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**Information technology — Font  
information interchange —**

Part 3:

**Glyph shape representation**

**AMENDMENT 2: Additional shape  
representation technology for Open Font  
Format**

*Technologies de l'information — Échange d'informations sur les  
fontes —*

*Partie 3: Représentation de la forme de glyphes*

**AMENDEMENT 2: Technologie de représentation de forme  
additionnelle pour format de fontes ouvert**

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## Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work. In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of the joint technical committee is to prepare International Standards. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO and IEC shall not be held responsible for identifying any or all such patent rights.

Amendment 2 to ISO/IEC 9541-3:2008 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC 34, *Document description and processing languages*.

This Amendment appends an additional glyph shape representation technology for the harmonization of ISO/IEC 9541-3 to Open Font Format (ISO/IEC 14496-22). In Open Font Format file, the scalable glyph shape representations by TrueType instructions or Adobe Type2 CharStrings are possible, but ISO/IEC 14496-22 does not include it. For the harmonization between ISO/IEC 9541 and ISO/IEC 14496-22, this Amendment covers Adobe Type2 CharStrings that is the half of the scalable glyph shape representation in ISO/IEC 14496-22. It is an extended version of ISO/IEC 9541-3 Type 1 glyph shape representation.

The additional interchange format is described by SGML (Standard Generalized Markup Language) conforming to ISO 8879:1986 and ISO 8879:1986/Cor.2:1999 (Annex K: Web SGML Adaptations).

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# Information technology — Font information interchange —

## Part 3: Glyph shape representation

### AMENDMENT 2: Additional shape representation technology for Open Font Format

Page 3, 1.7.2

**Replace the declaration with the following:**

```
<!-- (c) International Organization for Standardization 2004 Permission
to copy in any form is granted for use with conforming WebSGML
systems and applications as defined in ISO 8879:1986(WWW),
provided this notice is included in all copies. -->
<!-- Public document type definition. Typical invocation:
<!DOCTYPE gshapes PUBLIC "ISO 9541-3:1994 AM1:2004 and AM2:2008//DTD Glyph Shapes//EN">
-->

<!-- GLYPHSHAPES -->
<!ELEMENT gshapes (t1shapes | t2shapes | ot3shapes | niprop)+ >

<!-- Type 1 shape information. Typical invocation:
<!DOCTYPE t1shapes PUBLIC "ISO 9541-3:1994//DTD Type 1 Glyph Shapes//EN">
-->

<!-- Type 2 shape information. Typical invocation:
<!DOCTYPE t2shapes PUBLIC "ISO 9541-3:1994 AM1:2004//DTD Type 2 Glyph Shapes//EN">
-->

<!-- Open Type 3 shape information. Typical invocation:
<!DOCTYPE ot3shapes PUBLIC "ISO 9541-3:1994 AM2:2008//DTD Open Type 3 Glyph Shapes//EN">
-->
```

Add the following after Section 3:

## Section 4: Open Type 3 glyph shape representation

### 4.1 Scope

This section specifies the architecture and interchange format of one standard Glyph Shape Representation: ISO/IEC 9541 Standard OPEN TYPE 3. The Open Type 3 glyph shape representation is designed for the harmonization to the glyph shape representation used in “CFF” table in Open Font Format (ISO/IEC 14496-22). It is an extended version of ISO/IEC 9541-3 Type 1 glyph shape representation.

### 4.2 Definitions

This section uses the terms defined by 2.2. Extra terms in the following definitions are specific to this section.

### 4.3 Overview of Open Type 3 glyph shape representation architecture

#### 4.3.1 Difference from Type 1 glyph shape representation

The original ISO/IEC 9541-3 Type 1 glyph shape representation used graphical drawing operators that take single set of operands. For example, relative lineto operator (`rlineto`, described in 2.7.3.2.5) takes 2 number object operands of `dx` and `dy` that specifies relative displacements. To compress the glyph procedures, the Open Type 3 glyph shape representation enhances Type 1 glyph procedure operators to take multiple set of operands. The relative lineto operator in the Open Type 3 glyph shape representation recognizes the operand list as an array of pairs of `dx` and `dy`. By this enhancement, the glyph procedure repeating `rlineto` operator can be compressed to multiple sets of relative displacements and single `rlineto` operator. By the operator to be interpreted and the length of the operand list, the glyph procedure interpreter dynamically determines how many sets of operands are taken from the tail of operand list.

#### 4.3.2 Extended virtual machine

The interpretation of Open Type 3 glyph procedure is modelled by the virtual machine that is described in 2.7.1. To illustrate the interpretation of Open Type 3 glyph shape representation, “a transient array” to store any objects is introduced in state variables (described in 2.7.1.4). The Open Type 3 glyph procedure has no operators to allocate, free, initialize the transient array explicitly, it must be allocated dynamically or pre-allocated before the interpretation. The size of the transient array must at least 32 elements, although individual implementations may have longer array. As other state variables, the entries of the transient array are persistent only during the interpretation of each glyph procedures. The transient array has no default values. Therefore, it is possible that individual implementation resets all entries to number 0, or to random number, or keeps the objects stored in previous interpretation.

### 4.4 Open Type 3 glyph shape representation

By the comparison with original Type 1 glyph shape representation in the section 2, the Open Type 3 glyph procedure is classified into 4 groups.

- Original Type 1 glyph procedure operators that are not modified from the definition in section 2
- Enhanced Type 1 glyph procedure operators that the syntaxes are enhanced for Open Type 3
- Additional operators that are not defined in original Type 1 glyph shape representation
- Obsolete operators that described in section 2 but deprecated in Open Type 3

Some operators in the Open Type 3 glyph procedure take the multiple sets of the operands, and the number how many sets are taken is calculated dynamically from the length of the operand list when the operator is interpreted. For the description of such operators' syntax, following notation is used for enhanced operators. Other notations follow to the conventions defined in 2.7.3.

**Table 3 — Symbols preceding glyph procedure notation for Open Type 3 glyph shape representation**

operand-list notation	Meaning
{...}	indicates grouping for set of operands
?	takes zero or one operand or set of operands if available in the operand list.
+	takes the array of operand or set of operands as many as available from the operand list.

#### **4.4.1 Unchanged Type 1 glyph procedure operators**

The following operators are same as original Type 1 glyph procedures.

##### **4.4.1.1 Unchanged operators for starting and finishing**

###### **4.4.1.1.1 Reference point and escapement (rpe)**

This operator is same as that described in 2.7.3.1.1.

###### **4.4.1.1.2 Horizontal reference point and escapement (xrpe)**

This operator is same as that described in 2.7.3.1.2.

###### **4.4.1.1.3 End glyph (endglyph)**

This operator is same as that described in 2.7.3.1.3.

###### **4.4.1.1.4 Standard indexed accented glyph (siag)**

This operator is same as that described in 2.7.3.1.4.

##### **4.4.1.2 Unchanged path construction operators**

For the following graphical operators, the expected result by giving multiple sets of operands is identical to the result by the final set of operands only. The detailed behaviours of the operators are described in 2.7.3.

###### **4.4.1.2.1 Closepath (closepath)**

This operator is same as that described in 2.7.3.2.1.

###### **4.4.1.2.2 Horizontal moveto (hmoveto)**

This operator is same as that described in 2.7.3.2.3.

###### **4.4.1.2.3 Relative moveto (rmoveto)**

This operator is same as that described in 2.7.3.2.6.

###### **4.4.1.2.4 Vertical moveto (vmoveto)**

This operator is same as that described in 2.7.3.2.10.

#### 4.4.1.2.5 Set current point (setcurrentpoint)

This operator is same as that described in 2.7.3.2.11.

#### 4.4.1.3 Unchanged arithmetic operator

For the following arithmetic operator, giving multiple sets of operators is impossible because the result is stacked per each set.

##### 4.4.1.3.1 Divide (div)

This operator is same as that described in 2.7.3.4.1.

#### 4.4.1.4 Unchanged subroutine and subroutine-related operators

The following operators switch the sequence of glyph procedure tokens to be interpreted. Therefore it is impossible to take multiple sets of operands.

##### 4.4.1.4.1 Call subroutine (callsubr)

This operator is same as that described in 2.7.3.5.1.

##### 4.4.1.4.2 Return (return)

This operator is same as that described in 2.7.3.5.2.

##### 4.4.1.4.3 Call utility subroutine (callutilsubr)

This operator is same as that described in 2.7.3.5.3.

#### 4.4.2 Enhanced Type 1 glyph procedure operators

The following operators are enhanced to take multiple set of operands.

##### 4.4.2.1 Enhanced path construction operators

The following operators are designed to draw a line or curve between the current point memorized by interpreter and the parameters given by the set of operands. The interpretation updates the current point in the interpreter. In the description of glyph outline, they are most frequently used. Collecting all control point and omitting similar operator can reduce the size of the procedure. In most enhancements, the operators are enhanced to draw the zig-zag kinked line.

##### 4.4.2.1.1 Horizontal lineto (hlineto)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.2.3. This operator in Open Type 3 glyph shape representation is interpreted by most appropriate syntax in following syntaxes:

\* dx1 dy2 ... dxN **hlineto** (00/06) \*

\* dx1 dy2 ... dxN dy(N+1) **hlineto** (00/06) \*

and appends the alternating horizontal and vertical line from the current point. The first line is horizontal and the second line is vertical (specified by only *dy1*).



#### 4.4.2.1.2 Horizontal-vertical curveto (hvcurveto)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.2.4. This operator in Open Type 3 glyph shape representation is interpreted by most appropriate syntax in following syntaxes:

```
* dx1 dx2 dy2 dy3 dx3? hvcurveto (01/15) *
```

```
* dx1 dx2 dy2 dy3 {dya dxb dyb dxc dx dxe dye dyf}+ dx f? hvcurveto (01/15) *
```

```
* {dxa dxb dxb dyc dyd dxe dye dx f}+ dy f? hvcurveto (01/15) *
```

and appends one or more Bézier curves to the current point. The specification of a Bézier curve from currentpoint requires 6 operands. They are  $x_1$ ,  $y_1$ ,  $x_2$ ,  $y_2$ ,  $x_3$ ,  $y_3$  in figure 6,  $x_0$  and  $y_0$  are given by currentpoint. In the first and second syntax of this operator, the tangent of beginning of first Bézier curve must be horizontal ( $y_1=y_0$ ), and that of ending of first Bézier curve must be vertical ( $x_3=x_2$ ). By this restriction, the number of operands for the first Bézier curve is reduced to 4 ( $dx1$ ,  $dx2$ ,  $dy2$ ,  $dy3$ ). The restriction of beginning and ending tangents are alternating. The second Bézier curve must start with vertical tangent and finish with horizontal tangent specified by 4 operands ( $dya$ ,  $dxb$ ,  $dyb$ ,  $dxc$ ), because the first Bézier curve finishes with vertical tangent. The ending tangent of the final Bézier curve is not restricted. In basic syntax, although the final Bézier curve is specified by 4 operands ( $dx d$ ,  $dxe$ ,  $dye$ ,  $dy f$  in the first and second syntax, or  $dy d$ ,  $dxe$ ,  $dye$ ,  $dx f$  in the third syntax), extra operand ( $dx f$  in the first and second syntax, or  $dy f$  in the third syntax) makes sloping end of final Bézier curve. The standard order of 2 operands is x and y to specify a point, but the order of extra operand is non-standard y and x order in the first and second syntax.

#### 4.4.2.1.3 Relative lineto (rlineto)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.2.5.

```
* {dxa dya}+ rlineto (00/05) *
```

This operator appends one or more lines to the current point. Each set of 2 operands is interpreted by original **rlineto** operator syntax.

#### 4.4.2.1.4 Relative-relative lineto (rrcurveto)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.2.7.

```
* {dxa dya dxb dyb dxc dyc}+ rrcurveto (00/08) *
```

This operator in Open Type 3 glyph shape representation appends one or more Bézier curves to the current point. Each set of 6 operands is interpreted by original **rrcurveto** operator syntax described in 2.7.3.2.7.

#### 4.4.2.1.5 Vertical-horizontal curveto (vhcurveto)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.2.8. This operator in Open Type 3 glyph shape representation is interpreted by most appropriate syntax in following syntaxes:

```
* dy1 dx2 dy2 dx3 dy f? vhcurveto (01/14) *
```

```
* dy1 dx2 dy2 dx3 {dxa dxb dyb dyc dyd dxe dye dx f}+ dy f? vhcurveto (01/14) *
```

```
* {dya dxb dyb dxc dx dxe dye dy f}+ dx f? vhcurveto (01/14)
```

and appends one or more Bézier curves to the current point. The syntax is same as enhanced **hvcurveto** except x, y coordinates are exchanged for initial and final curves.

#### 4.4.2.1.6 Vertical lineto (vlineto)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.2.9. This operator in Open Type 3 glyph shape representation is interpreted by most appropriate syntax in following syntaxes:

```
* dy1 dx2 ... dyN vlineto (00/07) *
```

```
* dy1 dx2 ... dyN dx(N+1) vlineto (00/07) *
```

and appends the alternating vertical and horizontal line from the current point. The first line is vertical and the second line is horizontal (specified by only *dx1*).

#### 4.4.2.2 Enhanced hint operators

In Type 1 glyph shape representation, the following operators take two operands that specifies the zone to apply the hinting parameter. To set hinting parameters to parallel lines, the following operators are enhanced to take multiple sets of operands.

##### 4.4.2.2.1 Horizontal stem (hstem)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.3.2. This operator in Open Type 3 glyph shape representation is interpreted by most appropriate syntax in following syntaxes:

```
* y dy hstem (00/01) *
```

```
* y dy {dya dyb}+ hstem (00/01) *
```

and specifies one or more horizontal stem hints. The initial set of 2 operands specifies the zone from *y* to *y+dy* as the original syntax described in 2.7.3.3.2. Following operands are interpreted as relative values to preceding zone. For example, the second set of 2 operands *dya dyb* are interpreted to specify the zone from *y+dy+dya* to *y+dy+dya+dyb*. This syntax is different from **hstem3** described in 2.7.3.3.3 that specifies all zones by a set of absolute height and relative height.

##### 4.4.2.2.2 Vertical stem (vstem)

The original syntax of this operator in Type 1 glyph shape representation is described in 2.7.3.3.4. This operator in Open Type 3 glyph shape representation is interpreted by most appropriate syntax in following syntaxes:

```
* x dx vstem (00/03) *
```

```
* x dx {dxa dxb}+ vstem (00/03) *
```

and specifies one or more vertical stem hints. The interpretation is same as **hstem** except of a point the operands are *x* coordinate values for vertical hints.

#### 4.4.3 Additional glyph procedure operators

The following operators are introduced in Open Type 3 glyph shape description.

##### 4.4.3.1 Additional path construction operators

###### 4.4.3.1.1 Flex (flex)

\* dx1 dy1 dx2 dy2 dx3 dy3 dx4 dy4 dx5 dy5 dx6 dy6 fd **flex** (00/12 02/03) \*

This operator appends a flex line (described in 2.8.3) consists of 2 Bézier curves, from current point (the position of current point is described by x0, y0). 2 Bézier curves are joint so 12 operands are required to specify 2 Bézier curves, and an operand is used to specify the flex depth.

###### 4.4.3.1.2 Flex 1 (flex1)

\* dx1 dy1 dx2 dy2 dx3 dy3 dx4 dy4 dx5 dy5 d6 **flex1** (00/12 02/05) \*

This operator appends a flex line (described in 2.8.3) consists of 2 Bézier curves, from current point (the position of current point is described by x0, y0). In comparison with **flex**, the flex depth is fixed to 0.5 (it corresponds to the case  $fd=50$  is given to flex). The interpretation of the final operand d6 is dependent with the geometry of 5 control points. If  $\text{abs}(dx1+dx2+dx3+dx4+dx5) > \text{abs}(dy1+dy2+dy3+dy4+dy5)$ , the ending point of the flex line is defined by  $(x0+d6, y0+dy1+dy2+dy3+dy4+dy5)$ . Otherwise, the ending point of the flex line is defined by  $(x0+dx1+dx2+dx3+dx4+dx5, y0+d6)$ .

###### 4.4.3.1.3 Horizontal flex (hflex)

\* dx1 dx2 dy2 dx3 dx4 dx5 dx6 **hflex** (00/12 02/02) \*

This operator appends a flex line (described in 2.8.3) consists of 2 Bézier curves from current point. In comparison with **flex**, the heights of the beginning and ending points of the flex must be same ( $dy6=0$ ), and the tangents at beginning, ending and joining point must be horizontal ( $dy2=dy3=dy4, dy1=dy5=0$ ). By these restrictions, the number of operands to specify 2 Bézier curves is reduced to 6. The flex depth is fixed to 0.5 (it corresponds to the case that  $fd=50$  is given to **flex**).

###### 4.4.3.1.4 Horizontal Flex 1 (hflex1)

\* dx1 dy1 dx2 dy2 dx3 dx4 dx5 dy5 dx6 **hflex1** (00/12 02/04) \*

This operator appends a flex line (described in 2.8.3) consists of 2 Bézier curves from current point. In comparison with **flex**, the heights of the beginning and ending points of the flex must be same ( $dy6=0$ ) and the tangents at joining point must be horizontal ( $dy2=dy3=dy4$ ). By these restrictions, the number of operands to specify 2 Bézier curves is reduced to 9. The flex depth is fixed to 0.5 (it corresponds to the case that  $fd=50$  is given to flex).

###### 4.4.3.1.5 Horizontal-horizontal curveto (hhcurveto)

\* dy1? {dxa dxb dyb dxc}+ **hhcurveto** (01/11) \*

This operator appends one or more Bézier curves from current point. Except of the first Bézier curve, beginning and ending tangents of all Bézier curves must be horizontal to specify a Bézier curve by only 4 operands. If extra operand (dy1) is given, the beginning tangent of first Bézier curve is slanted.

#### 4.4.3.1.6 Relative curveto-lineto (**rcurveline**)

\* {dxa dya dxb dyb dxc dyc}+ dxd dyd **rcurveline** (01/08) \*

The preceding sets of 6 operands are interpreted by the syntax for enhanced **rrcurveto** operator described in 4.4.2.1.4. The final pair of operands is interpreted by the syntax for **rlineto** described in 2.7.3.2.5.

#### 4.4.3.1.7 Vertical-vertical curveto (**vvcurveto**)

\* dx1? {dya dxb dyb dyc}+ **vvcurveto** (01/10) \*

This operator appends one or more Bézier curves from current point. Except of the first Bézier curve, beginning and ending tangents of all Bézier curves must be vertical to specify a Bézier curve by only 4 operands. If extra operand (*dx1*) is given, the beginning tangent of first Bézier curve is slanted.

#### 4.4.3.2 Additional hint operators

In Open Type 3 glyph shape description, new hint operators are introduced to define multiple overlapping hint zones and apply a part of hint zones to each stems. The overlapping hint zones are defined by **hstemhm** and **vstemhm** operators. The operator **hintmask** selects non-overlapping hint zones from defined hint zones. Also the priorities of the hint zones can be set by **cntrmask** operator.

##### 4.4.3.2.1 Hintmask (**hintmask**)

This operator is interpreted by most appropriate syntax in following syntaxes:

\* **hintmask** (01/03) bitmask \*

and enable/disable the hint zones (previously declared by **hstemhm** and **vstemhm**) by *bitmask*. The *bitmask* object following to **hintmask** operator is a part of the operator. The length of *bitmask* object must be the minimum octet length to cover all hint zones, by bit per zone. In the *bitmask*, the most significant bit of the first octet enables/disables the first hint zone which is previously declared. If the corresponding bit is set to 1, the hint zone is enabled. Otherwise, the hint zone is ignored. The hint zones must not overlap.

##### 4.4.3.2.2 Counter mask (**cntrmask**)

\* **cntrmask** (01/04) bitmask \*

This operator specifies the priorities of the hint zones (previously declared by **hstem**, **hstemhm**, **vstem** and **vstemhm**) by *bitmask*. The *bitmask* object following to **cntrmask** operator is a part of the operator. The length of *bitmask* object must be the minimum octet length to cover all hint zones, by bit per zone. In the *bitmask*, the most significant bit of the first octet enables/disables the first hint zone which is previously declared. If the corresponding bit is set to 1, preceding **cntrmask** operator specifies the priority of the hint zone. The hint zones specified by the first **cntrmask** has the highest priority. The hint zones specified by the second **cntrmask** have the second priority. The hint zones may overlap.

##### 4.4.3.2.3 Horizontal stem hintmask (**hstemhm**)

This operator is interpreted by most appropriate syntax in following syntaxes:

\* y dy **hstemhm** (01/02) \*

\* y dy {dya dyb}+ **hstemhm** (01/02) \*

and specifies hint zones in the syntax same as enhanced **hstem**. **hstemhm** is used to register the hint zone information to be used by **hintmask** operator.

#### 4.4.3.2.4 Vertical stem hintmask (**vstemhm**)

This operator is interpreted by most appropriate syntax in following syntaxes:

```
* y dy vstemhm (01/07) *
```

```
* y dy {dya dyb}+ vstemhm (01/07) *
```

and specifies hint zones in the syntax same as enhanced **vstem**. **vstemhm** is used to register the hint zone information to be used by **hintmask** operator.

#### 4.4.3.3 Additional arithmetic operators

The operators **drop**, **dup**, **exch**, **index**, **roll** are introduced to manipulate the objects in the operand list. It should be noted that some of following operators are not equal to the operators defined in ISO/IEC 10180:1995 21.1 with same name. For example, **index** operator of Open Type 3 glyph procedure operator is different from **index** operator in ISO/IEC 10180:1995 21.1.13, but it is identical to **copy** operator specified by ISO/IEC 10180:1995 21.1.8.

##### 4.4.3.3.1 Absolute (**abs**)

```
num1 abs (00/12 00/09) num2
```

This operator calculates the absolute value (*num2*) of *num1*.

##### 4.4.3.3.2 Add (**add**)

```
num1 num2 add (00/12 00/10) sum
```

This operator calculates the sum of two numbers (*num1* and *num2*) in operands.

##### 4.4.3.3.3 Drop (**drop**)

```
any drop (00/12 01/02)
```

This operator discards an object on the head of the operand list.

##### 4.4.3.3.4 Duplicate (**dup**)

```
any dup (00/12 01/11) any any
```

This operator duplicates the top object in the operand list.

##### 4.4.3.3.5 Exchange (**exch**)

```
any1 any0 exch (00/12 01/12) any0 any1
```

This operator exchanges the top 2 objects in the operand list.

##### 4.4.3.3.6 Index (**index**)

```
num(N-1) ... num0 i index (00/12 01/13) num(N-1) ... num0 num(i)
```

This operator duplicates the object stored the depth *i* in the operand list and puts it in the head of the operand list.

#### 4.4.3.3.7 Multiply (mul)

`num1 num2 mul (00/12 01/08) product`

This operator calculates the product of *num1* and *num2*. The operand *num1* is pushed on the operand list, followed by *num2*. After execution, the result (*product*) is pushed on the head of the operand list. The operands *num1* and *num2* are of type Number, and the product is of type number. If overflow occurs, the result is undefined. If underflow occurs, the result is zero.

#### 4.4.3.3.8 Negate (neg)

`num1 neg (00/12 00/14) -num1`

This operator puts the negative of *num1* on the operand list.

#### 4.4.3.3.9 Random (random)

`random (00/12 01/07) num`

This operator generates a pseudo random number *num*. The range of *num* is greater than 0 and less than or equal to 1.

#### 4.4.3.3.10 Roll (roll)

`num(N-1) ... num0 N J roll (00/12 01/14) num((J-1) mod N) ... num0 num(N-1) ... num(J mod N)`

This operator performs a circular shift of the elements *num(N-1) ... num0* on the operand list by the amount *J*. Positive *J* indicates upward motion of the stack; negative *J* indicates downward motion. The value *N* must be a non-negative integer, otherwise the result is undefined.

#### 4.4.3.3.11 Square root (sqrt)

`num sqrt (00/12 01/10) square_root`

This operator calculates the square root of *num*. The operand *num* is pushed on the head of the operand list. After execution, the result is pushed on the head of the operand list. The operand *num* is of type Number, and the result (*square\_root*) is of type Number. If *num* is negative number, the result is undefined.

#### 4.4.3.3.12 Subtract (sub)

`num1 num2 sub (00/12 00/11) difference`

This operator calculates the difference that *num2* is subtracted from *num1*.

#### 4.4.3.4 Additional conditional operators

The following operators are conditional operators that create and consume numerical objects on the stack. The non-zero numerical object is recognized as boolean true, and zero is recognized as boolean false. The operators *and*, *or*, and *not* are not bitwise operators.

##### 4.4.3.4.1 And (and)

`num1 num2 and (00/12 00/03) 1_or_0`

This operator puts a 1 on the operand list when both of *num1* and *num2* are not zero. Otherwise, a 0 is put.