STP-PT-022

COMPARISON AND VALIDATION OF CREEP-BUCKLING ANALYSIS METHODS

ASME STANDARDS TECHNOLOGY, LLC

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FOREWORD

This document was developed under a research and development project which resulted from ASME Pressure Technology Codes & Standards (PTCS) committee requests to identify, prioritize, and address technology gaps in current or new PTCS Codes, Standards and Guidelines. This project is one of several included for ASME fiscal year 2008 sponsorship which are intended to establish and maintain the technical relevance of ASME codes & standards products. The specific project related to this document is project 07-11 (BPVC#5), entitled "Comparison and Validation of Creep-Buckling Analysis Methods."

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ABSTRACT

This report provides comparisons of creep-buckling calculations and provides guidance on approximate methods which are feasible for design. This report includes a discussion of the various creep models, presents creep buckling analysis techniques, and provides several comparative example calculations.

The techniques discussed in this report include:

1. Baseline analysis. Finite element creep analysis with different creep models and full non-linear strain-displacement (geometrical) analysis.
2. Critical strain technique. Elastic buckling strain defines the creep buckling strain.
3. Tangent/secant modulus approaches. Combinations of tangent and secant moduli of the isochronous stress-strain curve are used in calculations that reduce to elastic buckling calculations in the elastic case.
4. Use of an isochronous stress-strain curve in a limit/instability analysis of the imperfect structure. An instability (buckling) analysis would be in principle the same as Technique 3, and should generate the same answer. Adding plastic collapse as a failure mode ensures that the yield strength of the structure is not exceeded. This analysis therefore reflects the failure modes which are covered by the baseline technique.

1 INTRODUCTION

This report provides comparisons between approximate and detailed creep-buckling calculations. The objective is to provide guidance on approximate methods which are feasible for design. This requires the efficient calculation of structural strength and time to (buckling) failure, so that calculation of margins between design and failure boundaries does not require multiple trial and error creep calculations. The definition of creep buckling is taken to be wide, including elastic and inelastic instability, bifurcation and acceleration of strain and deflection rates due to non-linear geometrical reduction in structural strength.

The techniques used in this report are:

1. Baseline analysis. Finite element creep analysis with different creep models and full non-linear strain-displacement (geometrical) analysis.
2. Critical strain technique. Elastic buckling strain defines the creep buckling strain. ([1], [2], [3])
3. Tangent/secant modulus approaches. Combinations of tangent and secant moduli of the isochronous stress-strain curve are used in calculations that reduce to elastic buckling calculations in the elastic case. ([4], [5], [6])
4. Use of an isochronous stress-strain curve in a limit/instability analysis of the imperfect structure. An instability (buckling) analysis would be in principle the same as technique 3, and should generate the same answer. Adding plastic collapse as a failure mode ensures that the yield strength of the structure is not exceeded. This analysis therefore reflects the failure modes which are covered by the baseline technique.

Techniques 2 and 3 do not have an explicit treatment of initial imperfection or out-of roundness. For simple structures such as cylinders and spheres, Technique 1 requires an initial imperfection to give a reasonable result. With no defined initial imperfection it may or may not give a result, and if there was a result, it may or may not bear any resemblance to reality. Technique 4 requires the same initial imperfection as 1 to give a reasonable result.

The selection of the initial imperfection is simple for the cases considered in this report. It is the first elastic buckling mode shape with a defined magnitude. For more complex structures, it may be necessary to examine a number of possible imperfection mode shapes, and to base the strength prediction on the mode which gives the most conservative result. This is conveniently done by using a range of elastic buckling mode shapes, but other plausible or defined imperfection shapes can easily be used.

A 0.5 mm radial imperfection with 100 mm radius corresponds to the ASME definition of 1% maximum acceptable out-of-round. This and 0.1 mm imperfections are considered in this report.

Plasticity is not included. The cases to be analyzed will represent reasonable design conditions in terms of stress, temperature and life. Under these circumstances significant plasticity would not be expected for the simple structures in this report, unless it occurred due to severe distortions late in life. It would be difficult to load these structures so that initial yielding occurred which did not lead to instantaneous elastic-plastic buckling. In this case there is no difference between the technique 4 limit/instability analysis and the Technique 1 baseline analysis. However, plasticity may be readily included in all the analyses if necessary. There is no reason why isochronous stress-strain curves constructed from tests or from full elastic-creep-plasticity properties should present any difficulties over and above those in this report.

Inclusion of plasticity in the full inelastic analysis and in the three approximate methods is not expected to change the conclusions based on the creep models. The ability of the approximate methods to capture time-dependent strength and instability is being tested. Plasticity adds another variable but no extra complexity to the problem.

Any realistic or practical creep-buckling assessment should use full elastic-inelastic isochronous data. This report distinguishes between primary, secondary and tertiary creep only to prove this.

2 CREEP MODELS

2.1 Primary and Secondary Creep

For modeling primary and secondary creep, for convenience with the Abaqus options, the time-hardening power law model for creep strain rate is used.

$$\dot{\varepsilon}_c = A\sigma^n t^m \qquad (1)$$

where $A = 1.26 \times 10^{-15}$

$n = 4.0$

$m = 0$ for secondary creep

stress σ is in MPa

time t is in hours.

This secondary creep law with m = 0 is an approximate model for Grade 22 steel at 515˚C.

To account for primary creep we use the form of the creep model in equation 1, with the following constants.

$A = 1.26 \times 10^{-12}$

$n = 4.0$

$m = -0.51$ for primary and secondary creep up to 1×10^6 hours

stress σ is in MPa

time t is in hours.

Figure 1 shows creep strain as function of time for 5 and 20 MPa.

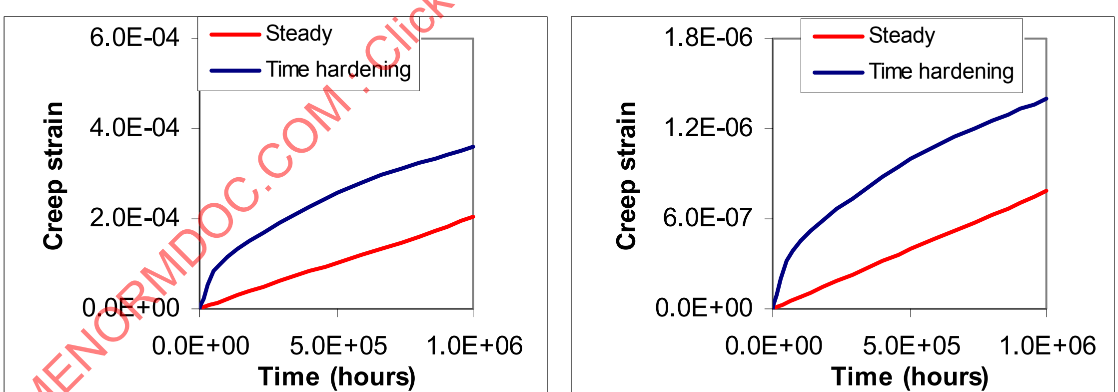

Figure 1 - Creep Strain with Steady (Original) and Time Hardening Models, 5 MPa And 20 MPa Stress, Respectively

2.2 Tertiary Creep

To account for tertiary creep, the "Omega" model in API 579-1/ASME FFS-1 [1] is used in this study. This model for creep gives the classical tertiary creep behavior, with creep strain rate increasing significantly and asymptotically for strains greater than 1/Ω, the Monkman-Grant strain. In this approach, a model (or data) for steady creep is modified as follows.

Creep strain rate is

$$\dot{\varepsilon}_c = \dot{\varepsilon}_{c0} \exp(\Omega \varepsilon_c) \tag{2}$$

where $\dot{\varepsilon}_{c0}$ = secondary creep rate from equation (1), with m = 0

For constant stress, creep strain at time t is

$$\varepsilon_c = -\frac{1}{\Omega} \ln(1 - \dot{\varepsilon}_{c0} \Omega t) \tag{3}$$

The isochronous secant modulus is

$$E_s = \frac{\sigma}{\sigma / \mathrm{E} + \varepsilon_c} \tag{4}$$

The isochronous tangent modulus defined in API 579-1/ASME FFS-1 is

$$E_t = \left[\frac{\partial \varepsilon_c}{\partial \sigma} \right]^{-1} \tag{5}$$

A definition of tangent modulus that includes elastic strain is

$$E_{t\,\mathrm{mod}} = \left[1 / E + \frac{\partial \varepsilon_c}{\partial \sigma} \right]^{-1} \tag{6}$$

This will be shown to provide a more accurate creep buckling prediction than equation 5.

The Omega parameter is the reciprocal of the Monkman-Grant strain, which is the product of steady state creep strain rate and time-to-rupture. Therefore the Omega parameter and the creep law define rupture time. If creep strains in a structure are less than the Monkman-Grant strain, tertiary creep and rupture should not affect structural behavior. In the cases examined in this report, it was necessary to consider an artificially high Omega (Ω = 2000) to generate a significant effect on buckling time. There may be situations where realistic values of Omega could affect creep buckling, and therefore it is worth having effective creep buckling analysis techniques which are able to deal with tertiary creep.

Figure 2 shows secondary and tertiary creep for constant stress = 42 MPa and Omega = 2000. The tertiary curve is calculated from equation 3.

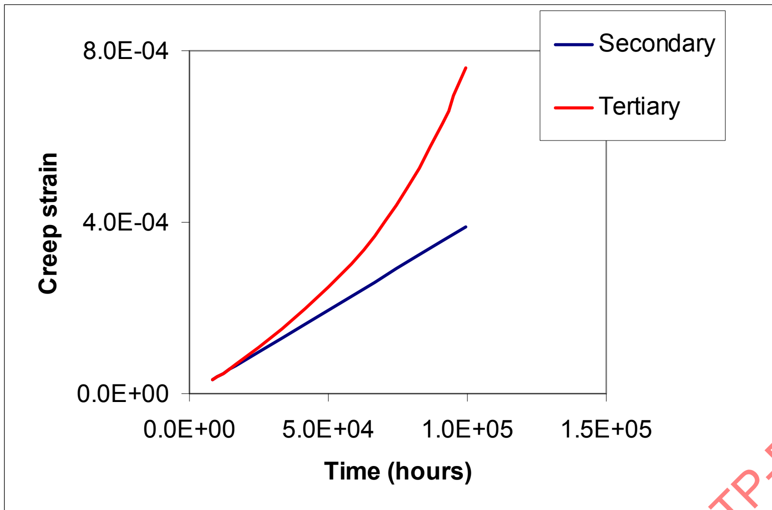

Figure 2 - Creep Strain at Constant Stress for 42 MPa, Omega = 2000, Secondary and Tertiary Creep Models

2.3 Isochronous Stress-Strain Curves

An isochronous curve is a representation of stress-strain data at a particular time. An isochronous curve generated from tests, by default, will include yielding and primary and secondary creep. The zero time isochronous curve is the elastic-plastic stress-strain curve. In this report, isochronous curves are generated from the elastic and creep models used herein. There are distinctions between primary, secondary and tertiary creep. This is because there is some interest in determining whether exclusion of primary or tertiary creep could affect buckling calculations. The only reason to separate the effects in this report is to establish their relative importance for creep buckling. In practice, distinguishing between primary and secondary creep, and plasticity should not be necessary. If the assessment data is presented in terms of particular models as in [1], then care has to be taken that an important effect such as primary creep is not ignored.

The isochronous curves in ASME III NH [7] are limited to typically 2.2% strain. This clearly excludes tertiary creep and rupture. This report uses two approaches to generate isochronous curves including tertiary creep. The constant stress approach is taken from [1]. The constant strain rate approach has been developed to reflect load shedding in a strain-controlled situation [8].

3 CREEP BUCKLING ANALYSIS TECHNIQUES

3.1 Baseline Finite Element Creep Analyses with Initial Imperfections

The analysis ramps up the pressure over 10 hours and then holds it until the analysis fails to converge due to small time increments. For the circular and spherical shell models in this report with radius = 100 mm, the definition of buckling was taken as 5 mm of creep deflection for the 1 mm thick shells and 1 mm of creep deflection for the 5 mm thick shells. This was intended to give similar creep bending strains for the different cases. It will be seen that this gave buckling times close to final collapse times for the slender (high r/t) cases. For the thicker shells, there is a greater difference between this definition and final collapse. There is some benefit in this approach. First, it is conservative. If the approximate calculations were calibrated against final collapse there may be some concern that severe distortions would be more likely than design factors indicate. Second, the modulus and isochronous curve limit calculations work on the basis of an instability which is associated with zero or initial imperfections. Comparing them with the onset of creep-buckling is therefore reasonable. For the cylinder under axial loading and the sphere under pressure, these definitions (5 mm and 1 mm deflections) were not useful for onset of buckling. It was found that initial deflection rates increasing by a factor of 5 was a reasonable estimate. The creep strains in these cases were much higher than the typical external pressure on cylinder cases.

The creep analyses routines have options for convergence criteria and damping. The creep analyses in this report all used a creep strain error tolerance of 0.0001 as required by the Abaqus code. The results are not sensitive to this value. The damping option was not used, since it was not clear when it was affecting the buckling time.

With shell elements, pressures may be applied at the mean radius (center of the element) or at the inner or outer surface. It is expected that there would be a difference in results for thick shells. In all cases in this report, the pressure was applied at the mean radius. This is because the approximate techniques are based on the same assumption. If approximate and accurate analyses are in agreement for pressure applied in this way, then they will be in agreement if pressure is applied on one or the other surface.

The full non-linear creep finite element analyses are not complex for the cases in this report, and so provide a reliable benchmark for comparison with approximate techniques.

3.2 Critical Strain Technique

The earliest reference to this approach appears to be by Gerard [3]. The calculations for the simple cases in this report are taken from API 579-1/ASME FFS-1 [1], which uses the work of Chern [2].

The steps are as follows:

i. Determine elastic buckling load P^{cr}.

ii. Define operating pressure P.

$Q = P^{cr}/P$.

σ = effective stress at load P.

E = modulus at operating temperature.

iii. Critical creep strain is

$$\varepsilon_c^{cr} = (2/3)(1+\nu)(\sigma/E)(Q-1) \qquad (7)$$

iv. Critical time for creep buckling is

$$t^{cr} = \frac{1 - \exp(-\varepsilon_c^{cr}\Omega)}{\dot{\varepsilon}_{c0}\Omega} \tag{8}$$

where Ω = Omega parameter,

$\dot{\varepsilon}_{c0}$ = initial (secondary) creep rate at stress σ.

Note that the values of Omega used in this report are intended to show the sensitivity of creep buckling to the Omega creep model. They do not reflect the material, multiaxility and other parameters defining the multiaxial value of Omega in [1].

v. For the time hardening creep model,

$$t^{cr} = \left[\frac{\varepsilon_c^{cr}(m+1)}{A\sigma^n}\right]^{\frac{1}{m+1}} \tag{9}$$

This calculation does not consider or require initial imperfections.

3.3 Modified Modulus Technique

The idea here is that the key material property for creep instability is the modulus of the isochronous stress-strain curve. Early references to this approach are by Shanley [4] and Gerard [6].

Figure 3 shows isochronous and tangent modulus stress-strain curves for the secondary creep law in Section 2.1, at 50,000 hours.

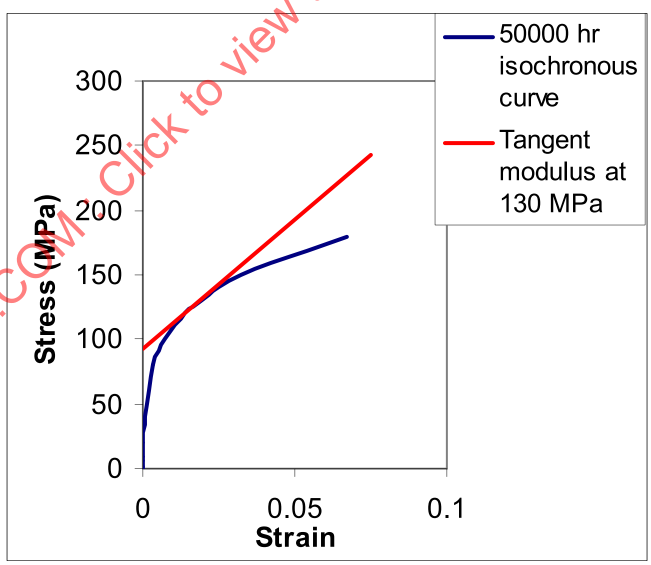

Figure 3 - Isochronous Stress-Strain Curve and Tangent Modulus

Table 1 shows expressions for buckling pressure for different simple geometries as functions of secant (E_s) and tangent (E_t) moduli quoted by Griffin [4].

Table 1 - WRC Calculations of Buckling Stress

Case	Buckling Stress
Cylinder – external pressure	$[E_t/\{4(1- {}^2)\}](t/r)^2$
Sphere – external pressure	$[E_t E_s/\{3(1- {}^2)\}]^{1/2}\, t/r$
Cylinder – axial compression	$[E_t E_s/\{3(1- {}^2)\}]^{1/2}\, t/r$

In Table 1, $\mu = \frac{1}{2} - (\frac{1}{2} - \nu)\, E_s/E$.

This procedure works naturally from the definition of buckling time as a starting point and then proceeds to calculate buckling pressure or stress. It is iterative, but this is easily accomplished with the Excel solver. To start from the definition of pressure and to calculate buckling time can lead to difficulties for cases where the isochronous stress-strain curve is insensitive to time. This can occur for short times.

This calculation also does not consider or require initial imperfections.

3.4 Isochronous Stress-Strain Curve Limit/Instability Analysis

i. Select an initial imperfection and a required buckling time. In this report we use the values from the creep buckling analysis.

ii. Use the buckling time to define isochronous stress-strain data.

iii. Strictly, this should be converted to true-stress, true strain for use with non-linear geometry analysis. This will make a significant difference if solid 3-d or 2-d elements are used and strains are high enough, (> ~ 5%).

iv. Use these data in an elastic-plastic limit/stability analysis of the imperfect structure with non-linear geometry active. The result is the maximum pressure or load the structure can tolerate, which may be compared with the pressure or load used in the creep analysis. Use of non-linear geometry means that both strength and stability (buckling) can define the maximum pressure or load. This makes the analysis different from a conventional limit analysis.

v. It is important to check the maximum stress or plastic strain in limit analysis and to ensure that the stress-plastic strain data used in the analysis includes higher values.

3.5 Effect of Tertiary Creep

The creep finite element calculations for secondary and tertiary creep were performed with a creep user routine, since Abaqus does not have a material model for tertiary creep. The code in this routine uses equations 1 and 2 with m = 0.

This implementation of the Omega model for tertiary creep in the creep finite element calculations was straightforward, although some restriction had to be placed on the maximum time increment.

Equation 3 gives the basis for calculating isochronous stress-strain curves based on constant stress tests. An isochronous curve based on constant strain rate tests may be defined as follows. Equation (2) gives the relation between creep strain, time and secondary creep strain rate:

$$\frac{\varepsilon_c}{t} = \dot{\varepsilon}_{c0} \exp(\Omega \varepsilon_c) \qquad (10)$$

Given a relation between stress and secondary creep rate

$$\sigma = f(\dot{\varepsilon}_{c0}) \qquad (11)$$

then the relation between stress, creep strain and time for constant creep strain rate is

$$\sigma = f(\frac{\varepsilon_c}{t}\exp(-\Omega\varepsilon_c)) \tag{12}$$

Figure 4 shows isochronous stress-strain curves with Omega = 2000, time = 100,000 hours for:

i. Plain secondary creep model.

ii. Omega model at constant stress.

iii. Omega model at constant strain rate.

The rupture stress under these conditions is 44.6 MPa.

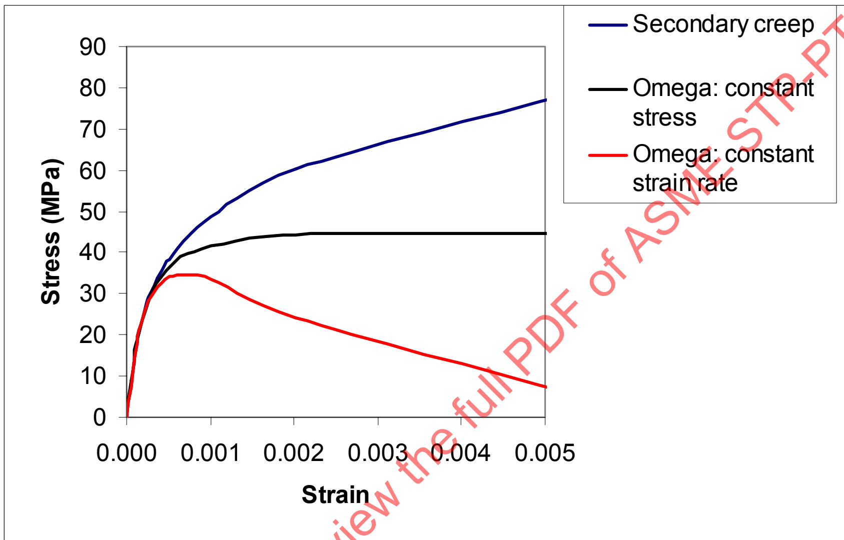

Figure 4 - Isochronous Curves for 100,000 Hours for Secondary Creep and Creep Rupture at 44.6 MPa

The relevance of the fixed strain rate curve is in circumstances which are not completely stress-controlled. This can occur in local constrained regions where damage leads to load shedding.

4 EXAMPLE 1 LONG (2-D) CYLINDER UNDER EXTERNAL PRESSURE

The primary, secondary and tertiary creep models are used to compare finite full creep element calculations with three approximate methods. The buckling formula in Table 1 is:

Buckling stress = $[E_t/\{4(1-\mu^2)\}](t/r)^2$.

For the r =100 mm, t = 1 mm and E = 1.7 x 10^5 MPa, the elastic buckling stresses is 4.67 MPa. The finite element mode 1 elastic buckling stress is 4.67 MPa.

The following analyses were performed.

1. Comparison of solid and shell elements for creep buckling.
2. Secondary creep analyses of R/t = 100 and R/t = 20 cases using second order thick shell reduced integration elements.
3. Spreadsheet analyses for critical strain and tangent modulus buckling calculations.
4. Time-independent limit analyses using isochronous curves.
5. Effect of primary creep.

In all cases, the deflected (buckled) shape is the same as indicated in Figure 5. The analysis model is a quarter of that shown. Boundary conditions represent the constraints associated with an infinitely long cylinder. Figure 6 shows a Section of a solid model used to check agreement with shell analyses.

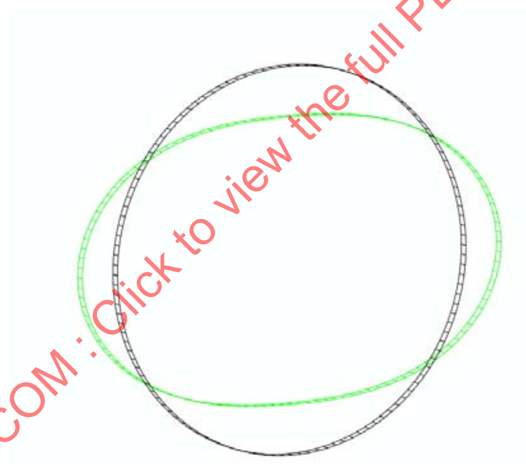

Figure 5 - Undeformed and Buckled Shapes of a Row of Shell Elements

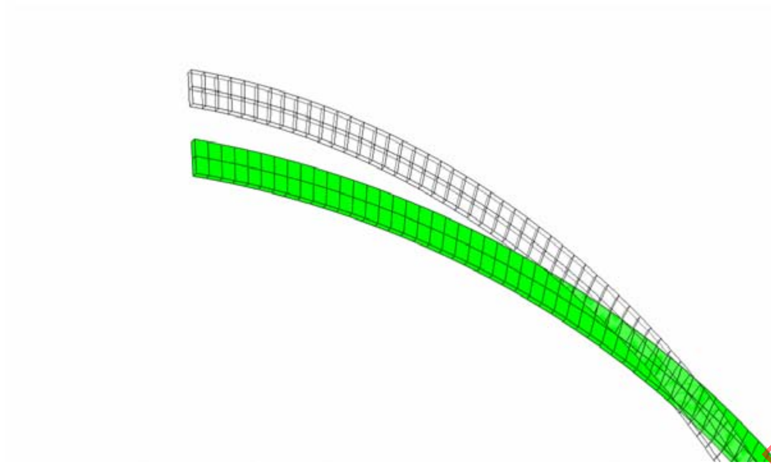

Figure 6 - Deflection in Part of Quarter Model Solid Section for R/T = 20

4.1 Comparison between Shell and Solid Models

Eight load cases were analyzed with the shell and solid models. Table 2 shows reasonable agreement between the shell and solid models. The buckling calculations for all subsequent analyses will be performed with shell models. These early analyses used different creep models from the rest of the work, so the results are not comparable with other results. These analyses confirmed that the S8R thick shell reduced integration elements and the analyses were reliable.

Table 2 - Comparison between Shell and Solid Models for Buckling Times of Cylinders under External Pressure

Units: mm, MPa, hours				Results: Buckling Time			
				Shell Elements		Solid Elements	Ratio: Times
Mean radius	Thickness	Pressure	Imperfection	Onset of Buckling	Final Buckling	Final Buckling	Solid/Shell
100.0	1.0	0.040	0.5	410	425	462	1.1
100.0	1.0	0.040	0.1	3.00E+04	3.10E+04	3.12E+04	1.0
100.0	1.0	0.030	0.5	7.54E+04	8.21E+04	7.19E+04	0.9
100.0	1.0	0.030	0.1	1.63E+06	1.67E+06	1.73E+06	1.0
100.0	5.0	3.500	0.5	2102	2213	2410	1.1
100.0	5.0	3.500	0.1	7500	8000	8325	1.0
100.0	5.0	2.000	0.5	2.85E+05	2.97E+05	3.39E+05	1.1
100.0	5.0	2.000	0.1	6.50E+05	6.80E+05	7.25E+05	1.1

4.2 Finite Element Creep Analyses with Initial Imperfections

Table 3 and Table 4 gives buckling times for eight load cases. Examples of displacement-time curves are given in Figure 7 and Figure 8.

4.3 Critical Strain and Modified Modulus Calculations

These followed the procedure outlined in Sections 3.2 and 3.3 for the secondary creep model with no effect of Omega or time hardening. The modulus calculations used two approaches. The first approach followed the definitions in equation 5, Section 3 and Table 1. The second approach used a tangent modulus definition including elastic strain as in equation 6. Table 4 contains the results.

4.4 Comparison between Shell Secondary Creep Analyses and Time-Independent Isochronous Limit Analyses

The procedure here was:

i. Select a pressure and perform a full creep buckling analysis.

ii. Use the time to onset of buckling to define isochronous stress-strain data.

iii. Use these data in an elastic-plastic limit analysis of the imperfect structure with non-linear geometry active.

iv. The result is a limit pressure which may be compared with the pressure used in the creep analysis.

Table 4 gives results of these analyses in terms of pressure ratios for the eight load cases. In each case the pressure to give creep buckling in the time obtained from the full creep analysis is expressed as a ratio of the pressure in the creep analysis. A value less than 1 is conservative. This would mean that the approximate technique under predicts buckling pressure for a given time.

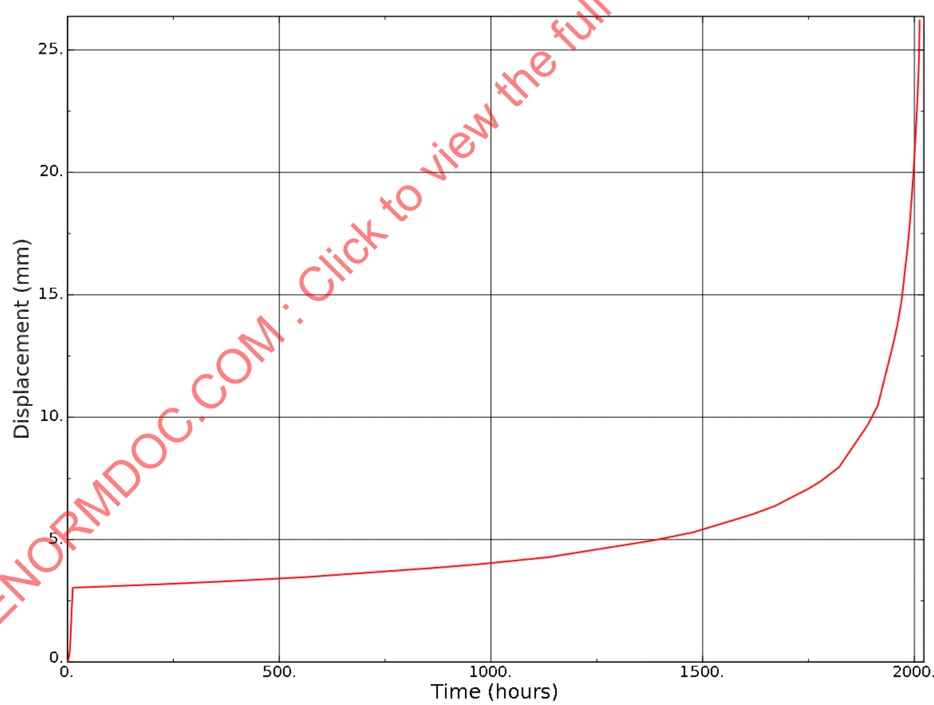

Figure 7 - Displacement-Time Plot for Case 1 in Table 3 and Table 4

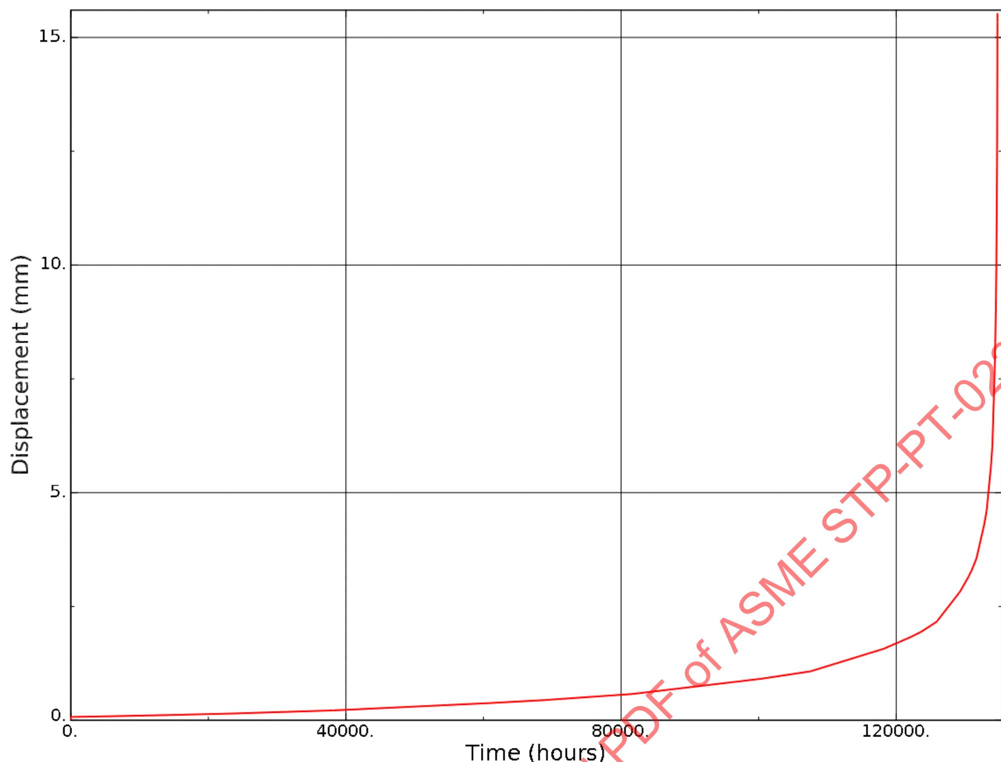

Figure 8 - Displacement-Time Plot for Case 8 in Table 3 and Table 4

Table 3 - Comparison between Shell Model Buckling Times and Critical Strain Buckling Times. Effects of Initial Imperfection are Given.

Units: mm, MPa, hours						
Secondary Creep Model				Buckling Time		Ratio: Times
Mean Radius	Thickness	Pressure	Imperfection	Shell Model	API Critical Strain	Calc/FE Model
100.0	1.0	0.040	0.5	1800	1.63E+07	9031
100.0	1.0	0.040	0.1	205,000	1.63E+07	79
100.0	1.0	0.030	0.5	268,000	1.28E+08	478
100.0	1.0	0.030	0.1	1.34E+07	1.28E+08	9.6
100.0	5.0	3.500	0.5	984	12,074	12.3
100.0	5.0	3.500	0.1	5548	12,074	2.2
100.0	5.0	2.000	0.5	35,570	185,888	5.2
100.0	5.0	2.000	0.1	127,000	185,888	1.5

Table 4 - Comparison between Finite Element Buckling Pressures and Approximate Techniques

Units: mm, MPa, hours						Buckling Pressure Ratios (< 1 is conservative)			
Secondary creep model				Shell Model		Isochronous Limit	Critical Strain	Full Isochr. Modulus	Creep Isochr. Modulus
Mean radius	Thickness	Pressure	Imperfection	Buckling Time	Creep strain	FE Model	FE Model	FE Model	FE Model
100	1	0.04	0.5	1800	2.7E-04	0.96	1.17	1.26	11.6
100	1	0.04	0.1	205,000	1.8E-04	0.94	1.16	1.25	3.6
100	1	0.03	0.5	268,000	5.6E-04	0.88	1.55	1.66	4.4
100	1	0.03	0.1	1.34E+07	3.7E-04	0.86	1.36	1.23	1.7
100	5	3.5	0.5	984	9.5E-04	0.92	1.44	1.29	1.7
100	5	3.5	0.1	5548	7.7E-04	0.88	1.14	0.95	1.1
100	5	2	0.5	35,570	1.12E-03	0.86	1.42	1.13	1.2
100	5	2	0.1	127,000	1.17E-03	0.85	1.09	0.85	0.9

4.5 Effect of Primary Creep

We use the form of the creep model in equation 1, with the following constants.

$A = 1.26 \times 10^{-15}$

$n = 4.0$

$m = -0.51$ for primary and secondary creep up to 1×10^6 hours

stress σ is in MPa

time t is in hours.

Table 5 shows creep buckling times for two cases showing the effect of primary creep. Table 6 shows comparisons of modified modulus and isochronous limit calculations.

Table 5 - Comparison of Steady and Primary Creep Buckling Times

Units: mm, MPa, hours						
Secondary Creep Model				Buckling Time		Ratio: Times
Mean Radius	Thickness	Pressure	Imperfection	Secondary Creep	Primary Creep	Primary/Secondary
100.0	1.0	0.030	0.5	268,000	18,500	0.07
100.0	5.0	2.000	0.1	127,000	6013	0.05

Table 6 - Comparison of Isochronous Limit and Tangent Moduli Calculations

Units: mm, MPa, hours					Buckling Pressure Ratios (< 1 is conservative)			
Secondary Creep Model				Buckling Time	Isochronous Limit	Critical Strain	Full Isochr. Modulus	Creep Isochr. Modulus
Mean Radius	Thickness	Pressure	Imperfection	Primary Creep	FE Model	FE Model	FE Model	FE Model
100	1	0.030	0.500	18,500	0.89	1.56	1.66	4.51
100	5	2.000	0.100	6013	0.80	1.97	0.83	0.88

4.6 Effect of Tertiary Creep

As noted above, we consider the effect of Omega = 2000 on the secondary creep model. Creep and isochronous limit/stability finite element analyses, critical strain and tangent modulus calculations were performed as before. Table 7 and Table 8 are summaries of the results.

Table 7 - Comparison of Finite Element and Isochronous Limit/Instability Analyses for Secondary and Tertiary Creep With Omega = 2000

				Full Creep Finite Element Analysis: Buckling Time and Creep Strain				Load Factors: Lim Pressure/F.E.. Creep Pressure		
Units: mm, MPa, hours				Secondary Creep		Tertiary Creep: Omega = 2000		Isochronous Limit/Instability		
Mean Radius	Thickness	Pressure	Imperfection	Onset of Buckling	Creep Strain	Onset of Buck.	Creep Strain	Sec. Creep	Const. Stress	Const.Strain Rate
100.0	1.0	0.040	0.5	1800	2.7E-04	1740	3.8E-04	0.96	0.96	0.95
100.0	1.0	0.040	0.1	204,000	1.8E-04	197,000	2.54E-04	0.94	0.94	0.94
100.0	1.0	0.030	0.5	268,000	5.6E-04	259,000	6.10E-04	0.88	0.88	0.89
100.0	1.0	0.030	0.1	1.3E+07	3.7E-04	1.3E+07	8.40E-04	0.86	0.86	0.86
100.0	5.0	3.500	0.5	984	9.5E-04	674	1.13E-03	0.92	0.92	0.87
100.0	5.0	3.500	0.1	5548	7.7E-04	4207	9.7E-04	0.88	0.88	0.84
100.0	5.0	2.000	0.5	35,570	1.12E-03	22570	1.29E-03	0.86	0.88	0.80
100.0	5.0	2.000	0.1	127,000	1.17E-03	79570	1.34E-03	0.85	0.85	0.79

Table 8 - Comparison of Finite Element, Constant Stress Isochronous Limit/Instability, Critical Strain and Tangent Modulus Predictions for Creep Rupture Model with Omega = 2000

Units: mm, MPa, hours					Buckling Pressure Ratios (< 1 is Conservative)			
Tertiary Creep Model				Buckling Time	Isochronous Limit	Critical Strain	Full Isochr. Modulus	Creep Isochr. Modulus
Mean Radius	Thickness	Pressure	Imperfection	Shell Model	FE Model	FE Model	FE Model	FE Model
100	1	0.04	0.5	1740	0.96	1.17	1.17	12.24
100	1	0.04	0.1	197,000	0.94	1.16	1.16	3.75
100	1	0.03	0.5	259,000	0.88	1.55	1.54	4.67
100	1	0.03	0.1	1.31E+07	0.86	1.36	1.18	1.75
100	5	3.5	0.5	674	0.92	1.48	1.29	1.84
100	5	3.5	0.1	4207	0.88	1.17	0.95	1.16
100	5	2	0.5	22,570	0.86	1.48	1.18	1.34
100	5	2	0.1	79,570	0.85	1.13	0.89	0.98

5 EXAMPLE 2 SPHERE UNDER EXTERNAL PRESSURE

The secondary creep model is compared with the critical strain predictions and with the formula in Table 1:

Buckling stress = $[E_t E_s/\{3(1-\mu^2)\}]^{1/2}$ t/r.

Elastic buckling pressures of a 3-d shell model and an axisymmetric model were obtained. For the r = 100 mm, t = 1 mm and E = 1.7 x 10^5 MPa, the elastic buckling stresses are 1023 MPa and 1024 MPa, respectively. This is an insignificant difference. The buckling formula yields buckling stress = 1028 MPa. Figure 9 shows the buckling modes, and the buckling mode obtained with the axisymmetric model.

Table 9 summarizes the results of the calculations.

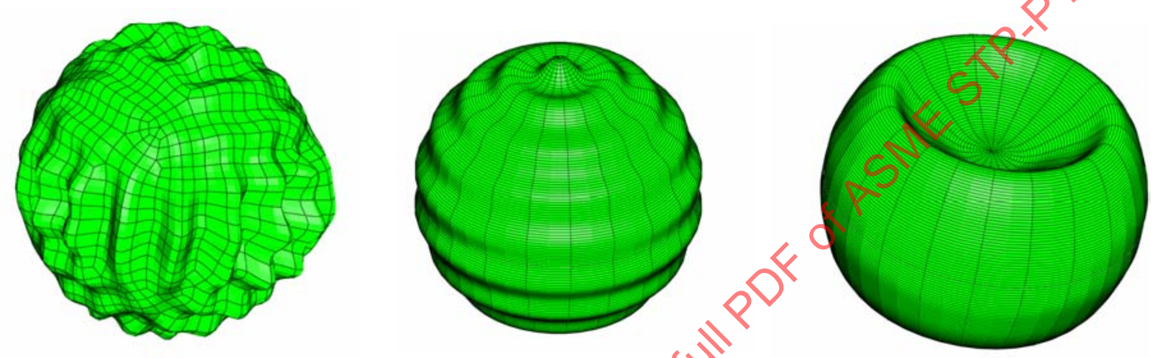

Figure 9 - First Elastic Buckling Modes for Sphere and Axisymmetric Model, with Buckling Stress = 1024 MPa. Typical Creep Buckling Mode.

The creep-buckling characteristics are different from the previous cases. Figure 10 shows the deflection history for case 1 in Table 9. It can be seen that a deflection of 5 mm, which was used for the corresponding case in Section 4, occurs well into the final instability. A deflection of 1 mm represents a rate increase of 5, and is more appropriate.

Table 9 shows the comparisons between the four analysis methods. As before, the isochronous limit/instability calculation is consistent and conservative. The critical strain and modulus methods change from unconservative to conservative as strain levels increase.

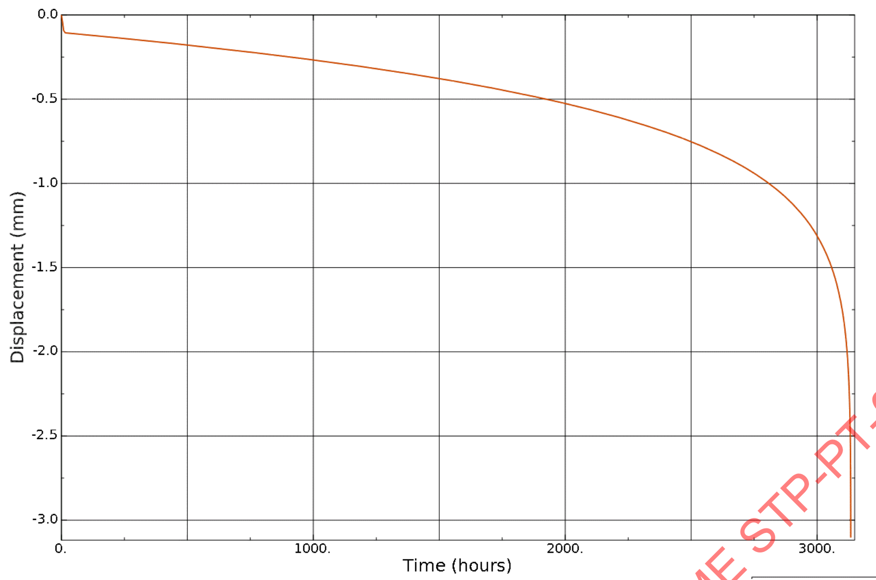

Figure 10 - Creep Buckling for Sphere Case 1

Table 9 - Comparison of Finite Element Secondary Creep and Approximate Analyses for Sphere Creep Buckling

Units: mm, MPa, hours				Full Creep Finite Element Analysis		Buckling Pressure Ratios (< 1 is Conservative)			
Secondary Creep Model				Secondary Creep		Isochronous Limit	Critical Strain	Full Isochr. Modulus	Creep Isochr. Modulus
Mean Radius	Thickness	Stress	Imperfection	Onset of buckling	Creep Strain	FE Model	FE Model	FE Model	FE Model
100.0	1	150	0.5	2900	0.04	0.82	1.23	1.09	1.15
100.0	1	100	0.1	54,500	0.042	0.79	0.91	0.81	0.83
100.0	5	500	0.5	600	0.10	0.83	0.84	0.75	0.77
100.0	5	500	0.1	1497	0.14	0.82	0.67	0.60	0.61

Units: mm, MPa, hours				Full Creep Finite Element Analysis			Buckling Pressure Ratios (< 1 is Conservative)
Secondary Creep Model				Secondary Creep		Critical Time	Critical Strain Time
Mean Radius	Thickness	Stress	Imperfection	Onset of Buckling	Creep Strain	API Critical Strain	FE Time
100.0	1.0	150.000	0.5	2900	0.04	6.99E+03	2
100.0	1.0	100.000	0.1	5.45E+04	0.042	3.74E+04	0.7
100.0	5.0	500.000	0.5	6.00E+02	0.1	2.96E+02	0.5
100.0	5.0	500.000	0.1	1.50E+03	0.14	2.96E+02	0.2