INTERNATIONAL STANDARD

ISO 12491

First edition
1997-05-01

Statistical methods for quality control of building materials and components

Méthodes statistiques de contrôle de la qualité des matériaux et éléments de construction

Reference number
ISO 12491:1997(E)

Contents

Page

International Organization for Standardization
Case postale 56 • CH-1211 Genève 20 • Switzerland
Internet central@iso.ch
X.400 c=ch; a=400net; p=iso; o=isocs; s=central

Printed in Switzerland

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 12491 was prepared by Technical Committee ISO/TC 98, *Bases for design of structures*, Subcommittee SC 2, *Reliability of structures*.

Annex A of this International Standard is for information only.

Introduction

Quality control of building materials and components is, according to ISO 2394, an indispensable part of an overall concept of structural reliability. As quality control is generally a time-consuming and expensive task, various operational techniques and activities have been developed to fulfil quality requirements in building. It appears that properly employed statistical methods can provide efficient, economic and effective means of quality control, particularly when expensive and destructive tests are to be performed. The purpose of this International Standard is to provide general techniques for quality control of building materials and components used in building or other civil engineering works.

Described techniques consist predominantly of classical statistical methods of common interest for all the participants in the building process. For other more sophisticated techniques and specific problems, existing statistical standards listed in annex A should be applied.

Statistical methods for quality control of building materials and components

1 Scope

This International Standard gives general principles for the application of statistical methods in the quality control of building materials and components in compliance with the safety and serviceability requirements of ISO 2394.

This International Standard is applicable to all buildings and other civil engineering work, existing or under construction, whatever the nature or combination of the materials used, for example concrete, steel, wood, bricks.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 2394:—[1], *General principles on reliability for structures.*

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:1993, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

3 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1 and ISO 3534-2, and the following definitions, apply.

NOTE - The terms and their definitions are listed in the order corresponding to their appearance in the main text. An alphabetic list of these terms with numerical references to subclauses where the terms appear is given in the index.

3.1 quality control: Operational techniques and activities that are used to fulfill requirements for quality.

3.2 statistical quality control: That part of quality control in which statistical methods are used (such as estimation and tests of parameters and sampling inspection).

[1] To be published. (Revision of ISO 2394:1986)

3.3 unit: Defined quantity of building material, component or element of a building or other civil engineering work that can be individually considered and separately tested.

3.4 population: Totality of units under consideration.

3.5 (random) variable, X: A variable which may take any of the values of a specified set of values and with which is associated a probability distribution.

NOTE - A random variable that may take only isolated values is said to be "discrete". A random variable which may take any value within a finite of infinite interval is said to be "continuous".

3.6 (probability) distribution: A function which gives the probability that a variable X takes any given value (in the case of a discrete variable) or belongs to a given set of values (in the case of a continuous variable).

3.7 distribution function, $\Pi(x)$: A function giving, for every value of x, the probability that the variable X is less than or equal to x:

$$\Pi(x) = P_r\,(X \leq x)$$

3.8 (probability) density function, $f(x)$: The derivative (when it exists) of the distribution function:

$$f(x) = \frac{\mathrm{d}\,\Pi(x)}{\mathrm{d}x}$$

3.9 (population) parameter: Quantity used in describing the distribution of a random variable in a population.

3.10 fractile, x_p: If X is a continuous variable and p is a real number between 0 and 1, the p-fractile is the value of the variable X for which the distribution function equals p. Thus x_p is a p-fractile if

$$P_r\,(X \leq x_p) = p$$

3.11 (population) mean, μ: For a continuous variable X having the probability density $f(x)$, the mean, if it exists, is given by

$$\mu = \int x\, f(x)\,\mathrm{d}x$$

the integral being extended over the interval(s) of variation of the variable X.

3.12 (population) variance, σ^2: For a continuous variable X having the probability density function $f(x)$, the variance, if it exists, is given by

$$\sigma^2 = \int (x-\mu)^2\, f(x)\,\mathrm{d}x$$

the integral being extended over the interval(s) of variation of the variable X.

3.13 (population) standard deviation, σ: Positive square root of the population variance σ^2.

3.14 standardized variable: A random variable, the mean of which equals zero and the standard deviation of which equals 1. If the variable X has a mean equal to μ and a standard deviation equal to σ, the corresponding standardized variable is given as

$$(X - \mu)/\sigma$$

NOTE - The distribution of the standardized variable is called "standardized distribution".

3.15 normal distribution: Probability distribution of a continuous variable X, the probability density function of which is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

3.16 log-normal distribution: Probability distribution of a continuous variable X which can take any value from x_0 to $+\infty$, or from $-\infty$ to x_0.

In the former, more frequent, case the probability density function is given as

$$f(x) = \frac{1}{(x-x_0)\sigma_Y\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x-x_0)-\mu_Y}{\sigma_Y}\right)^2\right]$$

where

$x \geq x_0$

μ_Y and σ_Y are, respectively, the mean and the standard deviation of the new variable;

$Y = \ln(X - x_0)$

In the latter, less frequent, case the sign of the brackets $(X - x_0)$ and $(x - x_0)$ is to be changed. Note that the variable Y has a normal distribution.

3.17 (random) sample: One or more sampling units taken from a population in such a way that each unit of the population has the same probability of being taken.

3.18 (sample) size, n: Number of sampling units in the sample.

3.19 sample mean, $\bar{x}$: Sum of n values x_i of sampling units divided by the sample size n:

$$\bar{x} = \frac{1}{n}\sum x_i$$

3.20 sample variance, s^2: Sum of n squared deviations from the sample mean $\bar{x}$ divided by the sample size n minus 1:

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

3.21 sample standard deviation, s: Positive square root of the sample variance s^2.

3.22 estimation: Operation of assigning, from observations on a sample, numerical values to the parameters of a distribution chosen as the statistical model of the population from which this sample was taken.

3.23 estimator: Function of a set of the sample random variables used to estimate a population parameter.

3.24 estimate: Value of an estimator obtained as a result of an estimation.

3.25 confidence level, γ : Given value of the probability associated with a confidence interval.

NOTE - In ISO 3534-1, it is designated $(1-\alpha)$.

3.26 two-sided confidence interval: When T_1 and T_2 are two functions of the observed values such that, θ being a parameter to be estimated, the probability P_r $(T_1 \le \theta \le T_2)$ is at least equal to the confidence level γ (where γ is a fixed number, positive and less than 1), the interval between T_1 and T_2 is a two-sided γ confidence interval for θ.

3.27 one-sided confidence interval: When T is a function of the observed values such that, θ being a population parameter to be estimated, the probability P_r $(T \ge \theta)$ or the probability P_r $(T \le \theta)$ is at least equal to the confidence level γ (where γ is a fixed number, positive and less than 1), the interval from the smallest possible value of θ up to T (or the interval from the T up to the largest possible value of θ) is a one-sided γ confidence interval for θ.

3.28 outliers: Observations in a sample, so far separated in value from the remainder as to suggest that they may be from a different population.

3.29 (statistical) test: Statistical procedure to decide whether a hypothesis about the distribution of one or more populations should be accepted or rejected.

3.30 (statistical) hypothesis: Hypothesis, concerning the population, which is to be accepted or rejected as the outcome of the test using sample observations.

3.31 significance level, α: Given value, which is the upper limit of the probability of a statistical hypothesis being rejected when this hypothesis is true.

3.32 number of degrees of freedom, ν : In general, the number of terms in a sum minus the number of constraints on the terms of the sum.

3.33 χ^2-distribution: Probability distribution of a continuous variable χ^2 which can take any value from 0 to ∞ , the probability density function of which is

$$f(\chi^2;\nu) = \frac{(\chi^2)^{(\nu/2)-1}}{2^{(\nu/2)}\,\Gamma(\nu/2)} \exp\left(-\frac{\chi^2}{2}\right)$$

where

$\chi^2 \geq 0$ with a parameter (number of degrees of freedom) ν = 1, 2, 3,...;

Γ is the gamma function.

3.34 *t*-distribution: Probability distribution of a continuous variable t which can take any value from $-\infty$ to $+\infty$, the probability density function of which is

$$f(t;\nu) = \frac{1}{\sqrt{\pi\nu}} \frac{\Gamma[(\nu+1)/2]}{\Gamma(\nu/2)} \frac{1}{(1+t^2/\nu)^{(\nu+1)/2}}$$

where

$-\infty < t < +\infty$ with a parameter (number of degrees of freedom) ν = 1, 2, 3,...;

Γ is the gamma function.

3.35 noncentral *t*-distribution: Probability distribution of a continuous variable t which can take any value from $-\infty$ to $+\infty$, the probability density function of which is

$$f(t;\nu,\delta) = \frac{1}{\sqrt{\pi\nu}} \frac{1}{2^{(\nu-1)/2}\,\Gamma(\nu/2)} \frac{1}{(1+t^2/\nu)^{(\nu+1)/2}} \times$$

$$\times \exp\left(-\frac{\nu\delta^2}{2(\nu+t^2)}\right) \int_0^\infty z^\nu \exp\left(-\frac{1}{2}\left(z-\frac{\delta t}{\sqrt{\nu+t^2}}\right)^2\right) dz$$

where

$-\infty < t < +\infty$ with two parameters; i.e. number of degrees of freedom ν and noncentrality parameter δ.

3.36 *F*-distribution: Probability distribution of a continuous variable F which can take any value from 0 to $+\infty$, the probability density function of which is

$$f(F;\nu_1,\nu_2) = \frac{\Gamma[(\nu_1+\nu_2)/2]}{\Gamma(\nu_1/2)\,\Gamma(\nu_2/2)} (\nu_1)^{\nu_1/2} (\nu_2)^{\nu_2/2} \frac{F^{(\nu_1/2)-1}}{(\nu_1 F+\nu_2)^{(\nu_1+\nu_2)/2}}$$

where

$F \geq 0$ with parameters (numbers of degrees of freedom) ν_1 ,ν_2 = 1, 2, 3,...;

Γ is the gamma function.

3.37 lot: Definite quantity of units, manufactured or produced under conditions which are presumed uniform.

NOTE - In statistical quality control in building, a lot is usually equivalent to a "batch" and is considered as a "population".

3.38 isolated lot: A lot separated from the sequence of lots in which it was produced or collected, and not forming part of a current sequence of inspection lots.

NOTE - In statistical quality control in building, lots are usually considered as "isolated lots".

3.39 conforming unit: Unit which satisfies all the specified requirements.

3.40 nonconforming unit: Unit containing at least one nonconformity which causes the unit not to satisfy specified requirements.

3.41 sampling inspection: Inspection in which decisions are made to accept or not accept a lot, based on results of a sample selected from that lot.

3.42 sampling inspection by variables: Method of sampling inspection which consists of measuring a quantitative variable X for each unit of a sample.

3.43 sampling inspection by attributes: Method of sampling inspection which consists of distinguishing between conforming and nonconforming units of a sample.

3.44 sampling plan: A plan according to which one or more samples are taken in order to obtain information and the possibility of reaching a decision concerning the acceptance of the lot.

NOTE - It includes the sample size n and the acceptance constants k_σ , k_s (in sampling inspection by variables), or the sample size n and the acceptance number Ac (in sampling inspection by attributes).

3.45 operating characteristic curve (OC curve): Curve showing, for a given sampling plan, the probability that an acceptance criterion is satisfied, as a function of the lot quality level.

3.46 producer: Any participant of the building process supplying a lot for further procedure or use.

3.47 consumer: Any participant of the building process purchasing a lot for further procedure or use.

3.48 producer's risk point (PRP): A point on the operating characteristic curve corresponding to a predetermined and usually low probability of non-acceptance.

NOTE - This probability is the producer´s risk (PR) when an isolated lot is considered.

3.49 consumer's risk point (CRP): A point on the operating characteristic curve corresponding to a predetermined and usually low probability of acceptance.

NOTE - This probability is the consumer´s risk (CR) when an isolated lot is considered.

3.50 producer's risk (PR): For a given sampling plan, the probability of non-acceptance of a lot when the lot quality has a value stated by the plan as acceptable.

NOTE - This quality is the producer's risk quality (PRQ) when an isolated lot is considered.

3.51 consumer's risk (CR): For a given sampling plan, the probability of acceptance of a lot when the lot quality has a value stated by the plan as unsatisfactory.

NOTE - This quality is the consumer's risk quality (CRQ) when an isolated lot is considered.

3.52 producer's risk quality (PRQ): A lot quality level which, in the sampling plan for an isolated lot, corresponds to a specified producer's risk (PR).

NOTE - When a continuing series of lots is considered, the acceptable quality level AQL is used instead of PRQ.

3.53 consumer's risk quality (CRQ): A lot quality level which, in the sampling plan for an isolated lot, corresponds to a specified consumer's risk (CR).

NOTE - When a continuing series of lots is considered, the limiting quality level LQL is used instead of CRQ.

3.54 acceptance constants, k_σ, k_s : In sampling inspection by variables, constants used in the criteria for accepting the lot, as given in the sampling plan.

NOTE 1 Both these constants are also used as coefficients in estimation of population fractiles.

NOTE 2 In ISO 3534-2, the acceptance constant is designated k.

3.55 acceptance number (Ac): In sampling inspection by attributes, the largest number of nonconforming units found in the sample that permits acceptance of the lot, as given in the sampling plan.

3.56 lower specification limit, L: Specified value of the observed variable X giving the lower boundary of the permissible value.

3.57 upper specification limit, U: Specified value of the observed variable X giving the upper boundary of the permissible value.

3.58 number of nonconforming units, z: Actual number of nonconforming units found in a sample.

4 Population and sample

4.1 General

Mechanical properties and dimensions of building materials and components are described by random variables (called variables in this International Standard) with a certain type of probability distribution. The popular normal distribution (Laplace-Gauss distribution) may be used to approximate many actual symmetrical distributions. When a remarkable asymmetry is observed, then another type of distribution reflecting this asymmetry shall be considered. Often, three-parametric log-normal distribution is used (see 4.3).

To simplify calculation procedures, standardized variables (see 3.14) are used, whose means are equal to zero and whose variances are equal to one, and which have standardized distributions for which numerical tables are available.

As a rule, only a limited number of observations constituting a random sample x_1, x_2, x_3,..., x_n of size n taken from a population (lot) is available. The aim of statistical methods for quality control is to make a decision concerning the required quality of a population using the information derived from one or more random samples.

4.2 Normal distribution

The well-known normal distribution of a continuous variable, X, is a fundamental type of symmetrical distribution defined on an unlimited interval, which is fully described by two parameters: the mean μ and the variance σ^2. Any normal variable may be easily transformed to a standardized variable $U = (X - \mu)/\sigma$, for which tables of probability density and distribution function are commonly available.

In quality control of building materials and components, the fractiles u_p are frequently used, where the following values for the probability p are most often applied: p = 0,95; 0,975; 0,99; 0,995. The corresponding values of the fractiles u_p are given in table 1. It is to be noted that for high ratios σ/μ there is a non-negligible probability of the occurrence of negative values of the variable X. If X must be positive (which may follow from some physical reasons), then other theoretical models for the probability distribution may be more suitable.

All the information derived from a given random sample x_1, x_2,...,x_n of the size n, taken from a normal population, is completely described by two sample characteristics only: the sample mean $\bar{x}$ and the sample variance s^2. These characteristics are specific values of the corresponding estimators of the population mean and variance, denoted by $\bar{X}$ and S^2. The mean estimator $\bar{X}$ is a random variable described by the normal distribution having the same mean μ as the population and the variance equal to σ^2/n. The variance estimator S^2 is a random variable described by transformed χ^2-distribution with $\nu = (n-1)$ degrees of freedom as

$$S^2 = \sigma^2\chi^2/(n-1)$$

This transformation allows any fractile of S^2 to be determined from the corresponding fractile of χ^2. As the χ^2-distribution is asymmetrical, the lower fractiles χ^2_{p1} as well as the upper fractiles χ^2_{p2} are given in table 2. The recommended probabilities to be used in building are as follows: p_1 = 0,05; 0,025; 0,01; 0,005 and p_2 =0,95; 0,975; 0,99; 0,995.

4.3 Log-normal distribution

The asymmetrical log-normal distribution, defined on a semi-infinite interval, is generally described by three parameters: the mean μ, the variance σ^2 and, as the third characteristic, the lower or upper limit value x_0 corresponding to a certain positive or negative asymmetry may be used. In building, the log-normal distribution with the lower limit x_0 (and positive skewness) is often considered. In this case, as indicated in 3.16, the distribution of a variable X may be easily transformed to the normal distribution of a variable Y given by the transformation

$$Y = \ln (X - x_0)$$

The new variable Y is then treated in the same way as the variable X in the previous case; this is valid also for the transformation to the standardized variable. (Generally, Y and y should be used instead of X and x.)

Moreover, in building, it may often be assumed that $x_0 = 0$ and then only two parameters (μ and σ^2) are involved. In this case, the normal variable Y is given as

$$Y = \ln X$$

and the original variable X is assumed to have positive asymmetry, which is dependent on the ratio σ/μ, where σ and μ are the standard deviation and the mean of the variable X, respectively.

4.4 Normality tests

The assumption of a normal distribution of the variable X (or variable Y when the variable X has a log-normal distribution) may be tested using various normality tests: a random sample is compared with the theoretical model of the normal distribution and observed deviations are tested to determine whether they are significant or not. If the deviations are insignificant, then the assumption of normal distribution is accepted, otherwise it is rejected. Various normality tests, as established in ISO 5479, may be used. The recommended significance level α to be used in building is 0,05 or 0,01 (then the risk of acceptance of a wrong hypothesis has a suitable value).

5 Methods of statistical quality control

5.1 Quality requirements

To control the quality of building materials and components, adequate requirements should be specified for observed variables. These requirements usually involve population parameters (the mean μ and/or the variance σ^2) or a fractile x_p. The most frequently applied quality requirements limit admissible values of the mean by specified lower and upper boundary and/or limit the variance by a specified upper boundary, or specify boundaries for a given fractile. Then, methods of estimation and tests of population parameters and fractiles have to be applied.

Special procedures of quality control use sampling inspection methods, the aim of which is to decide directly on acceptance of a population using sample data and not to estimate explicitly the population parameters.

A normal distribution of the variable X (or variable Y if the variable X has a log-normal distribution) is assumed in most of the methods described in this International Standard.

5.2 Basic statistical methods

Basic statistical methods used for the quality control of building materials and components consist of estimation techniques, tests of statistical hypotheses, and sampling inspection.

Two estimation techniques for population parameters are generally applied in building:

- point estimation, and
- interval estimation.

These two basic techniques using a classical approach are described in 6.2 and 6.3. A modified approach to estimation or prediction of population parameters using a Bayesian approach is introduced in 5.3 and 6.7.

Methods of tests of hypotheses concerning population parameters which are commonly applied in building may be also divided into two groups:

- comparison of sample characteristics and the corresponding population parameters, and
- comparison of the characteristics of two samples.

These methods are described in detail in 6.4 and 6.5.

An important statistical method, frequently used in quality control of building materials and components, concerns the estimation or prediction of fractiles for normal distribution; this technique is described in 6.6 and 6.7.

Methods of sampling inspection are used in those cases where a decision concerning population quality is to be made without explicit determination of population parameters. It is, however, recommended to combine methods of sampling inspection in building with the systematic collection of data for the purpose of further evaluation.

A number of sampling plans and criteria are used in building to control the quality of materials and components. It is, however, strongly recommended always to check the power of a chosen plan using the operating characteristic curve (OC curve). In practical cases, the operating characteristic curve may be substituted by two points only, the producer's risk point (PRP) and the consumer's risk point (CRP) corresponding to a specified producer's risk (PR) and consumer's risk (CR) respectively.

Recommended simple methods of sampling inspection, suitable for the purpose of quality control of building materials and components, are described in clause 7.

5.3 Bayesian approach

An alterative technique to the basic methods of estimation and tests in quality control procedures is provided by a Bayesian approach. This approach could be used effectively, especially when a large continuous production of building materials and components is to be checked.

The fundamental principles of the Bayesian approach to quality control differ from the principles of the classical statistical methods mentioned above. If the observed variable Y is a function of a sample variable X and a vector of distribution parameters Θ (say μ and σ), given as

$$Y = h\,(X,\Theta)$$

the Bayesian approach considers Θ as a random variable and not as a deterministic parameter vector, which is the case with classical methods. According to these statistical methods, which are described in detail in clause 6, estimation of the parameter vector Θ is made separately for each lot using information derived from sample data. The Bayesian approach investigates the probability distribution of the parameter vector Θ using its prior distribution as well as current sample data taken from the lot under consideration.

Two kinds of distribution function of the parameter vector Θ are generally distinguished: a prior distribution function $\Pi'(\Theta)$, based on prior information, and a posterior distribution function $\Pi''(\Theta \mid x_1, x_2, \ldots, x_n)$, derived from current data $x_1, x_2, \ldots, x_n$ after sampling. The Bayesian approach provides methods to derive conjugate distribution functions $\Pi'(\Theta)$ and $\Pi''(\Theta \mid x_1, x_2, \ldots, x_n)$, as well as the predictive distribution function of the observed variable Y. The posterior distribution function $\Pi''(\Theta)$ may be derived as

$$\Pi''(\Theta \mid x_1, x_2, \ldots, x_n) = C\,\Pi'(\Theta) f(x_1 \mid \Theta)\, f(x_2 \mid \Theta) \ldots f(x_n \mid \Theta)$$

where C denotes a normalizing factor and $f(x_i \mid \Theta)$, $i = 1, 2, \ldots, n$, denotes the probability density function of the variable X if Θ is known.

The crucial point in the Bayesian procedure is the choice of the prior distribution function $\Pi'(\Theta)$. Engineering judgement is often needed. In some cases there may be substantial information from previous tests of similar products which may be used to construct $\Pi'(\Theta)$.

In a continuous production process, in which the units belong to sequence of lots, the prior distribution for a new sample can often be taken as equal to the posterior distribution derived using previous sample data. If no relevant information is available, so-called "uninformative" or "vague" prior distributions should be used.

Furthermore, the predictive probability density function of the observed variable X itself, given the prior distribution function $\Pi'(\Theta)$ and sample $x_1, x_2, \ldots, x_n$, may be written as

$$f^*\left(x \mid x_1, x_2, \ldots, x_n\right) = \int_\Theta f\left(x \mid \Theta\right) \Pi''\left(\Theta \mid x_1, x_2, \ldots, x_n\right) \mathrm{d}\Theta$$

where $f^*\left(x \mid x_1, x_2, \ldots, x_n\right)$ for the predictive probability density function of the variable X given the sample data $x_1, x_2, \ldots, x_n$ is used to distinguish it from $f(x \mid \Theta)$, which denotes the probability density function of the variable X if Θ is known.

Various effective and economical techniques of sampling inspection based on a comparison of the prior and posterior distributions of the random vector Θ may be derived from the above general principles. If, for example, doubt remains concerning the decision to be taken as a result of the inspection of the lot, sample $x_1, x_2, \ldots, x_n$ may be increased to a new sample $x_1, x_2, \ldots, x_n, \ldots, x_m$ having a larger size and, using Bayesian approach, the quality requirements may be checked again. Such a procedure generally reduces sampling costs without loosing accuracy.

5.4 Additional methods

There are further statistical methods, besides those described above, which are applied only exceptionally in building and, consequently, are not included in this International Standard, but can be found in a general form in the other International Standards given in annex A. These methods consist of:

a) determination of sample size guaranteeing the required accuracy of an estimate of population parameters;

b) tests of outliers;

c) comparison of characteristics of three or more samples;

d) tests concerning accuracy, trueness and precision of measurements;

e) statistical process control;

f) determination of statistical tolerance intervals.

When applying the above methods using other International Standards, the confidence and significance levels recommended in this International Standard should be accepted.

6 Estimation and tests of parameters

6.1 Principles of estimation and tests

A point estimate of a population parameter is given by one number, which is the value of an estimator derived from a given sample. The best point estimate of a population parameter is unbiased (the mean of the estimator is equal to the corresponding population parameter) and efficient (the variance of the unbiased estimator is the minimum).

An interval estimate of a population parameter is given by two numbers and contains the parameter with a certain probability γ called the confidence level. The following selected values $\gamma = 0{,}90$; 0,95 or 0,99, in some cases also $\gamma = 0{,}75$, are recommended to be used in quality control in building, depending on the type of variable and the possible consequences of exceeding the estimated values. The interval estimates indicate the accuracy of an estimate and are therefore preferable to point estimates.

A test of a statistical hypothesis is a procedure that is used to decide whether a hypothesis about the distribution of one or more populations should be accepted or rejected. If results derived from random samples do not differ markedly from those expected under the assumption that the hypothesis is true, then the observed difference is said to be insignificant and the hypothesis is accepted; otherwise the hypothesis is rejected. The recommended significance level $\alpha = 0{,}01$ or 0,05 guarantees that the risk of acceptance of a wrong hypothesis has a suitable value.

Methods of estimation and tests of means and variances are covered in general in ISO 2854 and ISO 2602. The most suitable methods, adjusted for quality control of building materials and components, are described in 6.2 to 6.5. Also 6.6 describes the classical approach to the estimation of fractiles and 6.7 indicates the Bayesian approach to a prediction (a point estimate) of fractiles.

6.2 Estimation of the mean

The best point estimate of the population mean μ is the sample mean $\bar{x}$.

The interval estimate of the mean μ depends on knowledge of the population standard deviation σ.

If the standard deviation σ is known, then the two-sided interval estimate at the confidence level $\gamma = (2p-1)$ is given by

$$\bar{x} - u_p \sigma / \sqrt{n} \leq \mu \leq \bar{x} + u_p \sigma / \sqrt{n}$$

where u_p is the fractile of the standardized normal distribution corresponding to the probability p (close to 1) given in table 1. (For further information see ISO 2854.)

If the population standard deviation σ is unknown, then the two-sided interval estimate at the confidence level $\gamma = (2p-1)$ is given by

$$\bar{x} - t_p\, s / \sqrt{n} \leq \mu \leq \bar{x} + t_p\, s / \sqrt{n}$$

where

s is the sample standard deviation;

t_p is the fractile of the t-distribution for $\nu = (n-1)$ degrees of freedom;

p is the probability (close to 1) given in table 3.

(For further information see ISO 2854.)

In both the above cases only the one-sided interval estimate at the confidence level $\gamma = p$ may be used when only the lower or only the upper limit of the above estimates is considered. The value of p and corresponding fractiles u_p and t_p should be specified in accordance with chosen confidence level $\gamma = p$ (see 6.1).

6.3 Estimation of the variance

The best point estimate of the population variance σ^2 is the sample variance s^2.

The two-sided interval estimate for the variance σ^2 at the confidence level $\gamma = (p_2 - p_1)$ is given as

$$(n-1)s^2 / \chi^2_{p2} \leq \sigma^2 \leq (n-1)s^2 / \chi^2_{p1}$$

where χ^2_{p1} and χ^2_{p2} are fractiles of the χ^2-distribution for $\nu = (n-1)$ degrees of freedom corresponding to the probabilities p_1 (close to 0) and p_2 (close to 1) given in table 2. (For further information see ISO 2854.)

Often the lower limit of the above interval estimate for the variance σ^2 is considered to be 0 and then the confidence level γ of the estimate equals $(1-p_1)$.

The estimate for the standard deviation σ may be obtained by square root of the relationships derived for the variance σ^2.

6.4 Comparison of means

To test the difference between the sample mean $\bar{x}$ and the population mean μ if the population standard deviation σ is known, the test value u_0, given by

$$u_0 = |\bar{x} - \mu|\sqrt{n} \,/\sigma$$

is compared with the critical value u_p (table 1), which is the fractile of the standardized normal distribution corresponding to the significance level $\alpha = (1-p)$ (close to 0). If $u_0 \leq u_p$, then the hypothesis that the sample is taken from the population with the mean μ is accepted; otherwise it is rejected.

If the population standard deviation σ is unknown, then the test value t_0, given by

$$t_0 = |\bar{x} - \mu|\sqrt{n} \,/s$$

is compared with the critical value t_p (table 3), which is the fractile of the t-distribution for the $\nu = (n-1)$ degrees of freedom corresponding to the significance level $\alpha = (1-p)$ (close to 0). If $t_0 \leq t_p$, then the hypothesis that the sample is taken from population with the mean μ is accepted; otherwise it is rejected.

To test the difference between the means $\bar{x}_1$ and $\bar{x}_2$ of two samples of sizes n_1 and n_2, respectively, and which are taken from two populations having the same population standard deviation σ, the test value u_0, given by

$$u_0 = |\bar{x}_1 - \bar{x}_2| \sqrt{n_1 n_2} \,/\left(\sigma\sqrt{n_1 + n_2}\right)$$

is compared with the critical value u_p (table 1), which is the fractile of the standardized normal distribution corresponding to the significance level $\alpha = (1-p)$ (close to 0). If $u_0 \leq u_p$, then the hypothesis that the both samples are taken from the populations with the same (though unknown) mean μ is accepted; otherwise it is rejected.

If the standard deviation σ of both populations is the same, but unknown, then it is necessary to use sample standard deviations s_1 and s_2. The test value t_0, given by

$$t_0 = |\bar{x}_1 - \bar{x}_2| \sqrt{(n_1 + n_2 - 2)n_1 n_2} \,/\sqrt{\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right](n_1 + n_2)}$$

is compared with the critical value t_p (table 3), which is the fractile of the t-distribution for the $\nu = (n_1 + n_2 - 2)$ degrees of freedom corresponding to the significance level $\alpha = (1-p)$ (close to 0). If $t_0 \leq t$, then the hypothesis that the samples are taken from populations with the same (though unknown) mean μ is accepted; otherwise it is rejected.

For two samples of the same size $n_1 = n_2 = n$, for which observed values may be coupled (paired observations), the difference between the sample means may be tested using the differences between coupled values $w_1 = (x_{1i} - x_{2i})$. First the mean $\overline{w}$ and standard deviation s_w are determined and then the test value t_0, given by

$$t_0 = |\overline{w}| \sqrt{n} / s_w$$

is compared with the critical value t_p (table 3), which is the fractile of the t-distribution for the $\nu = (n-1)$ degrees of freedom corresponding to the significance level $\alpha = (1-p)$ (close to 0). If $t_0 \le t_p$, then the hypothesis that both samples are taken from populations with the same (though unknown) mean μ is accepted; otherwise it is rejected. (For further information, see ISO 3301.)

6.5 Comparison of variances

To test the difference between the sample variance s^2 and a population variance σ^2 the test value χ_0^2, given by

$$\chi_0^2 = (n - 1)\, s^2/\sigma^2$$

is first determined.

If $s^2 \le \sigma^2$, then the test value χ_0^2 is compared with the critical value χ_{p1}^2 (table 2) corresponding to $\nu = (n-1)$ degrees of freedom and to the significance level $\alpha = p_1$. When $\chi_0^2 \ge \chi_{p1}^2$, then the hypothesis that the sample is taken from the population with the variance σ^2 is accepted; otherwise it is rejected.

If $s^2 \ge \sigma^2$, then the test value χ_0^2 is compared with the critical value χ_{p2}^2 (table 2) corresponding to $\nu = (n-1)$ degrees of freedom and to the significance level $\alpha = (1-p_2)$. When $\chi_0^2 \le \chi_{p2}^2$, then the hypothesis that the sample is taken from the population with the variance σ^2 is accepted, otherwise it is rejected.

For two samples of sizes n_1 and n_2, the difference between the sample variances s_1^2 and s_2^2 (the lower subscripts are chosen such that $s_2^2 \le s_1^2$) may be tested comparing the test value F_0, given by

$$F_0 = s_1^2 / s_2^2$$

with the critical value F_p, which is the fractile of the F-distribution given in table 4 (for further information, see ISO 2854) for $\nu_1 = (n_1 - 1)$ and $\nu_2 = (n_2 - 1)$ degrees of freedom and for the significance level $\alpha = (1-p)$. If $F_0 \le F_p$, then the hypothesis that both samples are taken from populations with the same (though unknown) variance σ^2 is accepted; otherwise it is rejected.

6.6 Estimation of fractiles

Various methods for the estimation of fractiles are available for different assumptions concerning the type of probability distribution and available data. The most efficient and general methods of estimation of fractiles x_p, independent of the type of distribution, are based on the order statistic. According to the simplest procedure, a sample x_1, x_2, ..., x_n is first transformed into the ordered sample

$$x_1' \le x_2' \le, \ldots, \le x_n'$$

and then the fractile estimate $x_{p,\text{est}}$ is given as

$$x_{p,\text{est}} = x'_{k+1}$$

where the integer k follows from the inequality

$$k \le np < k + 1$$

The exact density function of the estimator $X_{p,\text{est}}$ of the p-fractile is given as

$$g\left(x_{p,\text{est}}\right) = \binom{n}{k}(n-k)\left[\Pi\left(x_{p,\text{est}}\right)\right]^k \left[1-\Pi\left(x_{p,\text{est}}\right)\right]^{n-k-1} f\left(x_{p,\text{est}}\right)$$

where $\Pi(x)$ denotes the distribution function and $f(x)$ denotes the density function of the population. With increasing values of n, the density $g(x_{p,\text{est}})$ tends to the normal density function with the mean equal to x_p and standard deviation equal to

$$\left(\sqrt{p(1-p)/n}\right)/f\left(x_p\right)$$

For a population having a normal distribution, the following simple technique, which depends on knowledge of the population standard deviation σ, is recommended.

If the population standard deviation σ is known, then the p-fractile estimate $x_{p,\text{est}}$ is given as

$$x_{p,\text{est}} = \bar{x} + k_\sigma \sigma$$

If σ is unknown, then

$$x_{p,\text{est}} = \bar{x} + k_s s$$

The coefficients k_σ and k_s depend on the sample size n, on the probability p corresponding to the desired fractile x_p, and, furthermore, on the confidence level γ.

The coefficients k_σ and k_s, derived from the normal and noncentral t-distribution respectively (for further information, see ISO 3207), are given in tables 5 and 6 for the probabilities p = 0,90; 0,95 or 0,99 (upper fractiles) and confidence levels γ = 0,05; 0,10; 0,25; 0,50; 0,75; 0,90 or 0,95. For the probabilities p = 0,10; 0,05; 0,01 (lower fractiles) tables 5 and 6 may be also used; in this case p shall be substituted by $(1-p)$ and the coefficients k_σ and k_s shall be taken with negative signs.

The confidence level γ that the estimate $x_{p,\text{est}}$ will lie on the safe side of the correct value x_p should be greater than 0,50. In order to take into account statistical uncertainty, the value γ = 0,75 is recommended.

6.7 Prediction of fractiles using the Bayesian approach

The Bayesian approach described in 5.3 can be analytically elaborated for a normal variable X and the prior distribution function $\Pi'(\mu,\sigma)$ of μ and σ, given as

$$\Pi'(\mu,\sigma) = C\,\sigma^{-(1+\nu'+\delta(n'))}\exp\left\{-\frac{1}{2\sigma^2}\left[\nu'(s')^2 + n'(\mu - m')^2\right]\right\}$$

where

C is the normalizing constant;

$\delta(n') = 0$ for $n' = 0$;

$\delta(n') = 1$ otherwise;

m', s', n', ν' are parameters asymptotically given as

$E(\mu) = m'$

$E(\sigma) = s'$

$$V(\mu) = \frac{s'}{m'\sqrt{n'}}$$

$$V(\sigma) = \frac{1}{\sqrt{2\nu'}}$$

while the parameters n' and ν' may be chosen arbitrarily. Here $E(.)$ denotes the expectation and $V(.)$ the coefficient of variation of the variable in brackets.

The posterior distribution function $\Pi''(\mu,\sigma)$ of μ and σ is of the same type as the prior distribution function, but with parameters m'', s'', n'' and ν'', given as

$n'' = n' + n$

$\nu'' = \nu' + \nu + \delta(n')$

$m''n'' = n'm' + n\,\bar{x}$

$\nu''(s'')^2 + n''(m'')^2 = \nu'(s')^2 + n'(m')^2 + \nu s^2 + n(\bar{x})^2$

where

$\bar{x}$ and s are the sample mean and standard deviation;

n is the size of the sample;

$\nu = n - 1$.

The predictive value $x_{p,\text{pred}}$ of a fractile x_p is then

$$x_{p,\text{pred}} = m'' + t_p\, s''\sqrt{1 + 1/n''}$$

where t_p is the fractile of the t-distribution (table 3) with ν'' degrees of freedom.

The values for t_p given in table 3 should be therefore taken for $\nu = \nu''$ and appropriate probabilities p, for example for p = 0,90; 0,95 or 0,99 (upper fractiles). For the probabilities p = 0,10; 0,05 or 0,01 (lower fractiles) table 3 may be also used, but p shall be substituted by $(1-p)$ and the t_p values shall be taken with negative signs.

If no prior information is available, then $n' = \nu' = 0$ and the characteristics m'', n'', s'', ν'' equal the sample characteristics $\bar{x}$, n, s, ν. The predictive value of the fractile is then

$$x_{p,\text{pred}} = \bar{x} + t_p\, s\sqrt{1 + 1/n}$$

where t_p again denotes the fractile of the t-distribution (table 3) with ν degrees of freedom.

Furthermore, if the standard deviation σ is known, then $\nu = \infty$ and s shall be replaced by σ.

7 Sampling inspection

7.1 Variables and attributes

Two basic methods of sampling inspection are applicable for quality control in building: inspection by variables and inspection by attributes. These methods are (irrespective of the field of application and its specific features) described in ISO 3951 (variables method) and ISO 2859-1 and ISO 2859-2 (attributes method). These standards are, however, primarily devoted to sampling inspection of a sequence of lots, and for sampling inspection of an isolated lot they may be used only under specified restrictions which are not always acceptable to both the producer and consumer. For the important case of an isolated lot, which is frequently encountered in building, the methods of sampling inspection described in this International Standard are recommended.

Inspection by variables assumes that the observed variable may be described (after suitable transformation, if needed) by a normal distribution. This assumption may be verified using various methods given in ISO 5479 (see also 4.4). Furthermore, the procedure of inspection by variables is dependent on whether the population standard deviation is known or not.

When the assumption of a normal distribution is rejected, inspection by attributes could be used. In that case it is only necessary to distinguish conforming and nonconforming units in a lot.

It should be noted, however, that due to several reasons, including economic aspects, the variables method should have priority over the attributes method whenever possible.

7.2 Inspection of an isolated lot

A single sample is taken from an isolated lot and the decision regarding lot acceptance or non acceptance is made in accordance with the sampling plan. It is strongly recommended to check the efficiency of any sampling plan using the operating characteristic curve (OC curve), or at least its two characteristic points, the producer's risk point (PRP) and consumer's risk point (CRP).

The sampling plans recommended in this International Standard are based on the assumption of equal respect for both the producer's as well as the consumer's concerns, assuming that the producer's risk (PR) and consumer's risk (CR) are equal to 5%. The producer's risk quality (PRQ) corresponding to the producer's risk (PR), and the consumer's risk quality (CRQ) corresponding to the consumer's risk (CR), are taken into account simultaneously. This approach guarantees that, when the recommended sampling plans are used, a lot with a given PRQ will not be accepted only with the probability PR and a lot with a given CRQ (> PRQ) will be accepted only with the probability CR.

The following three recommended methods for sampling inspection are described in detail in 7.3 to 7.5:

a) inspection by variables, assuming the lot standard deviation σ is known;

b) inspection by variables, assuming the lot standard deviation σ is unknown;

c) inspection by attributes.

The following PRQ and CRQ values, in percent, are recommended to be specified for sampling inspection in building:

PRQ = 0,15; 0,25; 0,40; 0,65; 1,0; 1,5; 2,5; 4,0

CRQ = 0,65; 1,0; 1,5; 2,5; 4,0; 6,5; 10,0; 15,0

In building, both the lower specification limit L and the upper specification limit U may be considered for the controlled variable X. In many cases, however, only one of these limits is specified. When sampling inspection by variables is applied, the PRQ and CRQ values should therefore be specified individually for both limits L and U.

It should be also specified in advance what shall be done with the lot which is not accepted. For example, the producer and consumer may agree that nonconforming units will be removed or a new inspection under less severe criteria will be performed.

7.3 Sampling inspection by variables: σ known

When sampling inspection by variables of an isolated lot is used and the lot standard deviation σ of the controlled variable X is known, the following input data shall be specified:

a) the lower specification limit L and/or the upper specification limit U;

b) the producer's risk quality (PRQ) and the consumer's risk quality (CRQ) for L and/or U.

Having the above input data, an appropriate sampling plan (i.e. the required sample size n and the acceptance constant k_σ for given PRQ and CRQ values) is to be determined using table 7, which has been derived using a normal distribution.

A sample of n units is taken from the lot and the sample mean $\bar{x}$ is determined using observed values $x_1, x_2, ..., x_n$.

If only the lower specification limit L is given, the lot is accepted when

$$\bar{x} - k_\sigma \sigma \geq L$$

and not accepted when this inequality is not satisfied.

If only the upper specification limit U is given, the lot is accepted when

$$\bar{x} + k_\sigma \sigma \leq U$$

and not accepted when this inequality is not satisfied.

If both limits L and U are given, both inequalities are to be satisfied to accept the lot; if one or both of the above inequalities is violated, the lot is not accepted.

To make a decision concerning population quality, the following simplified procedure may be used for the first assessment: both above inequalities are considered for an arbitrary n using the coefficient k_σ given in table 5 for a specified probability p (usually p = 0,95) of acceptable occurrence of values less than L and/or greater than U and for a chosen confidence level γ (the value γ = 0,75 is recommended). To check the efficiency of this procedure, the operating characteristic curve (OC curve) should be always used.

7.4 Sampling inspection by variables: σ unknown

When sampling inspection by variables of an isolated lot is used and the lot standard deviation σ of the controlled variable X is unknown, the following input data shall be specified:

a) the lower specification limit L and/or the upper specification limit U;

b) the producer's risk quality (PRQ) and consumer's risk quality (CRQ) for L and/or U.

Having the above input data an appropriate sampling plan (i.e. the required sample size n and the acceptance constant k_s for given PRQ and CRQ values) is to be determined using table 8, which has been derived using the noncentral t-distribution.

A sample of n units is taken from the lot and the sample mean $\bar{x}$ and the sample standard deviation s are determined using observed values $x_1, x_2, ..., x_n$.

If only the lower specification limit L is given, the lot is accepted when

$$\bar{x} - k_s\, s \geq L$$

and not accepted when this inequality is not satisfied.

If only the upper specification limit U is given, the lot is accepted when

$$\bar{x} + k_s\, s \leq U$$

and not accepted when this inequality is not satisfied.

If both limits L and U are given, both inequalities are to be satisfied to accept the lot; if one or both of the above inequalities is violated, the lot is not accepted.

To make a decision concerning population quality, the following simplified procedure may be used for the first assessment: both above inequalities are considered for an arbitrary value of n using the coefficient k_s given in table 6 for a specified probability p (usually p = 0,95) of acceptable occurrence of values less than L and/or greater than U and for chosen confidence level γ (the value γ = 0,75 is recommended). To check the efficiency of this procedure, the operating characteristic curve (OC curve) should be always used.

7.5 Sampling inspection by attributes

When sampling inspection by attributes of an isolated lot is used, the following input data are to be specified for the controlled variable:

a) the definition of the conforming and nonconforming unit;

b) the producer's risk quality (PRQ) and the consumer's risk quality (CRQ).

Having the above input data, an appropriate sampling plan (i.e. the required sample size n and the acceptance number Ac for given PRQ and CRQ values) is to be determined using table 9, which has been derived using relevant discrete distributions.

A sample of n units is taken from the lot and the number of nonconforming units z in the sample is determinated. The lot is accepted when

$$z \leq \mathrm{Ac}$$

and not accepted when this inequality is not satisfied.

Table 1 - Fractiles u_p of the standardized normal distribution

p	0,90	0,95	0,975	0,99	0,995
u_p	1,28	1,64	1,96	2,33	2,58

Table 2 - Fractiles χ^2_{p1} and χ^2_{p2} of the χ^2-distribution with ν degrees of freedom

ν	χ^2_{p1}					χ^2_{p2}				
	0,10	0,05	0,025	0,01	0,005	0,90	0,95	0,975	0,99	0,995
3	0,58	0,35	0,22	0,12	0,72	6,25	7,82	9,35	11,35	12,84
4	1,06	0,71	0,48	0,30	0,21	7,78	9,49	11,14	13,28	14,86
5	1,61	1,15	0,83	0,55	0,41	9,24	11,07	12,83	15,09	16,75
6	2,20	1,64	1,24	0,87	0,68	10,65	12,59	14,45	16,81	18,55
7	2,83	2,17	1,69	1,24	0,99	12,02	14,07	16,01	18,48	20,28
8	3,49	2,73	2,18	1,65	1,34	13,36	15,51	17,54	20,09	21,96
9	4,17	3,33	2,70	2,09	1,74	14,68	16,92	19,02	21,67	23,59
10	4,86	3,94	3,25	2,56	2,16	15,99	18,31	20,48	23,21	25,19
12	6,30	5,23	4,40	3,57	3,07	18,55	21,03	23,34	26,22	28,30
14	7,79	6,57	5,63	4,66	4,08	21,06	23,69	26,12	29,14	31,32
16	9,31	7,96	6,91	5,81	5,14	23,54	26,30	28,85	32,00	34,27
18	10,87	9,39	8,23	7,02	6,27	25,99	28,87	31,53	34,87	37,16
20	12,44	10,85	9,59	8,26	7,43	28,41	31,41	34,17	37,57	40,00
22	14,04	12,34	10,98	9,54	8,64	30,81	33,92	36,78	40,29	42,80
24	15,66	13,85	12,40	10,86	9,89	33,20	36,42	39,36	42,98	45,56
26	17,29	15,38	13,84	12,20	11,16	35,56	38,89	41,92	45,64	48,29
28	18,94	16,93	15,31	13,57	12,46	37,92	41,34	44,46	48,28	50,99
30	20,60	18,49	16,79	14,95	13,79	40,26	43,77	46,98	50,89	53,67

Table 3 - Fractiles t_p of the t-distribution with ν degrees of freedom

ν	t_p				
	0,90	0,95	0,975	0,99	0,995
3	1,64	2,35	3,18	4,54	5,84
4	1,53	2,13	2,78	3,75	4,60
5	1,48	2,02	2,57	3,37	4,03
6	1,44	1,94	2,45	3,14	3,71
7	1,42	1,89	2,36	3,00	3,50
8	1,40	1,86	2,31	2,90	3,36
9	1,38	1,83	2,26	2,82	3,25
10	1,37	1,81	2,23	2,76	3,17

ν	t_p				
	0,90	0,95	0,975	0,99	0,995
12	1,36	1,78	2,18	2,68	3,06
14	1,35	1,76	2,14	2,62	2,98
16	1,34	1,75	2,12	2,58	2,92
18	1,33	1,73	2,10	2,55	2,88
20	1,32	1,72	2,09	2,53	2,85
25	1,32	1,71	2,06	2,49	2,79
30	1,31	1,70	2,04	2,46	2,75
∞	1,28	1,64	1,96	2,33	2,58

Table 4 - Fractiles F_p of the F-distribution with ν_1 and ν_2 degrees of freedom for p = 0,95 (upper values) and p = 0,99 (lower values)

ν_2	ν_1								
	3	4	5	6	8	10	20	30	∞
3	9,28 29,46	9,12 28,71	9,01 28,24	8,94 27,91	8,84 27,49	8,79 27,23	8,66 29,69	8,62 26,50	8,53 26,12
4	6,59 16,69	6,39 15,98	6,26 15,52	6,16 15,21	6,04 14,80	5,96 14,55	5,80 14,02	5,75 13,84	5,63 13,46
5	5,41 12,06	5,19 11,39	5,05 10,97	4,95 10,67	4,82 10,29	4,74 10,05	4,56 9,55	4,50 9,38	4,36 9,02
6	4,76 9,78	4,53 9,15	4,39 8,75	4,28 8,47	4,15 8,10	4,06 7,87	3,87 7,40	3,81 7,23	3,67 6,88
7	4,35 8,45	4,12 7,85	3,97 7,46	3,87 7,19	3,73 6,84	3,64 6,62	3,44 6,16	3,38 5,99	3,23 5,65
8	4,07 7,59	3,84 7,01	3,69 6,63	3,58 6,37	3,44 6,03	3,35 5,82	3,15 5,36	3,08 5,20	2,93 4,86
9	3,86 6,99	3,63 6,42	3,48 6,06	3,37 5,80	3,23 5,47	3,14 5,26	2,94 4,81	2,86 4,65	2,71 4,31
10	3,71 6,55	3,48 5,99	3,33 5,64	3,22 5,39	3,07 5,06	2,98 4,85	2,77 4,41	2,70 4,25	2,54 3,91
12	3,49 5,95	3,26 5,41	3,11 5,06	3,00 4,82	2,85 4,50	2,75 4,30	2,54 3,86	2,47 3,70	2,30 3,36
14	3,34 5,56	3,11 5,04	2,96 4,69	2,85 4,46	2,70 4,14	2,60 3,94	2,39 3,51	2,31 3,35	2,13 3,00
16	3,24 5,29	3,01 4,77	2,85 4,44	2,74 4,20	2,59 3,89	2,49 3,69	2,28 3,26	2,19 3,10	2,01 2,75
18	3,16 5,09	2,93 4,58	2,77 4,25	2,66 4,01	2,51 3,71	2,41 3,51	2,19 3,08	2,11 2,92	1,96 2,57
20	3,10 4,94	2,87 4,43	2,71 4,10	2,60 3,87	2,45 3,56	2,35 3,37	2,12 2,94	2,04 2,78	1,84 2,42
30	2,92 4,51	2,69 4,02	2,53 3,70	2,42 3,47	2,27 3,17	2,16 2,98	1,93 2,55	1,84 2,39	1,62 2,01
40	2,84 4,31	2,61 3,83	2,45 3,51	2,34 3,29	2,18 2,99	2,08 2,80	1,84 2,37	1,74 2,20	1,51 1,80
50	2,79 4,20	2,56 3,72	2,40 3,41	2,29 3,19	2,13 2,89	2,03 2,70	1,78 2,27	1,69 2,10	1,44 1,68
100	2,70 3,98	2,46 3,51	2,31 3,21	2,19 2,99	2,03 2,69	1,93 2,50	1,68 2,07	1,57 1,89	1,28 1,43
∞	2,60 3,78	2,37 3,32	2,21 3,02	2,10 2,80	1,94 2,51	1,83 2,32	1,57 1,88	1,46 1,70	1,00 1,00

Table 5 - Coefficients k_σ for estimation of fractiles when the population standard deviation σ is known

n	$\gamma = 0,05$			$\gamma = 0,10$			$\gamma = 0,25$			$\gamma = 0,50$		
	$p = 0,90$	$p = 0,95$	$p = 0,99$	$p = 0,90$	$p = 0,95$	$p = 0,99$	$p = 0,90$	$p = 0,95$	$p = 0,99$	$p = 0,90$	$p = 0,95$	$p = 0,99$
3	0,33	0,70	1,30	0,54	0,91	1,59	0,89	1,26	1,94	1,28	1,64	2,33
4	0,46	0,82	1,50	0,64	1,00	1,69	0,94	1,31	1,99	1,28	1,64	2,33
5	0,55	0,91	1,59	0,71	1,07	1,75	0,98	1,34	2,02	1,28	1,64	2,33
6	0,61	0,97	1,65	0,76	1,12	1,80	1,01	1,37	2,05	1,28	1,64	2,33
7	0,66	1,02	1,70	0,80	1,16	1,84	1,03	1,39	2,07	1,28	1,64	2,33
8	0,70	1,06	1,74	0,83	1,19	1,87	1,04	1,41	2,09	1,28	1,64	2,33
9	0,73	1,10	1,70	0,85	1,22	1,90	1,06	1,42	2,10	1,28	1,64	2,33
10	0,76	1,12	1,01	0,90	1,24	1,92	1,07	1,43	2,11	1,28	1,64	2,33
12	0,81	1,17	1,05	0,91	1,27	1,96	1,09	1,45	2,13	1,28	1,64	2,33
14	0,84	1,20	1,09	0,94	1,30	1,98	1,10	1,46	2,15	1,28	1,64	2,33
16	0,87	1,23	1,92	0,96	1,32	2,00	1,11	1,48	2,16	1,28	1,64	2,33
18	0,89	1,26	1,94	0,98	1,34	2,02	1,12	1,49	2,17	1,28	1,64	2,33
20	0,91	1,28	1,96	1,00	1,36	2,04	1,13	1,58	2,18	1,28	1,64	2,33
25	0,95	1,32	2,00	1,03	1,39	2,07	1,15	1,51	2,19	1,28	1,64	2,33
30	0,90	1,34	2,03	1,05	1,41	2,09	1,16	1,52	2,20	1,28	1,64	2,33
40	1,02	1,39	2,07	1,08	1,44	2,12	1,17	1,54	2,22	1,28	1,64	2,33
50	1,05	1,41	2,09	1,10	1,46	2,15	1,19	1,55	2,23	1,28	1,64	2,33
100	1,12	1,40	2,16	1,15	1,52	2,20	1,21	1,58	2,26	1,28	1,64	2,33

n	$\gamma = 0,75$			$\gamma = 0,90$			$\gamma = 0,95$		
	$p = 0,90$	$p = 0,95$	$p = 0,99$	$p = 0,90$	$p = 0,95$	$p = 0,99$	$p = 0,90$	$p = 0,95$	$p = 0,99$
3	1,67	2,03	2,72	2,02	2,39	3,07	2,23	2,60	3,28
4	1,62	1,90	2,66	1,92	2,29	2,97	2,11	2,47	3,15
5	1,58	1,95	2,63	1,86	2,22	2,90	2,02	2,38	3,06
6	1,56	1,92	2,60	1,81	2,17	2,85	1,95	2,32	3,00
7	1,54	1,90	2,58	1,77	2,13	2,81	1,90	2,27	2,95
8	1,52	1,88	2,56	1,74	2,10	2,78	1,86	2,23	2,91
9	1,51	1,87	2,55	1,71	2,07	2,75	1,83	2,19	2,87
10	1,50	1,86	2,54	1,69	2,05	2,73	1,80	2,17	2,85
12	1,48	1,84	2,52	1,65	2,02	2,70	1,76	2,12	2,80
14	1,46	1,83	2,51	1,63	1,99	2,67	1,72	2,09	2,77
16	1,45	1,81	2,50	1,60	1,97	2,65	1,69	2,06	2,74
18	1,44	1,80	2,49	1,58	1,95	2,63	1,67	2,03	2,71
20	1,43	1,79	2,48	1,57	1,93	2,61	1,63	2,01	2,69
25	1,41	1,78	2,46	1,54	1,90	2,58	1,61	1,97	2,66
30	1,40	1,77	2,45	1,52	1,88	2,56	1,58	1,95	2,63
40	1,39	1,75	2,43	1,49	1,85	2,53	1,54	1,91	2,59
50	1,30	1,74	2,42	1,46	1,83	2,51	1,52	1,88	2,56
100	1,35	1,71	2,39	1,41	1,77	2,45	1,45	1,81	2,46

Table 6 - Coefficients k_s for estimation of fractiles when the population standard deviation σ is unknown

n	γ = 0,05			γ = 0,10			γ = 0,25			γ = 0,50		
	p = 0,90	p = 0,95	p = 0,99	p = 0,90	p = 0,95	p = 0,99	p = 0,90	p = 0,95	p = 0,99	p = 0,90	p = 0,95	p = 0,99
3	0,33	0,64	1,13	0,53	0,84	1,36	0,91	1,25	1,87	1,50	1,94	2,76
4	0,44	0,74	1,25	0,62	0,92	1,45	0,94	1,28	1,90	1,42	1,83	2,60
5	0,52	0,82	1,33	0,68	0,98	1,52	0,97	1,30	1,92	1,38	1,78	2,53
6	0,58	0,87	1,40	0,72	1,03	1,58	0,99	1,33	1,94	1,36	1,75	2,48
7	0,62	0,92	1,45	0,75	1,05	1,62	1,01	1,34	1,96	1,35	1,73	2,46
8	0,65	0,96	1,49	0,78	1,10	1,66	1,02	1,36	1,98	1,34	1,72	2,44
9	0,69	0,99	1,53	0,81	1,12	1,69	1,03	1,37	1,99	1,33	1,71	2,42
10	0,71	1,02	1,56	0,83	1,14	1,71	1,04	1,38	2,01	1,32	1,70	2,41
12	0,75	1,06	1,62	0,86	1,19	1,76	1,06	1,40	2,03	1,32	1,69	2,39
14	0,79	1,10	1,66	0,89	1,21	1,79	1,07	1,42	2,05	1,31	1,68	2,38
16	0,82	1,13	1,69	0,91	1,23	1,82	1,09	1,43	2,06	1,31	1,68	2,38
18	0,84	1,15	1,72	0,93	1,25	1,85	1,10	1,44	2,07	1,30	1,67	2,37
20	0,86	1,17	1,75	0,95	1,27	1,87	1,11	1,45	2,08	1,30	1,67	2,37
25	0,90	1,22	1,80	0,98	1,30	1,91	1,12	1,46	2,11	1,30	1,66	2,36
30	0,93	1,25	1,84	1,00	1,33	1,94	1,13	1,48	2,12	1,29	1,66	2,35
40	0,97	1,30	1,90	1,03	1,37	1,99	1,15	1,50	2,15	1,29	1,66	2,35
50	1,00	1,33	1,94	1,06	1,39	2,02	1,16	1,51	2,16	1,29	1,65	2,34
100	1,07	1,41	2,04	1,12	1,46	2,10	1,19	1,54	2,20	1,28	1,64	2,33

n	γ = 0,75			γ = 0,90			γ = 0,95		
	p = 0,90	p = 0,95	p = 0,99	p = 0,90	p = 0,95	p = 0,99	p = 0,90	p = 0,95	p = 0,99
3	2,50	3,15	4,40	4,26	5,31	7,34	6,16	7,66	10,55
4	2,13	2,68	3,73	3,19	3,96	5,44	4,16	5,14	7,04
5	1,96	2,46	3,53	2,74	3,40	4,67	3,41	4,20	5,74
6	1,86	2,34	3,24	2,49	3,09	4,24	3,01	3,71	5,06
7	1,79	2,25	3,13	2,33	2,89	3,97	2,76	3,40	4,64
8	1,74	2,19	3,04	2,22	2,75	3,78	2,58	3,19	4,35
9	1,70	2,14	2,98	2,13	2,65	3,64	2,45	3,03	4,14
10	1,67	2,10	2,93	2,07	2,57	3,53	2,36	2,91	3,98
12	1,62	2,05	2,85	1,97	2,45	3,37	2,21	2,74	3,75
14	1,59	2,00	2,80	1,90	2,36	3,26	2,11	2,61	3,59
16	1,57	1,98	2,76	1,84	2,30	3,17	2,03	2,52	3,46
18	1,55	1,95	2,72	1,80	2,25	3,11	1,97	2,45	3,37
20	1,53	1,93	2,70	1,77	2,21	3,05	1,93	2,40	3,30
25	1,50	1,90	2,65	1,70	2,13	2,95	1,84	2,29	3,16
30	1,47	1,87	2,61	1,66	2,08	2,88	1,78	2,22	3,06
40	1,44	1,83	2,57	1,60	2,01	2,79	1,70	2,13	2,94
50	1,43	1,81	2,54	1,56	1,97	2,74	1,65	2,07	2,86
100	1,38	1,76	2,46	1,47	1,86	2,60	1,53	1,93	2,68