

B.4.6 Second iteration

No second iteration is needed.

B.5 Calibration of parallelism of the measuring anvils

B.5.1 Task and target uncertainty

B.5.1.1 Measuring task

The measuring task consists of measuring the parallelism between two ∅ 6 mm measuring anvils of an external micrometer.

B.5.1.2 Target uncertainty

A target uncertainty (see 3.6) of 0,30 µm was chosen.

B.5.2 Principle, method, procedure and condition

B.5.2.1 Measurement principle

Light interference — Comparison with two parallel surfaces.

B.5.2.2 Measurement method

The following method applies.

— An optical parallel is placed between the two measuring anvils and adjusted parallel to one of the anvils.

— The number of interference lines on the other anvil is evaluated.

B.5.2.3 Measurement procedure

The following procedure applies.

— An optical parallel is wrung to the surface of one of the measuring anvils and adjusted to be parallel to the general direction of the surface of the anvil [symmetrical interference image — see Figure B.5 b)].

— The micrometer is "measuring" the optical parallel [see Figure B.5 a)] to bring the measurement force to the right level.

— The number of interference lines is observed on the image on the other anvil [see Figure B.5 c)].

— The deviation from parallelism is taken as number of lines times half the wavelength of the monochromatic light used.

B.5.2.4 Measurement conditions

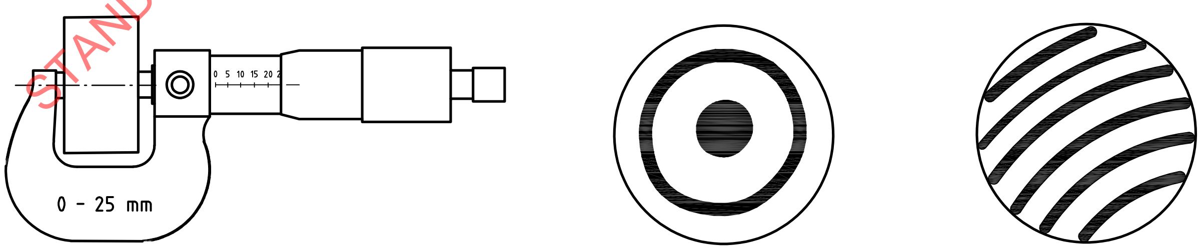
The following conditions apply.

— There are not any temperature conditions.

— The optical parallel shall be acclimatized for at least 1 h.

B.5.3 Graphical illustration of measurement set-up

See Figure B.5.



a) Measurement of parallelism between the measuring anvils **b) Images on the anvils** **c) Images on the anvils**

Figure B.5 — Measurement set-up

B.5.4 List and discussion of the uncertainty components

There are three significant uncertainty components in the calibration of the parallelism between the measuring anvils (see Table B.8):

a) the parallelism of the optical parallel;

b) the alignment of the optical parallel to the first measuring anvil;

c) the resolution of reading the interference image pattern on the second measuring anvil.

Table B.8 — Overview and comment table for uncertainty components for the calibration of the parallelism between the measuring anvils

Designation	Detailed designation	Name Uncertainty component	Comments
u_{SP}		Parallelism of optical parallel — MPE_{SP}	Diameter of the optical parallel is ∅ 31 mm. The area used is only ∅ 6 mm to ∅ 8 mm.
u_{OP}		Alignment to the first anvil	It is assumed that the maximum alignment error is 0,5 line.
u_{RR}		Resolution	The resolution is estimated to 1 line.

The two uncertainty components from the flatness of the two surfaces on the optical parallel have no influence because of the order of the flatness deviation compared to other components u_{OP} = 0,03 µm.

B.5.5 First iteration

B.5.5.1 First iteration — Documentation and calculation of the uncertainty components

u_{SP} — **Parallelism of optical parallel** **Type B evaluation**

The definitive value of MPE_{SP} has not yet been fixed. It is one of the tasks of the uncertainty budget. Initially MPE_{SP} is set to 0,05 µm for a ∅ 8 mm area in the middle of the surface. The limit value:

$$a_{SP} = 0{,}1\ \mu m$$

A rectangular distribution is assumed (b = 0,6):

$$u_{SP} = 0{,}1\ \mu m \times 0{,}6 = 0{,}06\ \mu m$$

u_{OP} — **Alignment to the first anvil** **Type B evaluation**

The wavelength of the light used is assumed to be 0,6 µm.

A maximum alignment error of 0,5 line is 0,15 µm.

$$a_{OP} = 0{,}15\ \mu m$$

A rectangular distribution is assumed (b = 0,6):

$$u_{OP} = 0{,}15\ \mu m \times 0{,}6 = 0{,}09\ \mu m$$

u_{RR} — **Resolution on the second anvil** **Type B evaluation**

The wavelength of the light used is assumed to be 0,6 µm.

The resolution is assumed to be one line = 0,3 µm.

