

Thermowells

Performance Test Codes

ASMENORMDOC.COM : Click to view the full PDF of ASME PTC 19.3 TW 2010

AN AMERICAN NATIONAL STANDARD



INTENTIONALLY LEFT BLANK

ASMENORMDOC.COM : Click to view the full PDF of ASME PTC 19.3 TW 2010

ASME PTC 19.3 TW-2010

Thermowells

Performance Test Codes

ASMENORMDOC.COM : Click to view the full PDF of ASME PTC 19.3 TW 2010

AN AMERICAN NATIONAL STANDARD



Three Park Avenue • New York, NY • 10016 USA

Date of Issuance: July 12, 2010

This Code will be revised when the Society approves the issuance of a new edition. There will be no addenda issued to PTC 19.3 TW-2010.

ASME issues written replies to inquiries concerning interpretations of technical aspects of this document. Periodically certain actions of the ASME PTC Committee may be published as Code Cases. Code Cases and interpretations are published on the ASME Web site under the Committee Pages at <http://cstools.asme.org> as they are issued.

ASME is the registered trademark of The American Society of Mechanical Engineers.

This code or standard was developed under procedures accredited as meeting the criteria for American National Standards. The Standards Committee that approved the code or standard was balanced to assure that individuals from competent and concerned interests have had an opportunity to participate. The proposed code or standard was made available for public review and comment that provides an opportunity for additional public input from industry, academia, regulatory agencies, and the public-at-large.

ASME does not "approve," "rate," or "endorse" any item, construction, proprietary device, or activity.

ASME does not take any position with respect to the validity of any patent rights asserted in connection with any items mentioned in this document, and does not undertake to insure anyone utilizing a standard against liability for infringement of any applicable letters patent, nor assumes any such liability. Users of a code or standard are expressly advised that determination of the validity of any such patent rights, and the risk of infringement of such rights, is entirely their own responsibility.

Participation by federal agency representative(s) or person(s) affiliated with industry is not to be interpreted as government or industry endorsement of this code or standard.

ASME accepts responsibility for only those interpretations of this document issued in accordance with the established ASME procedures and policies, which precludes the issuance of interpretations by individuals.

No part of this document may be reproduced in any form,
in an electronic retrieval system or otherwise,
without the prior written permission of the publisher.

The American Society of Mechanical Engineers
Three Park Avenue, New York, NY 10016-5990

Copyright © 2010 by
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
All rights reserved
Printed in U.S.A.

CONTENTS

Foreword	v
Acknowledgments	v
Committee Roster	vi
Correspondence With the PTC Committee	vii
Section 1 Object and Scope	1
1-1 Object	1
1-2 Scope	1
Section 2 Nomenclature	2
Section 3 Jurisdiction of Codes	4
3-1 Reference Standards and Governing Codes	4
3-2 Specification of Thermowells	4
Section 4 Dimensions	5
4-1 Configurations	5
4-2 Dimensional Limits	5
Section 5 Materials	10
5-1 General Considerations	10
Section 6 Stress Equations	11
6-1 General Considerations	11
6-2 Corrosion and Erosion	11
6-3 Flow-Induced Thermowell Stresses	12
6-4 Strouhal Number, Drag Coefficients, and Lift Coefficient	13
6-5 Natural Frequency of Thermowells	14
6-6 Mounting Compliance Factor	15
6-7 Unsupported Length, Diameter, and Fillet Radius	16
6-8 Frequency Limit	18
6-9 Magnification Factor	21
6-10 Bending Stresses	21
6-11 Pressure and Shear Stresses	24
6-12 Steady-State Static and Dynamic Stress Limits	24
6-13 Pressure Limit	27
Section 7 Overview of Calculations	28
7-1 Quantitative Criteria	28
7-2 Fluid Properties	28
7-3 Fluid Velocity	28
7-4 Material Properties and Dimensions	28
7-5 Reynolds and Strouhal Numbers	29
7-6 Natural Frequency at Operation Temperature	29
7-7 Natural Frequency at Expected Mode of Operation	29
7-8 Steady-State and Dynamic Stresses	29
7-9 Allowable Fatigue Limits	29
7-10 Pressure Rating	29
Section 8 Examples	30
8-1 Tapered, Welded Thermowell for a Steam-Header Application (U.S. Customary Units)	30
8-2 Step-Shank, Threaded Thermowell for a Hot Water Application (SI Units)	33

Section 9	Statement of Compliance	39
9-1	Specification of a Thermowell	39
9-2	Velocity and Pressure Ratings	39
Section 10	References	40
10-1	Referenced Documents	40
10-2	Referenced ASME Documents	40
Figures		
4-1-1	Schematic Diagram of a Thermowell	6
4-1-2	Examples of Straight-Shank Thermowells	7
4-1-3	Examples of Step-Shank Thermowells	8
4-1-4	Examples of Tapered Thermowells	9
6-3.1-1	Fluid-Induced Forces and Assignment of Axes for Calculation of Thermowell Stresses	12
6-6-1	Unsupported Length of Thermowells	17
6-8.1-1	Schematic Indicating Excitation of Resonances When Excitation Frequency Coincides With the Thermowell Natural Frequency	19
6-8.1-2	Schematic Showing the Amplitude Response of a Thermowell Subjected to Fluid-Induced Forces as Solid Lines, for In-Line and Transverse Excitation Modes	19
6-10.1-1	Bending Moment, Stress at the Support Plane, and Locations of Maximum Steady-State or Oscillating In-Line Stress	22
6-10.7-1	Mounting of a Thermowell in an Elbow, With the Tip Facing Downstream	24
6-10.7-2	Geometry to Be Used in Calculation of Thermowell Ratings	25
6-10.7-3	Mounting of a Thermowell in an Elbow, With the Tip Facing Upstream	25
Tables		
4-1-1	Dimensional Limits for Straight and Tapered Thermowells Within the Scope of This Standard	7
4-2-1	Dimensional Limits for Step-Shank Thermowells Within the Scope of This Standard	8
6-5.3-1	Parameters for Natural Frequency Calculation for Step-Shank Thermowells	15
6-12.3-1	Allowable Fatigue-Stress Amplitude Limits for Material Class A and Class B	26
Nonmandatory Appendix		
A	Conversion Factors	41

FOREWORD

In 1957, the ASME Performance Test Codes Committee 19.3 determined that the 1930 edition of the *Supplement on Temperature Measurement* dealing with thermowells was unsatisfactory. Since the design of thermowells requires both thermal and stress considerations, the ASME Boiler and Pressure Vessel Committee was approached for assistance. However, the special needs for the design of intrusive pipe fittings were deemed beyond the scope of what could be properly included in the vessel codes.

The PTC 19.3 Committee is charged with temperature measurement and thermowell design. The purpose of the thermowell is to facilitate temperature measurement while resisting fluid forces of the process. This committee undertook the task of providing guidance in this area, on the basis of a paper authored by J. W. Murdock [1], ultimately leading to the publication of PTC 19.3-1974, *Supplement on Instruments and Apparatus, Part 3, Temperature Measurement*. Prior to the acceptance of PTC 19.3-1974, the incidence of thermowell failures during the start-up testing of high-pressure steam turbines was unacceptable; its subsequent use in steam services has been highly successful at preventing catastrophic thermowell failure.

Since its publication, PTC 19.3 has received widespread acceptance and use in both steam and nonsteam applications outside the scope of the performance test codes. In 1971 an ASME ad hoc committee, PB51, under the jurisdiction of the PTC Board, was formed to assess the thermowell standard. This committee, designated PTC 19.3.1, produced a draft thermowell standard. In 1999, PTC 19.3 undertook the task of completing this draft. In the course of this effort, it was discovered that a number of thermowells designed to PTC 19.3-1974 but placed in nonsteam services suffered catastrophic failure. Review of the literature revealed that the PTC 19.3.1 draft did not incorporate recent, significant advances in our knowledge of thermowell behavior, and the committee decided to thoroughly rewrite the standard. The goals of the new Standard are to provide a thermowell rating method that can be used in a myriad array of services, including processes involving corrosive fluids; offer advice where fatigue endurance is critical; and establish criteria for insuring sensor reliability. These factors result in a more reliable basis for thermowell design than the PTC 19.3-1974 Supplement. It is intended that this edition of this Standard not be retroactive.

PTC 19.3 TW on thermowells was approved by the PTC Standards Committee on January 15, 2010, and approved and adopted as a Standard practice of the Society by action of the Board on Standardization and Testing on February 18, 2010. It was also approved as an American National Standard by the ANSI Board of Standards Review on April 22, 2010.

ACKNOWLEDGMENTS

The Committee gratefully acknowledges the special contributions of R. D. Blevins, D. R. Frikken, W. J. Koves, and A. Löbig.

ASME PTC COMMITTEE

Performance Test Codes

(The following is the roster of the Committee at the time of approval of this Code.)

STANDARDS COMMITTEE OFFICERS

M. P. McHale, *Chair*
J. R. Friedman, *Vice Chair*
J. H. Karian, *Secretary*

STANDARDS COMMITTEE PERSONNEL

P. G. Albert , General Electric Co.	M. P. McHale , McHale & Associates, Inc.
R. P. Allen , Consultant	P. M. McHale , McHale & Associates, Inc.
J. M. Burns , Burns Engineering	J. W. Milton , Reliant Energy
W. C. Campbell , Southern Company Services	S. P. Nuspl , Consultant
M. J. Dooley , Sigma Energy Solutions	R. R. Priestley , General Electric Co.
J. R. Friedman , Siemens Energy, Inc.	J. A. Rabensteine , Environmental Systems Corp.
G. J. Gerber , Consultant	J. A. Silvaggio, Jr. , Siemens Demag Delaval Turbomachinery, Inc.
P. M. Gerhart , University of Evansville	W. G. Steele, Jr. , Mississippi State University
T. C. Heil , Consultant	J. C. Westcott , Mustan Corp.
R. E. Henry , Sargent & Lundy	W. C. Wood , Duke Power Co.
J. H. Karian , The American Society of Mechanical Engineers	T. K. Kirkpatrick , <i>Alternate</i> , McHale & Associates, Inc.
D. R. Keyser , Service Engineering	J. A. Scavuzzo , <i>Alternate</i> , The Babcock & Wilcox Co.
S. J. Korellis , EPRI	

PTC 19.3 COMMITTEE — TEMPERATURE MEASUREMENT

D. C. Ripple , <i>Chair</i> , National Institute for Standards & Technology	S. M. Dale , Conax Technologies LLC
J. H. Karian , <i>Secretary</i> , The American Society of Mechanical Engineers	A. G. Gilson , Black & Veatch
D. S. Bartran , Consultant	A. Heisler , Pyromation, Inc.
D. Bauschke , Emerson Process Management	F. L. Johnson , JMS Southeast, Inc.
C. W. Brook , Wika Instruments Ltd.	D. Marra , Florida Power Light
M. Carugati , Alloy Engineering Co, Inc.	J. W. Stevens , University of Colorado

CORRESPONDENCE WITH THE PTC COMMITTEE

General. ASME Codes are developed and maintained with the intent to represent the consensus of concerned interests. As such, users of this Code may interact with the Committee by requesting interpretations, proposing revisions, and attending Committee meetings. Correspondence should be addressed to

Secretary, PTC Standards Committee
The American Society of Mechanical Engineers
Three Park Avenue
New York, NY 10016-5990
<http://go.asme.org/inquiry>

Proposing Revisions. Revisions are made periodically to the Code to incorporate changes that appear necessary or desirable, as demonstrated by the experience gained from the application of the Code. Approved revisions will be published periodically.

The Committee welcomes proposals for revisions to this Code. Such proposals should be as specific as possible, citing the paragraph number(s), the proposed wording, and a detailed description of the reasons for the proposal, including any pertinent documentation.

Proposing a Case. Cases may be issued for the purpose of providing alternative rules when justified, to permit early implementation of an approved revision when the need is urgent, or to provide rules not covered by existing provisions. Cases are effective immediately upon ASME approval and shall be posted on the ASME Committee Web page.

Request for cases shall provide a Statement of Need and background information. The request should identify the Code, paragraph, figure or table number(s), and be written as a Question and Reply in the same format as existing Cases. Requests for Cases should also indicate the applicable edition of the Code to which the proposed Case applies.

Interpretations. Upon request, the PTC Standards Committee will render an interpretation of any requirement of the Code. Interpretations can be rendered only in response to a written request sent to the Secretary of the PTC Standards Committee.

The request for interpretation should be clear and unambiguous. It is further recommended that the inquirer submit his request in the following format:

Subject: Cite the applicable paragraph number(s) and a concise description.
Edition: Cite the applicable edition of the Code for which the interpretation is being requested.
Question: Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. The inquirer may also include any plans or drawings that are necessary to explain the question; however, they should not contain proprietary names or information.

Requests that are not in this format will be rewritten in the appropriate format by the Committee prior to being answered, which may inadvertently change the intent of the original request.

ASME procedures provide for reconsideration of any interpretation when or if additional information that might affect an interpretation is available. Further, persons aggrieved by an interpretation may appeal to the cognizant ASME Committee. ASME does not “approve,” “certify,” “rate,” or “endorse” any item, construction, proprietary device, or activity.

Attending Committee Meetings. The PTC Standards Committee and its subcommittees, such as PTC 19.3, hold meetings or telephone conferences, which are open to the public. Persons wishing to attend any meeting or telephone conference should contact the Secretary of the PTC Standards Committee.

ASMENORMDOC.COM : Click to view the full PDF of ASME PTC 19.3 TW 2010

INTENTIONALLY LEFT BLANK

THERMOWELLS

Section 1 Object and Scope

1-1 OBJECT

The object of this Standard is to establish a mechanical design standard for reliable service of tapered, straight, and stepped-shank thermowells in a broad range of applications. This includes an evaluation of the forces caused by external pressure, and the combination of static and dynamic forces resulting from fluid impingement.

1-2 SCOPE

This Standard applies to thermowells machined from bar stock and includes those welded to or threaded into a flange as well as those welded into a process vessel or pipe with or without a weld adaptor. Thermowells

manufactured from pipe are outside the scope of this Standard.

Thermowells with specially designed surface structures (e.g., a knurled surface or a surface with spiral ridges) are beyond the scope of this Standard, due to the difficulty of providing design rules with broad applicability for these types of thermowells.

Thermowell attachment methods, standard dimensions, parasitic vibration of a sensor mounted inside the thermowell, and thermal equilibrium of the sensor relative to the process stream are beyond the scope of this Standard. In addition, thermowells fabricated by welding, including flame spray or weld overlays, at any place along the length of the shank or at the tip are outside the scope of this Standard.

Section 2

Nomenclature

For U.S. Customary units, *lb* denotes pound as a unit of mass, *lbf* denotes pounds-force, *kip* denotes 10^3 pounds-force, and *ksi* denotes 10^3 pounds-force per square inch or kips per square inch. When parameters are specified in mixed units within the U.S. Customary unit system (e.g., diameter *B* in inches, velocity *V* in feet per second), conversion factors between feet and inches will be needed in the calculations. See para. 6-4.1 and subsection 8-1 for examples.

- A* = outside diameter of thermowell at support plane or root, based on which point is closest to the thermowell tip, m (in.)
- A_p* = projected area of thermowell perpendicular to direction of flow and exposed to the flow stream, m² (in.²)
- a* = polynomial function used in eq. (6-8-4), dimensionless
- B* = outside diameter at tip of thermowell, m (in.)
- b* = fillet radius at the root of the thermowell shank, m (in.)
- b_s* = fillet radius at the base of the reduced-diameter length of a step-shank thermowell, m (in.)
- C_D* = coefficient for steady-state drag pressure, dimensionless
- C_d* = coefficient for oscillating-drag (in-line with flow) pressure, dimensionless
- C_l* = coefficient for oscillating-lift (transverse to flow) pressure, dimensionless
- c* = corrosion allowance, m (in.)
- c_i* = coefficients used in eq. (6-5-3), dimensionless
- D* = outside diameter at any cross section, m (in.)
- D_a* = average diameter of the thermowell, as defined in para. 6-5.3, Step 1, m (in.)
- d* = bore diameter of thermowell, m (in.)
- E* = modulus of elasticity at service temperature, Pa [psi or lb/(in.²·sec²)] (Refer to Nonmandatory Appendix A and para. 6-5.3 for a discussion of units of *E*.)
- E_{ref}* = reference value of modulus of elasticity, Pa (psi)
- F_D* = in-line static drag force on thermowell, due to fluid impingement, N (lbf)
- F_d* = in-line dynamic drag force on thermowell, due to fluid impingement, N (lbf)
- F_l* = transverse dynamic drag force on thermowell, due to fluid impingement, N (lbf)

- F_M* = magnification factor for thermowell oscillations transverse to fluid flow, dimensionless
- F'_M* = magnification factor for thermowell oscillations in-line with fluid flow, dimensionless
- f* = frequency, Hz
- f_a* = approximate resonance frequency of thermowell, Hz
- f_n^c* = resonance frequency of thermowell with compliant support, Hz
- f_n* = natural frequency with ideal clamping, Hz
- f_s* = vortex shedding frequency or rate, Hz
- G* = parameter defined in eq. (6-10-3), dimensionless
- G_{RD}* = parameter *G* appropriate for evaluation of stress at the base of a reduced-diameter shank, dimensionless
- G_{SP}* = parameter *G* appropriate for evaluation of stress at the support point, dimensionless
- G_β* = either *G_{RD}* or *G_{SP}*, dimensionless
- H_{a,f}* = factor to account for added fluid mass, dimensionless
- H_{a,s}* = factor to account for added sensor mass, dimensionless
- H_c* = frequency factor to account for support or foundation compliance, dimensionless
- H_f* = frequency factor to account for shear, rotation, taper, and tip-mass effects, dimensionless
- I* = moment of inertia of cross section, kg·m² (lb·in.²)
- K_M* = rotational stiffness of thermowell support, N·m/rad [(in.-lb)/rad]
- K_t* = stress concentration factor, dimensionless
- L* = unsupported length of thermowell, measured from the tip to the support plane, m (in.)
- L₀* = length of the thermowell shielded from fluid flow, m (in.)
- L_S* = length of reduced-diameter shank for a step-shank thermowell, m (in.)
- M* = bending moment, N·m (in.-lb)
- M_β* = bending moment for steady-state drag (for *β* = *D*), oscillating drag (for *β* = *d*), or lift (for *β* = *l*), N·m (in.-lb)

m = mass per unit length of a thermowell of uniform cross section, kg/m (lb/in.)	S_s = shear stress, Pa (psi)
N_s = Strouhal number, dimensionless	S_t = tangential pressure stress, Pa (psi)
N_{sc} = Scruton number or mass damping factor, dimensionless	S_z = longitudinal stress in the thermowell, Pa (psi)
P = operating pressure, Pa (psi)	T = operating temperature, °C (°F)
P_c = design static pressure of shank of thermowell, Pa (psi)	T_a = ambient temperature, °C (°F)
P_D = aerodynamic force per unit of projected area on thermowell, Pa (psi)	t = minimum tip thickness of the thermowell, m (in.)
P_d = oscillating-drag force per unit of projected area on thermowell, Pa (psi)	V = process fluid velocity, m/s (in./sec)
P_f = design pressure for flange supporting thermowell, Pa (psi)	V_{IR} = fluid velocity that excites the in-line resonance, m/s (in./sec)
P_l = oscillating-lift force per unit of projected area on thermowell, Pa (psi)	v = specific volume (reciprocal of the fluid density ρ), m ³ /kg (in. ³ /lb)
P_r = external pressure rating of the thermowell, Pa (psi)	\hat{x} = unit vector normal to the fluid velocity and to the axis of the thermowell
P_t = design pressure of tip of the thermowell, Pa (psi)	y = distance from thermowell axis, m (in.)
P_β = either P_D , P_d , or P_l , Pa (psi)	\hat{y} = unit vector pointing in the direction of the fluid flow
R_p = pipe radius, m (in.)	z = distance from the thermowell root along the thermowell axis, m (in.)
r = ratio of shedding frequency to natural frequency, dimensionless (lift resonance)	z_s = distance from the thermowell root to the plane where stress is evaluated, m (in.)
r' = ratio of shedding frequency to natural frequency, dimensionless (in-line resonance)	\hat{z} = unit vector along axis of the thermowell, pointing toward the tip
Re = Reynolds number, calculated on the basis of the tip diameter: $Re = BV/\mu$, dimensionless, or $Re = BV/\nu$, dimensionless	α = average coefficient of thermal expansion, m/(m·K) [in./(in.·°F)]
S = maximum allowable working stress, Pa (psi)	β = parameter used in eq. (6-5-3), dimensionless
S_a = axial pressure stress, Pa (psi)	μ = dynamic fluid viscosity, Pa·s [lb/(ft·sec)]
S_D = steady-state drag stress due to fluid impingement, Pa (psi)	
S_d = oscillating-drag stress due to fluid impingement, Pa (psi)	
S_f = fatigue endurance limit, in the high-cycle limit, Pa (psi)	
S_L = oscillating-lift stress due to fluid impingement, Pa (psi)	
S_r = radial pressure stress, Pa (psi)	

NOTE: Viscosity is often given in the literature in units of centipoise, abbreviation cP. Useful conversion factors are 1 cP = 0.67197×10^{-3} lb/(ft·sec) and 1 cP = 10^{-3} Pa·s.

ν = kinematic fluid viscosity, m²/s (ft²/sec)

ρ = fluid density, kg/m³ (lb/in.³)

ρ_m = mass density of the thermowell material, kg/m³ (lb/in.³)

ρ_s = average density of a temperature sensor, kg/m³ (lb/in.³)

ζ = damping factor, dimensionless

$\omega_s = 2\pi f_s$, rad/s (rad/sec)

Section 3

Jurisdiction of Codes

Thermowells are an integral part of the piping system and the process containment system, and as a result, they may be subject to requirements from the governing piping or pressure vessel code.

3-1 REFERENCE STANDARDS AND GOVERNING CODES

(a) ASME B40.9, *Thermowells for Thermometers and Elastic Temperature Sensors*, discusses the selection, fabrication, and installation of thermowells, as well as providing some standardized designs. Complementing B40.9, PTC 19.3 TW is limited in scope to mechanical design of thermowells.

(b) ASME Boiler and Pressure Vessel Code (BPVC) Section III Appendices, Appendix N provides guidance on the flow-induced vibration of banks or arrays of tubes and on the excitation of structural vibrations by turbulence. Both of these topics are outside the scope

of PTC 19.3 TW, which considers the vibration of single thermowells due to vortex shedding only.

(c) Guidance on minimizing temperature measurement errors in thermowell applications is found in the latest edition of PTC 19.3. Effects considered include heating of the thermowell by fluid impingement, errors due to thermal radiation and conduction along the thermowell, and heat transfer between the thermowell and the surrounding fluid.

3-2 SPECIFICATION OF THERMOWELLS

Specification of a thermowell and the materials of construction are the sole responsibility of the designer of the system that incorporates the thermowell. Sole responsibility for ensuring compatibility of the process fluid with the system rests with the end user. Thermowells may be stated to be in conformance to this Standard, subject to the requirements of Section 9 of this Standard.

Section 4 Dimensions

4-1 CONFIGURATIONS

Figure 4-1-1 shows a schematic diagram of a thermowell, along with its characteristic dimensions. Typical thermowell attachment configurations include threaded, socket weld, weld-in, lap-joint (Van Stone), and integral-flanged as shown in Figs. 4-1-2, 4-1-3, and 4-1-4 (see also Table 4-1-1). These figures are representative of common practice but do not display all allowable attachment configurations. The selection of a specific attachment method is subject to the governing piping or pressure vessel code. Use of ball joints, spherical unions, or packing gland installations are not permissible in Performance Test Code applications.

The dashed line in Fig. 4-1-1 indicates the support plane, which is an imaginary extension of the supporting-structure surface that passes through the shank of the thermowell. The unsupported length, L , is calculated as the distance from the tip of the thermowell to the intersection of the thermowell axis with this surface. For thermowells mounted on flanges or welded into weld adaptors, the support plane will be a flat plane. However, for thermowells mounted by direct welding into a pipe wall, the support plane will actually be a curved surface with the same curvature as the inner pipe wall. For this case, the support plane should be approximated as a plane located at a distance from the thermowell tip equal to the largest actual distance from the tip to any point on the true curved support surface. For thermowells welded to a flange or pipe wall at an angle, the support plane will not be normal to the thermowell axis.

For nonstandard attachments, this Standard covers the design requirements of the thermowell only. The designer shall account for the support compliance of the attachment (refer to subsection 6-6), and the attachment method shall meet all the requirements of the governing piping or pressure vessel code.

4-2 DIMENSIONAL LIMITS

This standard applies to straight and tapered thermowells within the dimensional limits given in Table 4-1-1, where

A = thermowell outer diameter at the root of the thermowell shank, or at the support plane if the thermowell is firmly supported along its shank

B = thermowell diameter at the tip

d = bore diameter

L = length of the thermowell from the tip to the support plane

t = minimum thickness of the tip

For the purpose of defining L and A , the support plane shall also be defined (see subsection 6-7). The root of the thermowell is located where the thermowell shank makes a transition to

- (a) a machined transition to a flange, socket weld collar, or threaded section of the thermowell
- (b) a weld-joint transition to other piping components

The Standard also applies to step-shank thermowells within the dimensional limits given in Table 4-2-1, where L_s is the length of the reduced-diameter section of thermowell shank, in addition to the dimensions defined for Table 4-1-1. Refer to Fig. 4-1-1.

Calculations should be made using the nominal dimensions provided that a corrosion allowance is not used (see subsection 6-2) and that the thermowell is fabricated with manufacturing tolerances of $\pm 1\%$ for lengths L and L_s and $\pm 3\%$ for diameters A , B , and d . If tolerances for A , B , or d are not met, calculations shall be made according to subsection 6-2, using as the corrosion allowance the linear sum of the actual tolerance and any corrosion allowance. If tolerances for L or L_s are not met, calculations shall be made assuming that the lengths L and L_s each equal the nominal length plus the respective manufacturing tolerance. External pressure calculations shall be made based on the minimum material condition, as discussed in subsection 6-13.

This Standard applies to thermowells with an as-new surface finish of $0.81 \mu\text{m}$ (32 $\mu\text{in.}$) Ra or better. Stress limits given in subsection 6-12 are not valid for thermowells manufactured with rougher surfaces.

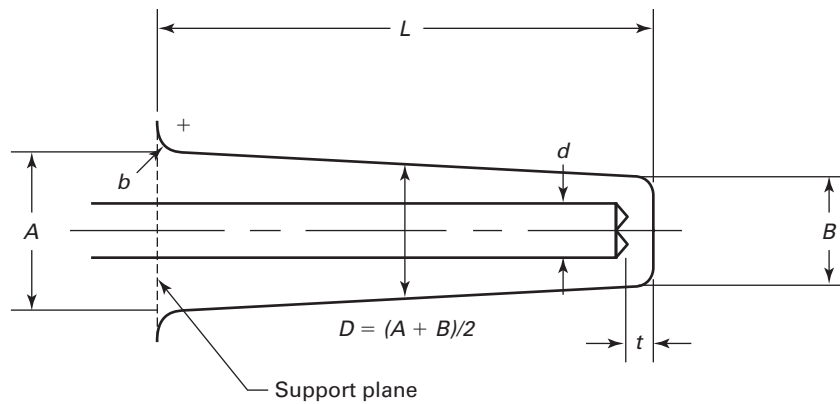
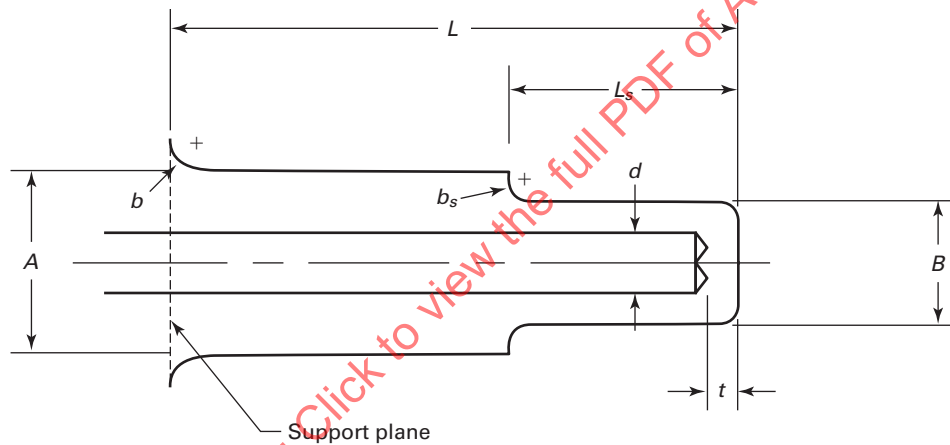
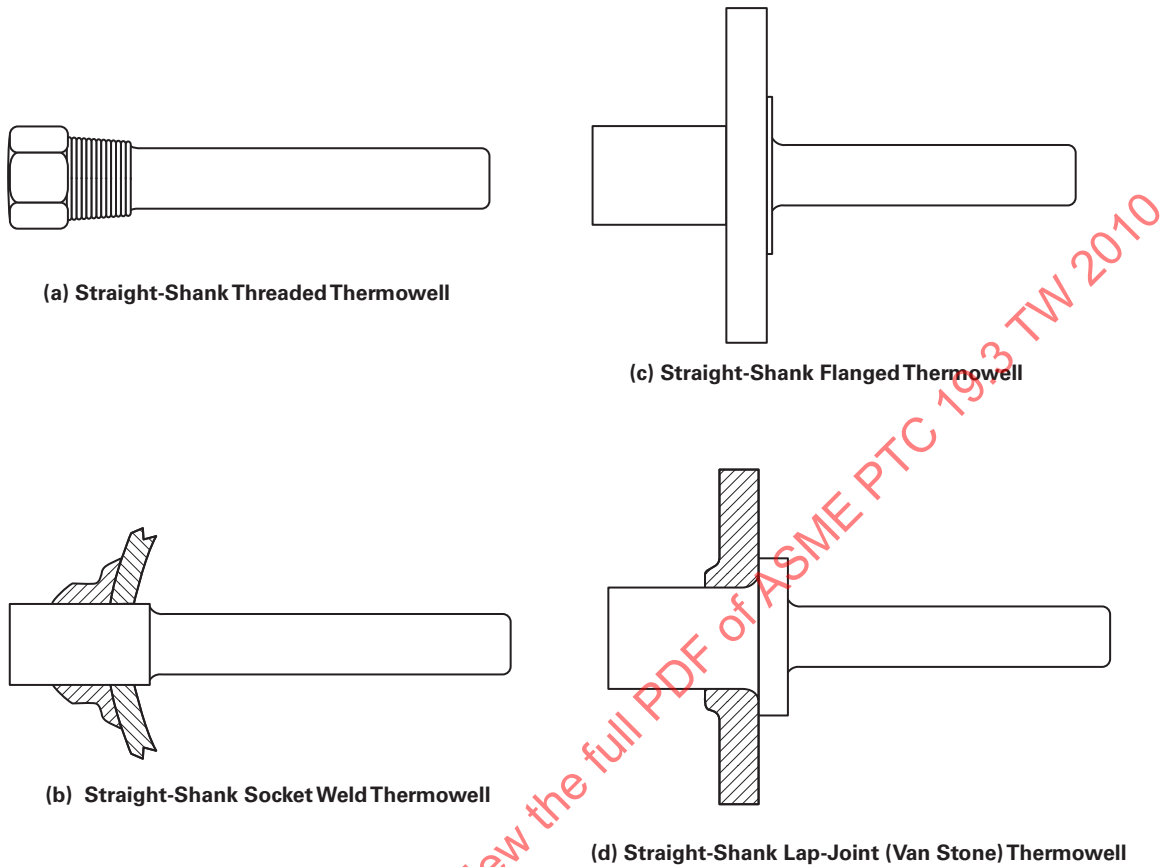
Fig. 4-1-1 Schematic Diagram of a Thermowell**(a) Schematic, Cross-Sectional View of a Thermowell****(b) Schematic, Cross-Sectional View of a Step-Shank Thermowell**

Fig. 4-1-2 Examples of Straight-Shank Thermowells**Table 4-1-1 Dimensional Limits for Straight and Tapered Thermowells Within the Scope of This Standard**

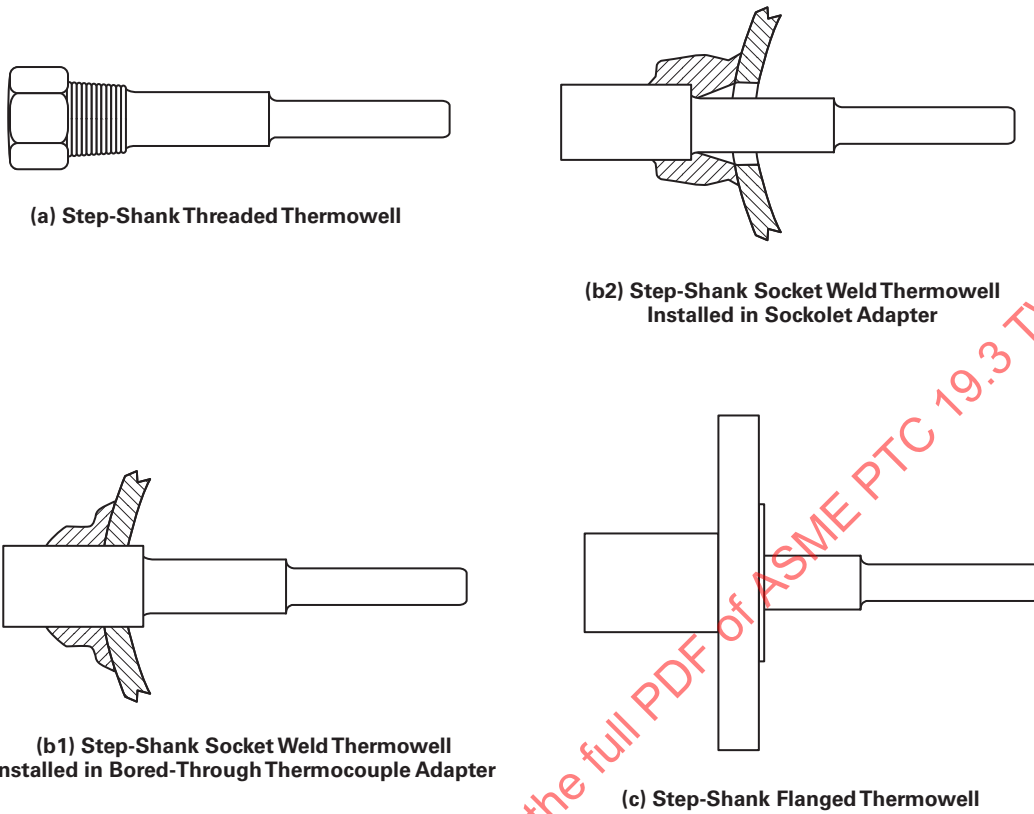
Description	Symbol	Minimum	Maximum
Unsupported length	L	6.35 cm (2.5 in.) [Note (1)]	60.96 cm (24 in.) [Note (2)]
Bore diameter	d	0.3175 cm (0.125 in.)	2.0955 cm (0.825 in.)
Tip diameter	B	0.92 cm (0.36 in.)	4.65 cm (1.83 in.)
Taper ratio	B/A	0.58	1
Bore ratio	d/B	0.16	0.71
Aspect ratio	L/B	2	...
Minimum wall thickness	$(B - d)/2$	0.30 cm (0.12 in.)	...

GENERAL NOTE: Limits in this table apply to the nominal dimensions of the thermowell.

NOTES:

(1) Thermowells of length less than the minimum specified require design methods outside the scope of this Standard.

(2) The equations in this Standard are valid for thermowells longer than the maximum indicated; however, only single-piece, drilled bar-stock shanks are covered by this Standard.

Fig. 4-1-3 Examples of Step-Shank Thermowells**Table 4-2-1 Dimensional Limits for Step-Shank Thermowells Within the Scope of This Standard**

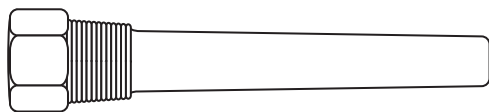
Description	Symbol	Minimum	Maximum
Unsupported length	L	12.7 cm (5 in.)	60.96 cm (24 in.)
Bore diameter	d	0.61 cm (0.24 in.)	0.67 cm (0.265 in.)
Step diameter ratio, for $B = 1.270$ cm (0.5 in.)	B/A	0.5	0.8
Step diameter ratio, for $B = 2.223$ cm (0.875 in.)	B/A	0.583	0.875
Length ratio	L_s/L	0	0.6
Minimum wall thickness	$(B - d)/2$	0.30 cm (0.12 in.)	...
Allowable Dimensions [Note (1)]			
Tip diameter	B	1.270 cm (0.5 in.) and 2.223 cm (0.875 in.)	

GENERAL NOTE: Limits in this table apply to the nominal dimensions of the thermowell.

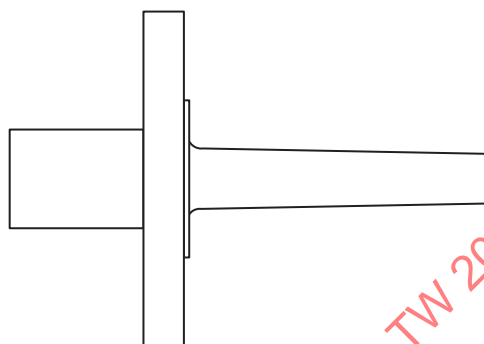
NOTE:

(1) The methods presented in this Standard apply for other tip diameters than those specified, but the correlation for natural frequency is supplied only for the given tip diameters.

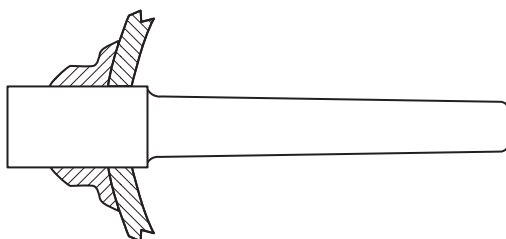
Fig. 4-1-4 Examples of Tapered Thermowells



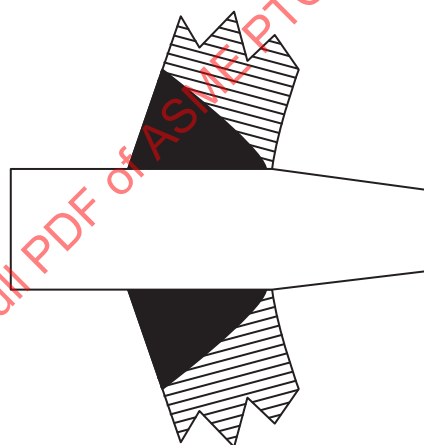
(a) Tapered-Shank Threaded Thermowell



(c) Tapered-Shank Flanged Thermowell



**(b) Tapered-Shank Socket Weld Thermowell
Installed in Bored-Through Thermocouple Adapter**



**(d) Tapered-Shank Weld-In Thermowell
Installed Directly Into Pipe Wall**

Section 5 Materials

5-1 GENERAL CONSIDERATIONS

The system designer (see subsection 3-2) shall carefully consider, among other environmental conditions, the characteristics of the following to determine the proper material for the thermowell:

- (a) process fluid
- (b) pressure
- (c) temperature
- (d) fluid velocity
- (e) application
- (f) weldability

In general, the choice of material shall be governed mainly by strength requirements and possible corrosion that the thermowell will encounter. Thermowells are subjected to sustained stress reversals with a very high number of cycles (see subsection 6-3), so the materials of construction shall be selected on the basis of resistance to corrosion and corrosion fatigue. Of particular concern

is use of materials susceptible to stress corrosion cracking or embrittlement at the service conditions.

The thermowell material used should be forged or bar stock and shall conform to the requirements of the governing code.

In the absence of a governing code, other materials, which may or may not be ASTM, ANSI, or ASME approved, may be used when necessary, subject to the following requirements:

(a) The specific materials shall be agreed to by the designer and supplier of the thermowell.

(b) Unlisted materials may be used provided they conform to a published specification covering chemical, physical, and mechanical properties; method and process of manufacture; heat treatment; and quality control, and otherwise meet the requirements of this Standard.

(c) Allowable stresses shall be determined in accordance with the applicable allowable stress basis of this Standard or a more conservative basis.

Section 6

Stress Equations

6-1 GENERAL CONSIDERATIONS

6-1.1 Overview of Design Criteria

Thermowells shall be designed to withstand static pressure, steady-state fluid impingement, turbulence, and dynamic excitation due to von Karman vortices. Excitation by structure-borne vibration is a possibility and should also be considered, but is not addressed by this Standard, since this type of excitation is determined by the design and support of the entire piping system. Consideration of these loads on a mechanical model of the thermowell results in pressure and velocity limits due to the combination of steady-state and oscillatory forces acting on the thermowell. In evaluating an existing design or in designing a thermowell for given applications, the complete range of operating conditions for the thermowell, from start-up to emergency conditions, shall be considered. Factors that reduce the thermal mass of the thermowell and measurement errors are those that tend to reduce strength. Thermowell design consists of achieving accurate and reliable temperature measurement without compromising mechanical integrity or fluid containment. In all cases, the mechanical strength requirements shall control.

6-1.2 Optimization of Thermowell Design

Proper design of a thermowell requires that the sensor mounted inside the thermowell attain thermal equilibrium with the process fluid. Thermal modeling of the sensor response is outside the scope of this Standard (refer to the latest version of PTC 19.3 for guidance). This Section briefly summarizes general design rules that will optimize the sensor performance within the constraints of the mechanical strength requirements.

A high fluid-velocity rating for the thermowell requires a sufficiently high natural frequency for the thermowell (subsection 6-8) and sufficiently low oscillatory stresses (subsection 6-10). Higher natural frequencies result from decreasing the unsupported length, L , increasing the support-plane diameter, A , and decreasing the tip diameter, B . Lower oscillatory stresses result from decreasing length L and increasing diameter A . A higher static pressure rating (subsection 6-13) requires increasing the value of tip diameter B .

In contrast, good thermal performance favors increasing length L and decreasing diameters A and B .

6-1.2.1 Factors Improving Mechanical Strength. Factors that improve mechanical strength with mini-

mal degradation of thermal performance include the following:

- (a) locating a larger fillet radius at the support plane
- (b) locating the support plane away from a weld or heat-affected zone of a weld
- (c) avoiding threaded installations

6-1.2.2 Factors Improving Thermal Performance.

Factors that improve thermal performance with minimal degradation of mechanical strength include the following:

- (a) use of the smallest practical bore size
- (b) insulation of the outside of the pipe to reduce heat flux along the sensor axis

6-2 CORROSION AND EROSION

Refer to subsection 5-1 for considerations on materials selection for corrosive environments.

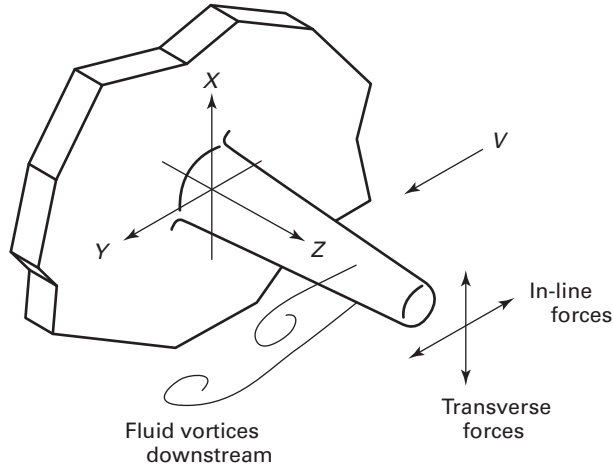
For applications where corrosion or erosion of the outer thermowell surface cannot be avoided, the designer shall establish a corrosion allowance, c . It is emphasized that the use of a corrosion allowance alone is insufficient at ensuring structural integrity of the thermowell in cases when stress corrosion is present.

When a corrosion allowance is included, thermowell ratings for maximum allowable pressure and maximum allowable fluid velocity shall be calculated for three cases:

- (a) initial thermowell dimensions
- (b) a thermowell design with tip thickness t and outer diameter at the support plane, A , reduced by c ; all other dimensions as in (a)
- (c) a thermowell design with tip thickness t and outer diameter at the tip, B , reduced by c ; all other dimensions as in (a)

Cases described in (b) and (c) are intended to approximate two extreme cases of corrosion and erosion: case (b), where the thermowell loses material at the root; and case (c), where the thermowell loses material at the tip. If finite element calculations are performed to determine this effect, assume that the value of c varies linearly along the length of the thermowell, from zero at the tip to c at the root for case (b), and from c at the tip to zero at the support plane for case (c).

Fig. 6-3.1-1 Fluid-Induced Forces and Assignment of Axes for Calculation of Thermowell Stresses



The maximum allowable pressure and maximum allowable fluid velocity shall be the minimum of the values obtained for the three cases above.

6-3 FLOW-INDUCED THERMOWELL STRESSES

6-3.1 Overview of Flow-Induced Stresses

The flow-induced stresses are modeled as a distributed force acting on a flexible beam. The total force on the beam is proportional to the projected area of the thermowell normal to the flow direction. While the hydrostatic-pressure stresses control rupture strength of the thermowell, the bending stresses and the possibility of flow-induced resonance dominate its velocity rating. The pressure stresses are primarily circumferential, while the flow-induced stresses are in the form of longitudinal bending stresses. These are greatest at the support plane of the thermowell and distributed about the neutral axis as with any transversely loaded beam.

The fluid forces acting on the thermowell are directed along the flow direction \hat{y} (drag) and transverse direction \hat{x} (lift) [2–4] as shown in Fig. 6-3.1-1. These can be represented as a vector acting on the centerline of the thermowell:

$$F(t) = [F_D + F_d \sin(2\omega_s t)]\hat{y} + F_l \cos(\omega_s t)\hat{x} \quad (6-3-1)$$

where $f_s = \omega_s/2\pi$ is the Strouhal frequency discussed in subsection 6-4. Forces acting along the flow direction are termed “in-line”; forces acting along a direction normal to the flow are termed “transverse.” Approximating the fluid forces as two orthogonal components normal to the thermowell axis greatly simplifies the fluid-structure

interaction and at the same time retains sufficient accuracy for the reliable calculation of velocity ratings of the thermowell.

6-3.2 Force Amplitudes

The force amplitudes should be expressed as a force per unit area, P_β , acting on the projected area, A_p , of the thermowell, for that portion of the thermowell that is exposed to the flow stream. There are three cases:

$$F_D = A_p P_D \quad F_d = A_p P_d \quad F_l = A_p P_l \quad (6-3-2)$$

where

- subscript D = conventional steady-drag forces
- subscript d = oscillating-drag (in-line) forces
- subscript l = oscillating-lift (transverse) forces

The symbol P_β is used below to denote any one of the three forces per unit area, P_D , P_d , or P_l . Each of the forces should be interpreted as effective pressures, P_D , P_d , and P_l , having the form

$$P_D = \frac{1}{2} \rho C_D V^2 \quad P_d = \frac{1}{2} \rho C_d V^2 \quad P_l = \frac{1}{2} \rho C_l V^2 \quad (6-3-3)$$

where

- C_D , C_d , and C_l = constants (see para. 6-4.2)
- V = velocity of the process fluid
- ρ = density of the process fluid

Summing the forces per unit area of eq. (6-3-3) over the thermowell projected area, while invoking a coherent vortex shedding process based on the vortex shedding rate at the tip, results in a conservative estimate

of the excitation forces and resultant bending stresses. These assumptions result in a lower bound estimate of the conditions that lead to stress failure.

6-3.3 Choice of Maximum Velocity Value

In all cases, design calculations shall take into account the possibility of flow increases above the design rating of the mechanical equipment and for process upset conditions. Specific flow maximums should be used where such data are available. Examples include start-up, shut-down, process upset, and pressure-relief conditions. Pre-start-up conditions such as steam blows for pipe clean out shall also be considered. In the case of high-pressure steam blows, the fluid velocities can greatly exceed 100 m/s (300 ft/sec), and thermowells shall be designed for these conditions.

6-3.4 Flow-Induced Vibration of Thermowell Arrays

This Standard addresses the vibration of a single thermowell in a fluid flow and does not address the interactions of multiple thermowells in close proximity. Flow-induced vibrations of arrays of tubes are discussed in ASME BPVC Section III-A, Appendix N, Section N-1300.

6-3.5 Turbulence-Induced Vibration of Thermowells

This Standard addresses the dynamic vibration caused by vortex shedding but does not address the incoherent excitation of structural vibrations by broadband, high-frequency turbulence. This excitation mechanism can be important for short, slender [e.g., 6 cm (2.5 in.) long by 1 cm (0.4 in.) diameter] thermowells in high flows. These cases require specialized analysis beyond the scope of the present Standard. ASME BPVC Section III-A, Appendix N, Section N-1340 provides guidance on turbulence-induced vibrations.

6-3.6 Low Fluid Velocities

At very low fluid velocities, the risk of thermowell failure is greatly reduced. The calculations of natural-frequency and corresponding-frequency limits (subsections 6-5 to 6-8), steady-state stress (para. 6-12.2), and oscillating stress (para. 6-12.3) do not need to be performed provided the following criteria are met:

- (a) The process fluid has a maximum velocity less than 0.64 m/s (2.1 ft/sec).
- (b) The thermowell dimensions satisfy the limits
 - (1) $A - d \geq 9.55 \text{ mm (0.376 in.)}$
 - (2) $L \leq 0.61 \text{ m (24 in.)}$
 - (3) $A \geq B \geq 12.7 \text{ mm (0.5 in.)}$
- (c) The thermowell material satisfies $S \geq 69 \text{ MPa (10 ksi)}$ and $S_f \geq 21 \text{ MPa (3 ksi)}$.

(d) The thermowell material is not subject to stress corrosion or embrittlement.

The calculation of the external pressure rating (subsection 6-13) shall still be performed.

Designers are cautioned that if the in-line resonance is excited at fluid velocities below 0.64 m/s (2.1 ft/sec), sustained operation on resonance may damage the temperature sensor even if the risk of mechanical thermowell failure is very small.

6-3.7 Pulsed Flow

The analysis of thermowell response to fluid flow in this document presumes a steady fluid velocity. Pulsating flows where the fluid velocity varies at a frequency close to the natural frequency of the thermowell can also excite thermowell vibrations. Thermowell failures have been attributed to the exposure of a thermowell to pulsating fluid flow (e.g., thermowell failures have been seen for installations close to the discharge of a centrifugal pump). Designers should consider possible sources of flow pulsations.

6-4 STROUHAL NUMBER, DRAG COEFFICIENTS, AND LIFT COEFFICIENT

6-4.1 Strouhal Number

The shedding of vortices by a thermowell subject to transverse fluid flow produces a periodic force on the thermowell [4, 5]. The frequency of the vortex shedding, f_s , is related to the fluid velocity by the dimensionless Strouhal number, N_s :

$$f_s = \frac{\omega_s}{2\pi} = N_s \frac{V}{B} \quad (6-4-1)$$

where

B = tip diameter of the thermowell

Machined thermowells of dimensions within the scope of this Standard have Strouhal numbers characteristic of rough-surfaced cylinders [6]. A correlation of available experimental data gives the Strouhal number as a function of the Reynolds number [7]:

$$N_s = \begin{cases} 0.22(1 - 22/Re) & \text{for } 22 \leq Re < 1,300 \\ 0.213 - 0.0248 [\log_{10}(Re/1,300)]^2 & \text{for } 1,300 \leq Re < 5 \times 10^5 \\ +0.0095 [\log_{10}(Re/1,300)]^3 & \text{for } 5 \times 10^5 \leq Re < 5 \times 10^7 \\ 0.22 & \text{for } 5 \times 10^7 \leq Re \end{cases} \quad (6-4-2)$$

In eq. (6-4-2), the Reynolds number is calculated using the tip diameter:

$$Re = \frac{VB\rho}{\mu} \quad \text{or} \quad Re = \frac{VB}{\nu} \quad (6-4-3)$$

where

μ = dynamic viscosity

ν = kinematic viscosity

ρ = fluid density at flowing conditions

For thermowell design, the Strouhal number may also be calculated from a simplified, conservative approximation of eq. (6-4-2):

$$N_s \cong 0.22 \quad (6-4-4)$$

For Reynolds numbers above approximately 100, the Strouhal number depends only weakly on the value of fluid viscosity. For a Reynolds number between 10^3 and 5×10^5 , the viscosity needs to be known only to within a factor of 2. For Reynolds numbers greater than 5×10^5 , the viscosity needs to be known only well enough to confirm that $Re > 5 \times 10^5$. References [8, 9, and 17] should be consulted for typical viscosity values. If the viscosity is difficult to determine, eq. (6-4-4) should be used for the Strouhal number.

6-4.1.1 Example. Superheated steam at a temperature of 1,000°F and a pressure of 2,000 psig flows through a pipe of 6-in. diameter at 100,000 lb/hr, past a thermowell with a tip diameter of 0.625 in. What is the Reynolds number?

(a) *Calculation in SI Units*

From steam tables [8, 9], the dynamic fluid viscosity $\mu = 3.079 \times 10^{-5}$ Pa·s.

Input parameters for the Reynolds number calculation are

$$B = 0.625 \text{ in.} \times (0.0254 \text{ m/in.}) = 0.015875 \text{ m}$$

$$\text{Pipe radius} = R_p = (6 \text{ in.}/2)(0.0254 \text{ m/in.}) = 0.0762 \text{ m}$$

$$\text{Pipe area} = \pi R_p^2 = 0.018241 \text{ m}^2$$

$$\begin{aligned} \rho V &= (\text{density}) \times (\text{flow velocity}) \\ &= (\text{mass flow rate}) / (\text{pipe area}) \\ &= (100,000 \text{ lb/hr})(0.454 \text{ kg/lb})(1 \text{ hr}/3,600 \text{ sec}) / (0.018241 \text{ m}^2) \\ &= 691.36 \text{ kg}/(\text{m}^2 \cdot \text{s}) \end{aligned}$$

Finally

$$\begin{aligned} Re &= BV\rho/\mu \\ &= 0.015875 \text{ m} \times 691.36 \text{ kg}/(\text{m}^2 \cdot \text{s}) / (3.079 \times 10^{-5} \text{ Pa} \cdot \text{s}) \\ &= 3.56 \times 10^5 \end{aligned}$$

(b) *Calculation in U.S. Customary Units*

From steam tables [8, 9], the dynamic fluid viscosity $\mu = 2.07 \times 10^{-5}$ lb/(ft·sec).

Input parameters for the Reynolds number calculation are

$$B = 0.625 \text{ in.} \times (1 \text{ ft}/12 \text{ in.}) = 0.052083 \text{ ft}$$

$$\text{Pipe radius} = R = (6 \text{ in.}/2)(1 \text{ ft}/12 \text{ in.}) = 0.25 \text{ ft}$$

$$\text{Pipe area} = \pi R^2 = 0.19635 \text{ ft}^2$$

$$\begin{aligned} \rho V &= (\text{density}) \times (\text{flow velocity}) \\ &= (\text{mass flow rate}) / (\text{pipe area}) \end{aligned}$$

$$\begin{aligned} &= (100,000 \text{ lb/hr})(1 \text{ hr}/3,600 \text{ sec}) / (0.19635 \text{ ft}^2) \\ &= 141.47 \text{ lb}/(\text{ft}^2 \cdot \text{sec}) \end{aligned}$$

Finally

$$\begin{aligned} Re &= BV\rho/\mu \\ &= 0.052083 \text{ ft} \times 141.47 \text{ lb}/(\text{ft}^2 \cdot \text{sec}) / (2.07 \times 10^{-5} \text{ lb}/(\text{ft} \cdot \text{sec})) \\ &= 3.56 \times 10^5 \end{aligned}$$

6-4.2 Drag and Lift Coefficients

For design purposes, the eq. (6-3-3) coefficients for conventional-drag, oscillating-drag, and oscillating-lift pressures shall be

$$\begin{aligned} C_D &= 1.4 \\ C_d &= 0.1 \\ C_l &= 1.0 \end{aligned} \quad (6-4-5)$$

6-5 NATURAL FREQUENCY OF THERMOWELLS

6-5.1 Transverse Vibrations

The natural frequency of transverse vibrations of a thermowell mounted to a support is a function of

- elastic properties of the thermowell
- mass per unit length
- shear and rotational inertia at small values of L/A
- support compliance
- added mass of the fluid
- added mass of the sensor

The formulas of subsection 6-5 establish a conservative estimate of the natural frequency of common industrial thermowells by applying a series of correction factors to an idealized beam having the mean dimensions of the actual thermowell. Nonuniform cross sections, shear, and rotational inertia are all accounted for using the frequency factor, H_f , in para. 6-5.3, Step 3. Foundation compliance, accounted for with the compliance factor, H_c , is treated in subsection 6-6. The added mass of the fluid is accounted for with the factor H_{af} , and added sensor mass is accounted for with H_{as} .

While there are an infinite number of vibrational modes for a thermowell, the lowest-order resonance (i.e., the natural frequency including effects of support compliance), f_n^c , controls the onset of flow-induced resonance.

6-5.2 Finite Element Methods

The natural frequency of a thermowell may be calculated using finite element methods, provided the software is validated by comparison of calculated frequencies with the results obtained in para. 6-5.3.

6-5.3 Calculations and Correction Factors

Step 1. Calculate an average outer diameter, D_a , for the thermowell. For straight thermowells, D_a is the outer shank diameter. For tapered thermowells, set $D_a = (A + B)/2$. For step-shank thermowells, set $D_a = A$.

Step 2. Calculate the approximate natural frequency of the thermowell as

$$f_a = \frac{1.875^2}{2\pi} \left(\frac{EI}{m} \right)^{1/2} \frac{1}{L^2} \quad (6-5-1)$$

where

E = elastic modulus at the operating temperature

$I = \pi(D_a^4 - d^4)/64$, the second moment of inertia

L = unsupported length of the thermowell

$m = \rho_m \pi (D_a^2 - d^2)/4$, the mass per unit length of the thermowell

When performing calculations with U.S. Customary units, and when E is given in units of pounds per square inch (equivalent to lbf/in.², or psi), the conversion factor 386.088 in.-lb = 1 lbf·sec² is used to convert the units of E to pounds per inch per second squared. See para. 8-1.2 for an example.

Step 3. Calculate the correction factor, H_f , for deviations from a solid beam of uniform cross section. For straight-shank or tapered thermowells, use the correlation

$$H_f = \frac{0.99 \left[1 + (1 - B/A) + (1 - B/A)^2 \right]}{1 + 1.1 (D_a/L)^{3[1 - 0.8(d/D_a)]}} \quad (6-5-2)$$

where

A = thermowell diameter at the support plane

B = thermowell diameter at the tip

D_a = average thermowell diameter = $(A + B)/2$

d = bore diameter

For step-shank thermowells of nominal 0.66-cm (0.26-in.) bore, and tip diameters of either 1.27 cm (0.5 in.) or 2.22 cm (0.875 in.), use the correlation

$$\begin{aligned} H_f &= (y_1^{-\beta} + y_2^{-\beta})^{-1/\beta} \\ y_1 &= [c_1(A/B) + c_2](L_s/L) + [c_3(A/B) + c_4] \\ y_2 &= [c_5(A/B) + c_6](L_s/L) + [c_7(A/B) + c_8] \\ \beta &= [c_9(A/B) + c_{10}] \end{aligned} \quad (6-5-3)$$

where the parameters c_i are given in Table 6-5.3-1, and L_s is the length of the reduced-diameter section of the thermowell. Interpolation is not allowed between the entries for Table 6-5.3-1, although the designer may use appropriate beam models or finite-element methods to determine the H_f for thermowells of other dimensions.

The value of H_f will be approximately 1 for slender thermowells with $L/A > 10$ and $A = B$. For short thermowells or for those for which $A \neq B$, values of H_f will depend in detail on the taper ratio, bore diameter, and existence of any step; values may vary from approximately 0.6 to 1.5.

Step 4. Calculate the added mass correction factor for the fluid, $H_{a,f}$:

$$H_{a,f} = 1 - \frac{\rho}{2\rho_m} \quad (6-5-4)$$

Table 6-5.3-1 Parameters for Natural Frequency Calculation for Step-Shank Thermowells

Parameters	$B = 2.22$ cm (0.875 in.)	$B = 1.27$ cm (0.50 in.)
c_1	1.410	1.407
c_2	-0.949	-0.839
c_3	-0.091	-0.022
c_4	1.132	1.022
c_5	-1.714	-2.228
c_6	0.865	1.594
c_7	0.861	1.313
c_8	1.000	0.362
c_9	9.275	8.299
c_{10}	-7.466	-5.376

or alternatively, set $H_{a,f} = 1.0$ exactly for steam service or similar low-density gas, or $H_{a,f} = 0.94$ for liquid water. For a highly dense liquid, $H_{a,f}$ may be considerably lower (e.g., $H_{a,f} = 0.90$ for a fluid density of 1 600 kg/m³ and a thermowell density of 8 000 kg/m³).

Step 5. Calculate the sensor-mass correction factor $H_{a,s}$:

$$H_{a,s} = 1 - \frac{\rho_s}{2\rho_m} \left[\frac{1}{(D_a/d)^2 - 1} \right] \quad (6-5-5)$$

where ρ_s = average density of the temperature sensor to be inserted in the thermowell. For a sensor with compacted, mineral-insulated, metal-sheathed construction (either resistance thermometer or thermocouple), a typical sensor density is $\rho_s = 2\,700$ kg/m³ (169 lb/ft³), and this value should be used in the absence of detailed information on the sensor design. Alternatively, set $H_{a,s} = 0.96$ for a 0.25-in. nominal sensor diameter, or $H_{a,s} = 0.93$ for a 0.375-in. nominal sensor diameter.

Step 6. The lowest-order natural frequency of the thermowell with ideal support is given by

$$f_n = H_f H_{a,f} H_{a,s} f_a \quad (6-5-6)$$

6-6 MOUNTING COMPLIANCE FACTOR

The natural frequency, f_n , of a cantilever beam is calculated assuming an ideal, rigid base. In practice, however, this ideal is never achieved, and it is necessary to account for a significant reduction in natural frequency that results from flexibility of the thermowell mount or support [10]. The in situ natural frequency of the mounted thermowell is expressed in terms of a support flexibility or compliance factor, H_c , as

$$f_n^c = H_c f_n \quad (6-6-1)$$

The foundation compliance is highly sensitive to the radius of curvature b of the thermowell shank and support transition (see Fig. 4-1-1).

For cases where the support plane for the thermowell is at the thermowell root with fillet radius b [e.g., see Fig. 4-1-4, illustration (c) or (d)], the general form of the mounting-compliance frequency factor is

$$H_c = 1 - \frac{1}{(K_M / E)} \frac{\pi(A^4 - d^4)}{32L[1 + 1.5(b/A)]^2} \quad (6-6-2)$$

where

A = root diameter of the thermowell

b = fillet radius at the root of the thermowell

E = elastic modulus of the thermowell material

K_M = rotational stiffness of the thermowell support (discussed below)

L = unsupported length of the thermowell

When the fillet radius at the root of the thermowell, b , is not known, it shall be set to zero. For weld-in installations where the weld fillet is not located directly at the root of the thermowell, the fillet radius b is not equivalent to the fillet radius of the weld. Instead, the value of b shall be determined from the fillet geometry at the root. For cases in which the support plane of the thermowell has a geometry without a clear fillet at the support plane (e.g., see the indicated unsupported length in Fig. 6-6-1 for socket-weld or weld-in thermowells), set b equal to zero.

The stiffness, K_M , relates the angular displacement, $\delta\theta$, of the thermowell at its support plane to a bending moment, M , applied to the thermowell:

$$K_M \cdot \delta\theta = M \quad (6-6-3)$$

For a beam of uniform circular cross section with outside diameter D supported by a semi-infinite base of the same modulus as the thermowell material, K_M is given by

$$K_M = \frac{E}{0.787} \left(\frac{D}{2} \right)^3 \quad (6-6-4)$$

Since the base compliance depends predominantly on the root diameter, eq. (6-6-2) should be applied to thermowells of general geometry by replacing D with A . Approximating $A^4 - d^4$ by $0.99A^4$, one obtains for a semi-infinite base

$$H_c = 1 - (0.61) \frac{(A/L)}{[1 + 1.5(b/A)]^2} \quad (6-6-5)$$

The value of K_M attained in practice may be significantly less than that of eq. (6-6-4), due either to the flexibility of the supporting piping or to the flexibility of the thermowell attachment to the piping [11]. Reference [11], models of the piping system under static load, or literature results should be used to determine K_M . For thermowells installed in thin-wall pipes with outer connection heads, the mass of the head will cause a significant perturbation on the resonance frequency of the

thermowell. The increased susceptibility of small-bore fittings to vibration fatigue is well known, and practices designed to minimize the risk of cantilevered small-bore fittings also apply to thermowell connections and electrical-connection-head top works.

6-7 UNSUPPORTED LENGTH, DIAMETER, AND FILLET RADIUS

For the purpose of calculating the natural frequency of a thermowell, the unsupported length L shall be taken as the axial distance from the tip of the thermowell to the point where the thermowell is rigidly supported. The effect of support compliance (flexibility) is included by a series of correction factors applied to the ideal case.

In some installations, or for some varieties of thermowells, the definition of the unsupported length and the corresponding diameter A and fillet radius b is not obvious. Guidance for a variety of thermowell types is given below and illustrated in Fig. 6-6-1.

(a) *Lap-Joint and Flanged Thermowells.* For flanged thermowells, the unsupported length extends from the tip of the thermowell to the flanged face that is part of the machined thermowell.

(b) *Threaded Thermowells.* A threaded connection has greater compliance than a semi-infinite base. If the unsupported length is taken as the distance between the tip of the thermowell and the first engagement of the thread, then the increased compliance of the threaded joint (not including any additional compliance of the piping beyond the joint) should be accounted for by using

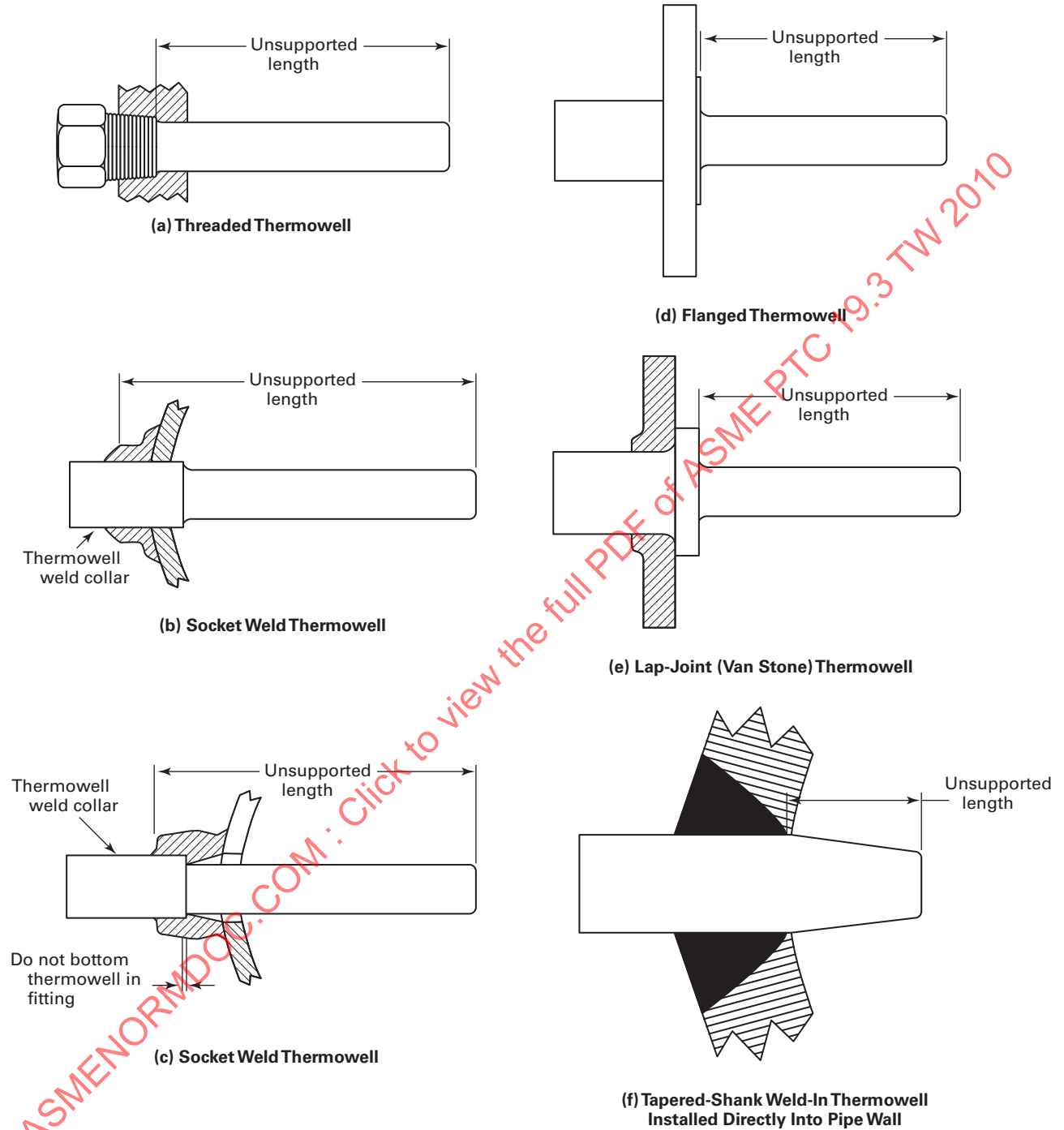
$$H_c = 1 - 0.9(A/L) \quad (6-7-1)$$

The support-point diameter, A , shall be equal to the diameter of the thermowell shank at the beginning of the transition to the threaded section of the thermowell. Although there may be a fillet between the shank and the threaded portion of the thermowell, this fillet does not effectively reduce the bending compliance of the thermowell or reduce stress concentration at the threads. Consequently, the fillet radius shall be taken as $b = 0$.

(c) *Socket-Weld Thermowells.* The clearance between a socket adaptor and the thermowell wall is sufficiently large that the joint between the adaptor and thermowell wall cannot be treated as an interference fit. In this case, the unsupported length extends from the tip of the thermowell to the point on the thermowell where the socket is welded to the adaptor. For design purposes, this point shall be taken as the midpoint of the thermowell weld collar, as indicated in Fig. 6-6-1. The base diameter, A , shall be taken as the diameter of the thermowell shank at the transition to the weld collar, and the fillet radius shall be taken as $b = 0$.

(d) *Weld-In Thermowells.* The unsupported length will depend on how far the thermowell is inserted into the pipe and on the degree of penetration of the weld. Weld

Fig. 6-6-1 Unsupported Length of Thermowells



specifications and tolerances for those specifications shall be taken into account when determining the unsupported length, which shall be taken as the longest length possible within the weld and location tolerances. Eqs. (6-6-2) and (6-6-5) for thermowell bending compliance and eq. (6-12-4) for stress-concentration factor apply only for weld fillets on the inside of the pipe. Consequently, the fillet radius shall be taken as $b = 0$ even if there is a substantial fillet on the outside of the pipe.

(e) *Thermowells With Support Collars*. Support collars or other means of support are outside the scope of the Standard. The use of support collars is not generally recommended, as rigid support can be obtained only with an interference fit between the support collar and the installed piping. In special cases, small support-collar gaps filled with a viscous process fluid may add significant damping, thereby suppressing thermowell resonances, and engineering models that account for the degree of support and fluid damping may be useful. Such designs require methods beyond the scope of this Standard. If a section of thermowell shank of increased diameter is used in the support-collar design, the added mass will shift the natural frequency of the thermowell, and the correlations for natural frequency supplied in this Standard do not apply.

Note that for Fig. 6-6-1, illustrations (b) and (c), the support plane for the thermowell is located at the intersection of the seal weld and the clearance gap between the thermowell shank and the adaptor. Such cases where the thermowell root is geometrically similar to a crack have reduced fatigue strength and should be avoided when the limiting factor for the thermowell velocity rating is fatigue strength.

6-8 FREQUENCY LIMIT

6-8.1 Overview

When a thermowell is immersed in a flowing fluid, the shedding of vortices produces the following two types of force on the thermowell (see Fig. 6-3.1-1):

- (a) an oscillating-lift force, transverse to the fluid flow at frequency f_s
- (b) an oscillating-drag force, in-line with the fluid flow at frequency $2f_s$

As the fluid velocity is increased, the rate of vortex shedding increases linearly while the magnitude of the forces increases with the square of the fluid velocity. The thermowell responds elastically according to the force distribution and its variation in time. Should the vortex shedding rate coincide with the natural frequency of the thermowell, resonance occurs and is attended with a dramatic increase in the dynamic bending stresses. The fluid velocity at which this takes place is referred to as a velocity critical. There are a minimum of two velocity criticals for each natural frequency of the thermowell: one describing the lift and the other describing the in-

line response. These should not be confused with the critical velocity marking the transition to turbulent boundary-layer flow.

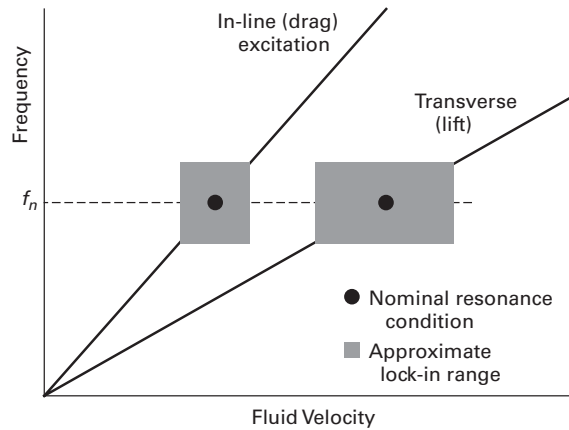
Since the in-line force fluctuates at twice the frequency of the lift excitation, the corresponding velocity critical is approximately one-half that required for lift resonance. For any given fluid velocity, both forces are acting on the thermowell with the result that the tip of the thermowell sweeps out an orbital (Lissajou figure) that changes shape as the fluid velocity is increased.

If the natural frequency of the thermowell overlaps with either f_s or $2f_s$, a large resonant buildup in vibration amplitude can occur, resulting in failure of either the thermowell or the temperature sensor mounted in the thermowell (see Fig. 6-8.1-1). Because f_s is proportional to fluid velocity V , the in-line resonance occurs at half the velocity of the transverse resonance. Although the in-line force is only weakly excited, large vibration amplitudes may still be encountered due to the low damping of typical thermowells [3, 12–15]. The vibration amplitude is proportional to the force per unit area exerted by the fluid (see para. 6-3.2 and subsection 6-10) and the magnification factor (see subsection 6-9). Figure 6-8.1-2 illustrates the variation of vibration amplitude with fluid velocity.

Because the elastic response of the thermowell and the vortex shedding process are so closely coupled, the actual vortex shedding process is extremely nonlinear and can be expected to be captured or locked onto the structural resonance of the thermowell [2]. This capture takes place as the vortex shedding rate approaches a natural frequency of the thermowell. As the beam responds, the vortex shedding rate tends to settle onto the resonant frequency of the beam and remains locked in for a considerable range of fluid velocities (refer to paras. 6-8.4 and 6-8.5). The natural frequency of thermowells may be as high as several thousand hertz; together with the lock-in phenomenon, it is possible for a thermowell to encounter many thousands of fatigue cycles in a single start-up process, even if the vortex shedding rate does not coincide with the natural frequency of the thermowell during steady-state process conditions.

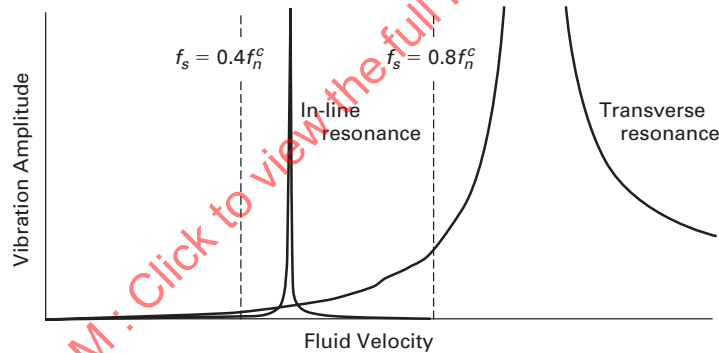
To prevent the occurrence of lock-in phenomena and to limit the buildup of vibration amplitudes to a safe value, the resonant frequency of the installed thermowell shall be sufficiently higher than either the in-line or the transverse resonance condition. Operation of the thermowell through the in-line resonance is allowed only if the cyclic stresses at the resonance condition are acceptably small (see paras. 6-8.4 and 6-8.5). The user is cautioned, however, that even if a thermowell is sufficiently strong to withstand in-line resonance, tip vibration at an in-line resonance may be extreme, leading to sensor degradation or destruction. In all cases, operation near the transverse resonance condition shall be avoided completely, other than exceptions discussed in para. 6-12.5.

Fig. 6-8.1-1 Schematic Indicating Excitation of Resonances When Excitation Frequency Coincides With the Thermowell Natural Frequency



GENERAL NOTE: Lock-in between the fluid vortices and the thermowell mechanical resonances can cause a resonance condition within the approximate boundaries indicated by the gray boxes.

Fig. 6-8.1-2 Schematic Showing the Amplitude Response of a Thermowell Subjected to Fluid-Induced Forces as Solid Lines, for In-Line and Transverse Excitation Modes



GENERAL NOTE: The frequency limits discussed in paras. 6-8.2 to 6-8.4 are shown as dotted lines. The figure ignores lock-in effects, which can shift the locations of the resonances, as shown in Fig. 6-8.1-1.

6-8.2 Frequency Limit for Low-Density Gases

For fluids of sufficiently low density and with $Re < 10^5$, the intrinsic damping of the thermowell sufficiently suppresses the in-line vibrations due to vortex shedding. The intrinsic damping factor, ζ , of common thermowells should be conservatively set at 0.0005 [2]. Values of ζ known from direct modal measurements should be used, although the designer is cautioned that the damping factor is not highly reproducible among multiple thermowells of similar design and may depend on details of the thermowell mounting, such as gasket choice for flanged thermowells.

Calculate the mass damping factor, or Scruton number, as

$$N_{Sc} = \pi^2 \zeta (\rho_m / \rho) [1 - (d/B)^2] \quad (6-8-1)$$

If $N_{Sc} > 2.5$ and $Re < 10^5$, in-line resonance is suppressed, and the installed natural frequency of the thermowell shall satisfy

$$f_s < 0.8f_n^c \quad (6-8-2)$$

If $N_{Sc} > 64$ and $Re < 10^5$, both transverse and in-line resonances are suppressed. Designs for a fluid velocity beyond the limit of eq. (6-8-2) are possible but shall consider the excitation of higher-order thermowell resonances. These calculations are beyond the scope of this Standard.

If $N_{Sc} \leq 2.5$ or $Re \geq 10^5$, the limits of 6-8.3 shall apply.

6-8.3 Frequency Limit for the General Case

If the conditions in para. 6-8.2 do not apply, establish the frequency limit of the thermowell as described in the following four steps:

Step 1. If the damping factor is known, set the magnification factor F'_M for in-line resonance to $1/(2\zeta)$. Otherwise, set the amplification factor for in-line resonance to $1/(2\zeta) = 1,000$, an upper limit for amplification exactly on resonance. The requirement that the natural resonance coincides with the in-line resonance also fixes the amplification factor for the transverse resonance to a value of $4/3$ [obtained by evaluating eq. (6-9-1) with $r = 0.5$].

If the Strouhal number is calculated with the correlation of eq. (6-4-2), set the fluid velocity for the in-line resonance to

$$V_{IR} = \begin{cases} \frac{Bf_n^c}{2N_s} \left(1 - \frac{22\mu}{B\rho V} \right) + \frac{22\mu}{B\rho} & \text{for } 22 \leq Re < 1,300 \\ \frac{Bf_n^c}{2N_s} \left[1 - \frac{a(R)}{N_s} \log_{10} \left(\frac{Bf_n^c}{2N_s V} \right) \right] & \text{for } 1,300 \leq Re < 5 \times 10^5 \\ \frac{Bf_n^c}{2N_s} & \text{for } 5 \times 10^5 \leq Re < 10^7 \end{cases} \quad (6-8-3)$$

where

$$\begin{aligned} a(R) &= 0.0285R^2 - 0.0496R \\ R &= \log_{10}(Re / Re_0) \\ Re_0 &= 1,300 \end{aligned} \quad (6-8-4)$$

and N_s is evaluated at the design velocity V , and not at the value V_{IR} [the factors to the right of $(Bf_n^c)/(2N_s)$ correct for the difference in N_s at V and at V_{IR}].

If the Strouhal number is calculated with the simplified relation of eq. (6-4-4), set the fluid velocity for the in-line resonance to

$$V_{IR} = \frac{Bf_n^c}{2N_s} \quad (6-8-5)$$

Step 2. Evaluate the cyclic drag stress following subsection 6-12. The cyclic lift stress should be neglected in evaluating the peak oscillatory bending stress [see eq. (6-10-6) and paras. 6-10.2 to 6-10.5].

Step 3. If the thermowell passes the cyclic stress condition [eq. (6-12-1)] for operation at the in-line resonance condition, the installed natural frequency, f_n^c , shall satisfy

$$f_s < 0.8f_n^c \quad (6-8-6)$$

Step 4. If the thermowell fails the cyclic stress condition for operation at the in-line resonance condition, the installed natural frequency, f_n^c , shall be high enough to limit excitation of the in-line resonance, as shown in Figs. 6-8.1-1 and 6-8.1-2. In this case, f_n^c shall satisfy

$$f_s < 0.4f_n^c \quad (6-8-7)$$

6-8.4 Frequency Limit When the In-Line Resonance Does Not Limit Operation

In cases where the thermowell passes the cyclic stress condition for operation at the in-line resonance condition, care shall still be taken that in steady state the flow condition will not coincide with the thermowell resonance. The steady-state fluid velocity should meet one of the following conditions:

$$f_s(\text{steady state}) < 0.4f_n^c \quad (6-8-8)$$

or

$$0.6f_n^c < f_s(\text{steady state}) < 0.8f_n^c \quad (6-8-9)$$

Graphically, these conditions are equivalent to operation at a fluid velocity intermediate between the two gray boxes in Fig. 6-8.1-1.

6-8.5 Passing Through the In-Line Critical

In cases where the thermowell design fails the cyclic stress condition for steady-state operation, transient exposure to the in-line resonance condition may be allowable, provided that certain criteria are met. A thermowell with a natural-frequency intermediate between the steady-state Strouhal frequency (which excites transverse vibrations) and twice the Strouhal frequency (which excites in-line vibrations) is subjected to large-amplitude vibration only for limited periods on start-up or shutdown, as the in-line vibrations are excited only when twice the Strouhal frequency coincides with the natural frequency of the thermowell.

Passage through the in-line resonance is allowed only if all of the following conditions are met:

- The process fluid is a gas.
- The thermowell is exposed to the in-line resonance condition only on start-up, shutdown, or other infrequent transient variations in fluid velocity.
- The sustained or steady-state peak stress is less than the fatigue limit for the number of cycles.
- The process fluid is known to not cause metallurgical changes to the thermowell material that would significantly reduce the fatigue resistance.
- The potential consequences of thermowell failure to equipment or personnel are sufficiently limited to be acceptable.

The number of cycles sustained for each flow-velocity transient shall be calculated assuming that lock-in phenomena occurs for a range of forcing frequencies equal to 20% of the natural frequency. If the criteria above are met, the designer shall evaluate the maximum stresses when the thermowell is excited at its natural frequency, in accordance with subsection 6-12, and determine whether the thermowell has sufficient fatigue strength for the expected number of start-up and shutdown events encountered by the thermowell in its lifetime.

Note that the design rules of PTC 19.3 TW ensure only the mechanical integrity of the thermowell. Passage through the in-line resonance may cause a severe vibration of the thermowell tip resulting in unacceptable sensor damage or drift.

6-9 MAGNIFICATION FACTOR

6-9.1 Magnification Factor Away From Resonance

The magnification factor, F_M , equals the ratio of thermowell deflection and stress at a given frequency to the deflection and stress at zero frequency. For frequencies outside the lock-in band of the vortex frequency [the lock-in band is equivalent to an r value in eqs. (6-9-1) and (6-9-2) in the range $0.8 < r < 1.2$] and at frequencies low enough that only the natural frequency of the thermowell is appreciably excited, the simple expressions below accurately model the magnification factor.

In the case of transverse lift resonance, the magnification factor is

$$F_M = \frac{1}{1-r^2} \quad (6-9-1)$$

$$r = \frac{f_s}{f_n^c}$$

f_n^c being the natural frequency of the thermowell including the reduction in the thermowell vibration frequency due to compliance of the foundation or support. For the case of in-line resonances, the magnification factor is

$$F_M' = \frac{1}{1-(r')^2} \quad (6-9-2)$$

$$r' = \frac{2f_s}{f_n^c}$$

6-9.2 Magnification Factor Near Resonance

When the natural frequency of the thermowell falls within the lock-in band of the vortex frequency, the thermowell deflection is limited by the intrinsic damping factor, ζ , of the thermowell. At resonance, the maximum magnification factor is

$$F_{M, \max} = \frac{1}{2\zeta} \quad \text{for excitation of a transverse (lift) resonance}$$

$$F_{M, \max}' = \frac{1}{2\zeta} \quad \text{for excitation of an in-line (drag) resonance} \quad (6-9-3)$$

6-10 BENDING STRESSES

6-10.1 Point of Maximum Stress

The peak stresses occur on the outside surface of the thermowell at the support plane for taper and straight-shank thermowells, and at either the support plane or the base of the reduced-diameter section of shank for step-shank thermowells. Except

in special cases where a thermowell is supported along its shank, the support plane will be at the thermowell root. As shown in Fig. 6-10.1-1, stresses for fluid-flow-induced forces are obtained from the relation between the second moment of the beam, M , the moment of inertia, I , and the longitudinal stress in the thermowell:

$$S_z = \begin{cases} -yM/I & \text{for steady-state stresses} \\ -yMF_M/I & \text{for lift resonance stresses} \\ -yMF_M'/I & \text{for drag resonance stresses} \end{cases} \quad (6-10-1)$$

Eq. (6-10-1) is evaluated at $x = 0$, $y = D(z_s)/2$ (steady-state and oscillating-drag stresses) or $x = D(z_s)/2$, $y = 0$ (oscillating-lift stresses), and $z = z_s$, where z_s is equal to either zero for evaluation of stress at the support plane, or to the distance from the support plane of the thermocouple to the cross section where the stress is evaluated. [For the common case of a thermowell supported at its root, eq. (6-10-1) is evaluated at $z = 0$ and either $x = 0$, $y = A/2$, or $x = A/2$, $y = 0$.] The general equation relating peak-second moment for each type of force acting on the thermowell is

$$M_\beta = P_\beta \int_{z_s}^L D(z)(z-z_s)dz \quad (6-10-2)$$

where P_β denotes either P_D , P_d , or P_l and is equal to the force per unit area applied transverse to the beam. For a thermowell shielded from fluid flow for a distance L_0 from the thermowell root, L_0 replaces z_s as the lower limit of the integration in eq. (6-10-2), and the result for M_β is used in eq. (6-10-1) to obtain the value of S_z .

For calculation purposes, it is convenient to define G as

$$G \equiv \frac{M_\beta D(z_s)}{2P_\beta I(z_s)} = \frac{32D(z_s)}{\pi(D(z_s)^4 - d^4)} \int_{z_s}^L D(z)(z-z_s)dz \quad (6-10-3)$$

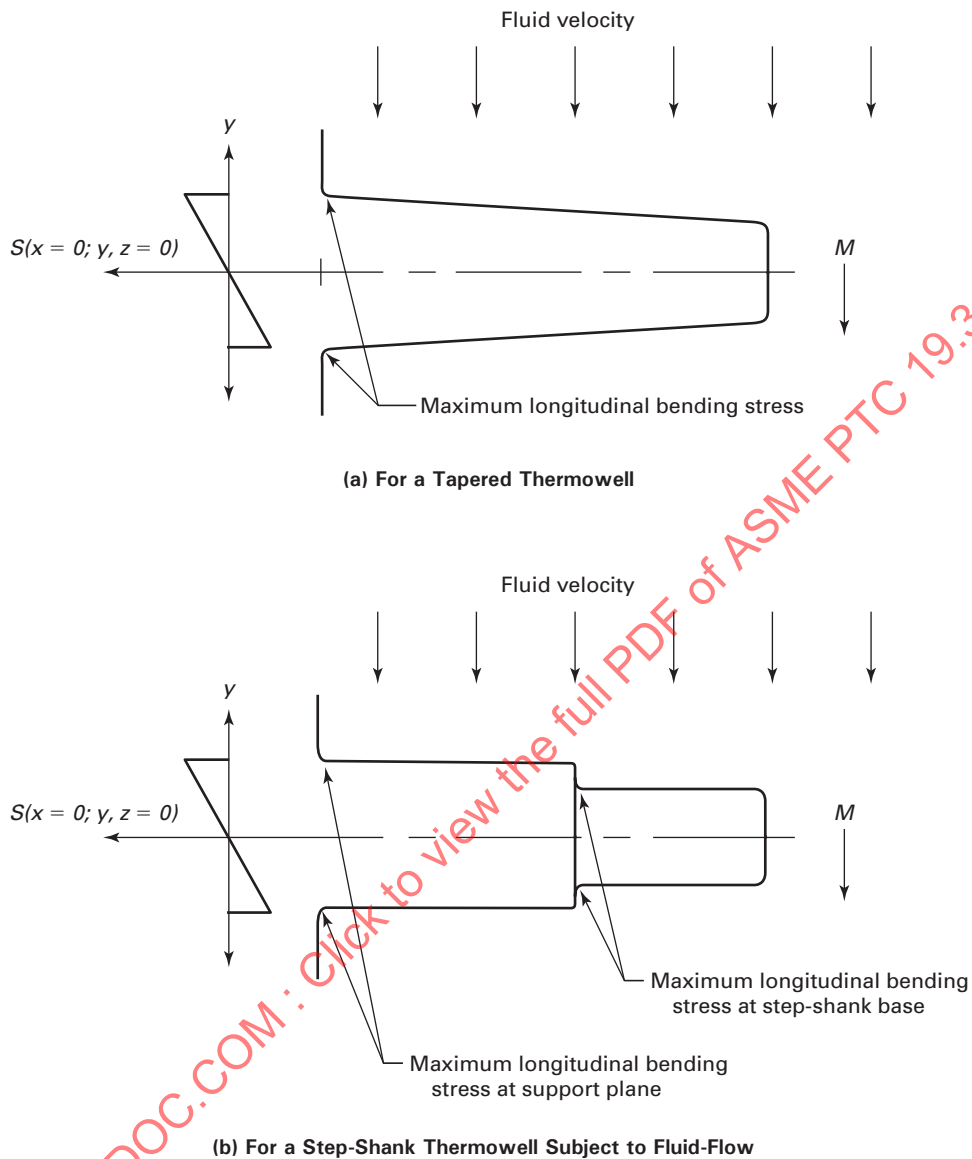
The dimensionless quantity G depends only on the thermowell geometry. At the thermowell support point, G is evaluated with the lower limit of integration $z_s = \max(0, L_0)$ and shall be denoted G_{SP} . For step-shank thermowells, peak stress amplitudes need to be evaluated at the base of the reduced-diameter shank, of diameter B , as well as at the support point. In this case, G is evaluated with the lower limit of integration $z_s = \max(L - L_S, L_0)$ and shall be denoted G_{RD} .

At the thermowell support point, the steady-state drag stress on the downstream side of the thermowell is

$$S_D = G_\beta P_D = \frac{G_\beta C_D \rho V^2}{2} \quad (6-10-4)$$

where the equation uses the convention that compressive stresses have a positive sign and G_β is a placeholder for either G_{SP} for stress evaluated at the support point or G_{RD} for stress evaluated at the reduced-diameter

Fig. 6-10.1-1 Bending Moment, Stress at the Support Plane, and Locations of Maximum Steady-State or Oscillating In-Line Stress



GENERAL NOTE: Locations for maximum oscillating transverse stress are located in the same plane, but at points rotated 90 deg about the thermowell axis.

shank (see paras. 6-10.2 to 6-10.5). The amplitudes for oscillating-lift and oscillating-drag stresses are

$$S_L = G_\beta F_M P_l = \frac{G_\beta C_l F_M \rho V^2}{2} \quad (6-10-5)$$

$$S_d = G_\beta F_M P_d = \frac{G_\beta C_d F_M \rho V^2}{2} \quad (6-10-6)$$

Equation (6-4-5) gives coefficients C_D , C_l , and C_d ; eqs. (6-9-1) through (6-9-3) give magnification factors F_M and F_M' . The stress amplitudes are used in estimating the combined bending stress in subsection 6-12.

For step-shank thermowells, peak stress amplitudes shall be evaluated at the base of the reduced-diameter shank, of diameter B , as well as at the support point. In this case, evaluating G at $D(z_s) = B$ for use in eqs. (6-10-4) through (6-10-6) gives the stress amplitudes at the base of the reduced-diameter shank.

6-10.2 Maximum Stress for Tapered or Straight Thermowells, No Shielding From Flow

With no shielding for a tapered or straight thermowell subject to a constant force per unit area, eq. (6-10-2) is integrated along the whole thermowell length. The result for the parameter G is

$$G_{SP} = \frac{16L^2}{3\pi A^2 [1 - (d/A)^4]} [1 + 2(B/A)] \quad (6-10-7)$$

The steady-state drag stress and oscillating-lift and oscillating-drag stress amplitudes are evaluated using eqs. (6-10-4) through (6-10-6) and the value of G_{SP} from eq. (6-10-7).

6-10.3 Maximum Stress for Tapered or Straight Thermowells, Shielded From Flow

If the thermowell is shielded from fluid flow from the thermowell support plane to a distance L_0 along the shank, the point of maximum stress shall be evaluated at the support plane for a straight or tapered thermowell. The parameter G is given as

$$G_{SP} = \frac{16L^2}{3\pi A^2 [1 - (d/A)^4]} \left\{ 3 \left[1 - (L_0/L)^2 \right] + 2(B/A - 1) \left[1 - (L_0/L)^3 \right] \right\} \quad (6-10-8)$$

The steady-state drag stress and oscillating-lift and oscillating-drag stress amplitudes are evaluated using eqs. (6-10-4) through (6-10-6) and the value of G_{SP} from eq. (6-10-8).

The majority of installed thermowells have some degree of shielding from fluid flow near the support point. However, the total second moment, and consequently the value of G , varies little from the shielded case when $L_0/L < 1$. For typical thermowells, the values of G_{SP} evaluated using eqs. (6-10-7) and (6-10-8) will differ by less than approximately 10% if $L_0/L < 0.3$.

6-10.4 Maximum Stress for Step-Shank Thermowells, No Shielding From Flow

For a step-shank thermowell, the peak stresses may occur either at the support plane of the thermowell or at the base of the reduced-diameter step shank. At the support plane, the parameter G is given as

$$G_{SP} = \frac{16L^2}{\pi A^2 [1 - (d/A)^4]} \left\{ (B/A) + [1 - (B/A)] [1 - (L_s/L)]^2 \right\} \quad (6-10-9)$$

where the reduced-diameter step has length L_s and diameter B . At the reduced-diameter shank step, the parameter G is given as

$$G_{RD} = \frac{16L_s^2}{\pi B^2 [1 - (d/B)^4]} \quad (6-10-10)$$

At the support plane of the thermowell, the steady-state drag stress and oscillating-lift and oscillating-drag stresses are evaluated using eqs. (6-10-4) through (6-10-6) and the value of G_{SP} from eq. (6-10-9). At the base of the reduced-diameter step shank, stress amplitudes are evaluated using eqs. (6-10-4) through (6-10-6) and the value of G_{RD} from eq. (6-10-10).

6-10.5 Maximum Stress for Step-Shank Thermowells, Shielded From Flow

If the thermowell is shielded from fluid flow from the thermowell support plane to a distance L_0 along the shank, the parameter G at the support plane for a step-shank thermowell is given as

$$G_{SP} = \frac{16L^2}{\pi A^2 [1 - (d/A)^4]} \left\{ (B/A) + [1 - (B/A)] \left[1 - (L_s/L) \right]^2 - (L_0/L)^2 \right\} \quad \text{for } L_0 < L - L_s$$

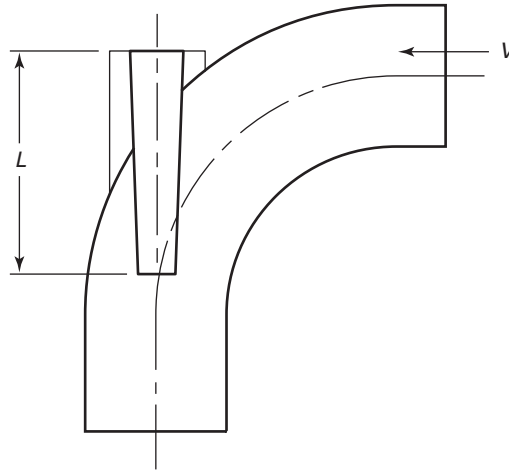
$$G_{SP} = \frac{16BL^2}{\pi A^3 [1 - (d/A)^4]} \left[1 - (L_0/L)^2 \right] \quad \text{for } L_0 \geq L - L_s \quad (6-10-11)$$

At the reduced-diameter shank step, eq. (6-10-10) applies for shielding from fluid flow with $L_0 < L - L_s$. For $L_0 > L - L_s$, the parameter G is given as

$$G_{RD} = \frac{16L^2}{\pi B^2 [1 - (d/B)^4]} \left[1 - (L_0/L) \right] \left[2(L_s/L) - 1 + (L_0/L) \right] \quad (6-10-12)$$

At the support plane of the thermowell, the steady-state drag stress and oscillating-lift and oscillating-drag stresses are evaluated using eqs. (6-10-4) through (6-10-6) and the appropriate value of G_{SP} from eq. (6-10-11). At the base of the reduced-diameter step shank, stress amplitudes are evaluated using eqs. (6-10-4) through (6-10-6) and the appropriate value of G_{RD} from eq. (6-10-10) or (6-10-12).

Fig. 6-10.7-1 Mounting of a Thermowell in an Elbow, With the Tip Facing Downstream



6-10.6 Partial Exposure to Fluid Flow

For thermowells with only partial exposure to the fluid flow not covered by paras. 6-10.3 or 6-10.5, the bending moment should be calculated by integrating the moment created by the pressure acting on the projected area exposed to fluid flow.

6-10.7 Mounting of Thermowells in an Elbow

For thermowells mounted in an elbow and pointing downstream, as shown in Fig. 6-10.7-1, the exact flow path is difficult to model. Thus, the projected area shall be conservatively estimated as the projected area of the thermowell if the flow were to be normal to the thermowell axis along the length of the thermowell exposed to fluid flow. The geometry to be used in the calculation of thermowell ratings is given in Fig. 6-10.7-2.

Thermowells mounted in an elbow with the tip pointing upstream, as shown in Fig. 6-10.7-3, are often preferable to a mounting with the tip pointing downstream. Provided that the flow lines in the upstream pipe are closely approximated as lines parallel to the pipe axis, there is minimal transverse fluid flow near the tip of the thermowell, with a consequent reduction of the bending moment. Tip effects are important, and the effective Strouhal number varies with the angle of flow with respect to the thermowell axis [16]. For such an installation, calculation of the bending moment is beyond the scope of this Standard. Predictions of the bending moment and Strouhal number should be made by using computational fluid dynamics or experimental measurements to determine the fluid flow pattern, including the perturbations of upstream piping elements, and consulting reference [16] to determine the forces on the thermowell.

6-11 PRESSURE AND SHEAR STRESSES

In addition to the bending stresses, there are the following stresses:

- (a) radial pressure stress, S_r
- (b) tangential pressure stress, S_t
- (c) axial pressure stress, S_a
- (d) shear stress due to flow impingement

Shear stresses are small relative to the other stresses and should be neglected.

For an external operating pressure P , the radial and hoop stresses at the root are given by

$$S_r = P \quad (6-11-1)$$

$$S_t = P \frac{1 + (d/A)^2}{1 - (d/A)^2} \quad (6-11-2)$$

where d is the bore diameter of the thermowell and compressive stresses have a positive sign.

The axial pressure stress is given by

$$S_a = \frac{P}{1 - (d/A)^2} \quad (6-11-3)$$

6-12 STEADY-STATE STATIC AND DYNAMIC STRESS LIMITS

6-12.1 Overview

In addition to the hydrostatic pressure limit of subsection 6-13, thermowells shall meet strength criteria to prevent fatigue failure. For conditions of low fluid velocity, as described in para. 6-3.6, the fluid does not impart sufficient momentum to the thermowell to cause fatigue

Fig. 6-10.7-2 Geometry to Be Used in Calculation of Thermowell Ratings

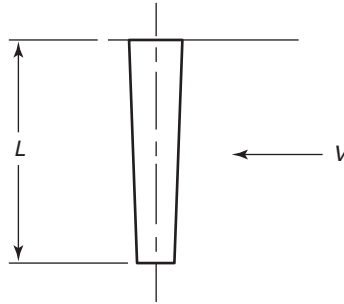
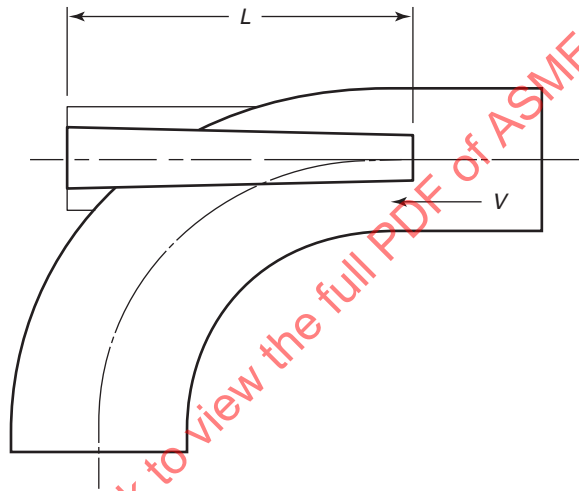


Fig. 6-10.7-3 Mounting of a Thermowell in an Elbow, With the Tip Facing Upstream



failure and only the steady-state stress limit in para. 6-12.2 shall be met. For higher fluid velocities, the thermowell shall meet the requirements described in para. 6-12.2 for steady-state stresses and paras. 6-12.3 and 6-12.4 for dynamic stresses. Paragraph 6-12.5 describes the special case of thermowells designed for operation at fluid velocities where the Strouhal frequency exceeds the natural frequency of the mounted thermowell.

6-12.2 Steady-State Stress Limits

The steady-state loading from the combined effects of hydrostatic fluid pressure and nonoscillating drag produces a point of maximum stress, S_{\max} , in the thermowell located on the outer surface of the thermowell, at the downstream side of the base of the thermowell, along the axial direction of the thermowell. For design, S_{\max} is given by

$$S_{\max} = S_D + S_a \quad (6-12-1)$$

Using the Von Mises criteria for failure, the applied stresses S_{\max} , S_r , and S_t should satisfy

$$\sqrt{\frac{(S_{\max} - S_r)^2 + (S_{\max} - S_t)^2 + (S_t - S_r)^2}{2}} \leq 1.5S \quad (6-12-2)$$

where

S = maximum allowable stress of the material, as specified by the governing code

For combinations of materials and operating temperature not covered by the governing code, stress limits shall be established by test. Note that for service at elevated temperatures for extended periods, creep rate and creep rupture limit the allowable stress to values significantly below the stress limits obtained from short-term yield-strength tests.

6-12.3 Dynamic Stress Limits

The dynamic stresses are the result of periodic drag forces that cause the thermowell to oscillate in the direction of the stream and periodic lift forces that cause it to oscillate in the transverse direction. The dynamic stress amplitude shall not exceed the high-cycle maximum allowable stress amplitude. The peak oscillatory bending stress amplitude, $S_{o,\max}$, is taken as the amplitude of

Table 6-12.3-1 Allowable Fatigue-Stress Amplitude Limits for Material Class A and Class B

Thermowell Material Class	Metal State at Location of Maximum Stress	Value of S_f , ksi/MPa
A	As-welded [Note (1)] or threaded	3.0 / 20.7
A	Welded, then machined [Note (2)]	4.7 / 32.4
A	No welds [Note (3)]	7.0 / 48.3
B	As-welded [Note (1)] or threaded	5.4 / 37.2
B	Welded, then machined [Note (2)]	9.1 / 62.8
B	No welds [Note (3)]	13.6 / 93.8

GENERAL NOTES:

(a) Class A is carbon, low-alloy, series 4XX, and high-alloy steels not covered in class B.

(b) Class B is series 3XX high-alloy steels, nickel–chromium–iron alloy, nickel–iron–chromium alloy, and nickel–copper alloys.

NOTES:

(1) Location of maximum stress coincides with either a welded joint or associated heat-affected zone [e.g., a weld-in thermowell, as shown in Fig. 4-1-4, illustration (d)].

(2) Location of maximum stress coincides with either a welded joint or associated heat-affected zone, which has been machined to a smooth surface subsequent to welding [e.g., a flanged thermowell, as shown in Fig. 4-1-4, illustration (c)]. The welded joint must be a full-penetration weld, and visual and magnetic-particle or liquid-dye-penetrant examination is required after machining. In the absence of full-penetration welds and/or weld inspection, “as welded” values for S_f should be used.

(3) Location of maximum stress is a smooth, machined surface and does not coincide with either welded joint or associated heat-affected zone (e.g., a lap-joint thermowell).

the two components added in quadrature, amplified by a stress concentration factor, K_t :

$$S_{o,max} = K_t (S_d^2 + S_L^2)^{1/2} \quad (6-12-3)$$

The thermowell design should be evaluated for fatigue in accordance with ASME BPVC, Section VIII, Division 2, Part 5, independent of the requirements of para. 6-12.3. Alternatively, the thermowell design should be evaluated for fatigue in accordance with the requirements of para. 6-12.3.

In the absence of more directly applicable data, the following should be used:

(a) A stress concentration factor of $K_t = 2.2$ should be used in the absence of specific dimensional details of the fillet at the base of the thermowell.

(b) Threaded connections shall use a stress concentration factor of $K_t = 2.3$ as a minimum.

(c) For known fillet radii b and root diameter A , K_t shall be obtained from

$$\begin{aligned} K_t &= 1.1 + 0.033(A/b) \text{ for } A/b < 33 \\ K_t &= 2.2 \text{ for } A/b \geq 33 \end{aligned} \quad (6-12-4)$$

The peak oscillatory bending stress amplitude $S_{o,max}$ shall not exceed the fatigue-endurance limit, adjusted for temperature and environmental effects:

$$S_{o,max} < F_T \cdot F_E \cdot S_f \quad (6-12-5)$$

where S_f is the allowable fatigue-stress amplitude limit in air at room temperature. F_E is an environmental factor ($F_E \leq 1$) allowing designers to adjust fatigue limits, when appropriate, for environmental effects such as corrosive service. ASME B31.1, *Power Piping*, Appendices IV

and V provide guidance on corrosion control and piping corrosion. F_T is a temperature correction factor given by

$$F_T = E(T)/E_{ref} \quad (6-12-6)$$

where $E(T)$ is the elastic modulus at the operation temperature. For material class A (see Table 6-12.3-1), E_{ref} equals 202 GPa (29.3×10^6 psi), except for the low-Cr alloys, for which E_{ref} equals 213 GPa (30.9×10^6 psi). For material class B (see Table 6-12.3-1), E_{ref} equals 195 GPa (28.3×10^6 psi), except for the nickel–copper alloys, for which E_{ref} equals 179 GPa (26.0×10^6 psi).

Because the natural frequency of thermowells is typically hundreds of hertz, the total number of fatigue cycles can readily exceed 10^{11} during the thermowell lifetime and S_f should be evaluated at the design-cycle limit. In the absence of more directly applicable data, the values of S_f in the high-cycle limit in Table 6-12.3-1 should be used for a design life of 10^{11} cycles for typical hydrocarbon and steam environments. Use of the values in Table 6-12.3-1 for other environments or at temperatures in excess of 427°C (800°F) may require a reduction in the environmental factor F_E to account for corrosion or related effects.

When thermowells are welded into a pipe or adaptor of different composition than the thermowell, the value of S_f shall be the smaller of the values for the two metals. The designer shall consider the effects on fatigue strength of the dissimilar weld and thermal-expansion mismatch between the materials.

For materials not covered by Table 6-12.3-1 or ASME BPVC Section VIII, Division 2, fatigue-strength amplitude limits shall be established by test. Testing shall be in accordance with the provisions of ASME BPVC Section VIII, Division 2, Annex 5.F. A fatigue analysis in

accordance with ASME BPVC Section VIII, Division 2, Part 5 (latest edition) is required for conditions outside the scope of Table 6-12.3-1.

The values in Table 6-12.3-1 are evaluated for 10^{11} fatigue cycles. For thermowells subject to a lower number of cycles over the design lifetime, the designer should use fatigue data for the appropriate number of cycles.

6-12.4 Maximum-Stress Locations for Step-Shank Thermowells

For step-shank thermowells, the stress criteria [eqs. (6-12-2) and (6-12-5)] shall be evaluated for the following two locations:

- (a) at the support plane of the thermowell
- (b) at the root of the reduced-diameter portion of the shank

First, evaluate the stress at the thermowell support plane, using the procedures described in subsection 6-10. Second, evaluate the stress at the root of the reduced-diameter portion of the shank, substituting (B/b_s) for (A/b) in eq. (6-12-4), where b_s is the fillet radius at this root.

6-12.5 Supercritical Operation

It is recognized that where the fluid density is low, namely low-pressure gases with densities less than 1 kg/m^3 (0.06 lb/ft^3), it is possible to design a thermowell for supercritical operation, defined as an operation where the Strouhal frequency exceeds the natural frequency of the mounted thermowell: $f_s > f_n^c$. Finite-element and modal analysis methods are generally required, but in principle, if the stresses of the lowest-order mode at the lift resonance condition are well below both the maximum allowable stress (static loads) and the fatigue allowable stress (dynamic loads), then the second order mode should be considered as a basis for thermowell design and selection. The success of such operation is dependent on many factors and shall be handled on a case-by-case basis. Supercritical operation is discouraged in performance testing of rotating equipment.

6-13 PRESSURE LIMIT

The external pressure rating of the thermowell shall be determined as follows:

Step 1. For pressure ratings less than 103 MPa (15 ksi), use UG-28 of Section VIII, Division 1 of the ASME BPVC to calculate the allowable external pressure, P_c , (as defined in UG-28) for a cylinder of outer diameter B , inner diameter d , and length L for straight and tapered thermowells, or length L_s for step-shank thermowells.

As a simplified alternative, and for materials not covered by UG-28, the allowable external pressure should be calculated as

$$P_c = 0.66S \left[\frac{2.167}{2B/(B-d)} - 0.0833 \right] \quad (6-13-1)$$

where

S = the maximum allowable stress of the governing code

The value of P_c calculated by eq. (6-13-1) may be as much as 17% lower than the value calculated by UG-28 for some materials at some temperatures.

At temperatures beyond the limits established by UG-28, designers should use eq. (6-13-1). In determining the appropriate value of S , designers should consider the possibility of creep buckling, especially for the larger values of d/B allowed by Tables 4-1-1 and 4-2-1.

For high-pressure [$> 103 \text{ MPa}$ (15 ksi)] service, use ASME BPVC Section VIII, Division 3, or ASME B31.3, Chapter IX.

Step 2. The minimum tip thickness, t , (see Fig. 4-1-1) shall always be equal to or greater than the minimum wall thickness of the shank. (Refer to Tables 4-1-1 and 4-2-1 for minimum allowed wall thickness.) Calculate the allowable pressure, P_t , for the thickness t using

$$P_t = \frac{S}{0.13} \left(\frac{t}{d} \right)^2 \quad (6-13-2)$$

where

d = thermowell bore diameter

S = maximum allowable stress

Step 3. The external pressure rating, P_r , of the thermowell is the minimum of P_c , P_t , and P_f for flange thermowells, or the minimum of P_c and P_t for other types of thermowells. The external pressure rating shall exceed the maximum operating pressure, P . For a flanged or a lap-joint (Van Stone) thermowell, determine the allowable pressure of the flange, P_f , in accordance with ASME B16.5, *Pipe Flanges and Flange Fittings*, or the governing code. De-rate the flange to the minimum of P_t and P_c if $P_f > P_t$ or P_c . The maximum allowable operating pressure of the flange, P_f , shall be in accordance with ASME B16.5 or the governing code unless $P_t < P_f$ or $P_c < P_f$, in which case P_f shall be reduced to the minimum of P_t and P_c .

The design pressure shall be calculated at the temperature of the operating condition. More than one operating condition may require calculations at multiple temperatures.

Section 7

Overview of Calculations

7-1 QUANTITATIVE CRITERIA

There are four quantitative criteria that the thermowell shall meet to be fit for service.

(a) *Frequency Limit.* The resonance frequency of the thermowell shall be sufficiently high so that destructive oscillations are not excited by the fluid flow.

(b) *Dynamic Stress Limit.* The maximum primary dynamic stress shall not exceed the allowable fatigue stress limit.

(c) *Static Stress Limit.* The maximum steady-state stress on the thermowell shall not exceed the allowable stress, as determined by the Von Mises criteria.

(d) *Hydrostatic Pressure Limit.* The external pressure shall not exceed the pressure ratings of the thermowell tip, shank, and flange.

In addition, the suitability of the thermowell material for the process environment (section 5) shall be considered.

7-2 FLUID PROPERTIES

Collect the following fluid properties:

P = operating pressure, Pa (psi)

T = operating temperature, °C (°F)

V = process-fluid velocity, m/s (in./sec)

ν = specific volume (reciprocal of the fluid density ρ), m³/kg (in.³/lb)

μ = dynamic fluid viscosity, Pa·s (lb·sec/ft²), or

ν = kinematic fluid viscosity, m²/s (ft²/sec)

ρ = fluid density, kg/m³ (lb/in.³)

As noted in para. 6-4.1, viscosity is not needed if eq. (6-4-4) is used to specify the Strouhal number. In determining the process-fluid velocity, the designer should consider variations in the fluid velocity due to start-up or shutdown conditions (para. 6-3.3), valve operations, or other deviations from steady-state operation.

7-3 FLUID VELOCITY

Determine whether the fluid velocity is sufficiently low that no calculations other than the external pressure limit need to be performed (para. 6-3.6).

7-4 MATERIAL PROPERTIES AND DIMENSIONS

Collect thermowell material properties and dimensions.

7-4.1 Necessary Dimensions

Necessary dimensions include

A = outside diameter of thermowell at support plane or root, m (in.)

B = outside diameter at tip of thermowell, m (in.)

b = fillet radius at root base of thermowell, m (in.)

b_s = fillet radius at the base of the reduced-diameter length of a step-shank thermowell, m (in.)

d = bore diameter of thermowell, m (in.)

L = unsupported length of thermowell, m (in.)

L_s = length of reduced-diameter shank for a step-shank thermowell, m (in.)

t = minimum tip thickness of the thermowell, m (in.)

Values of A , b , and L may depend on details of the installation (see subsection 6-7), such as the weld geometry or the use of weld adaptors.

7-4.2 Material Properties

Necessary material properties of the thermowell include

E = modulus of elasticity at service temperature, Pa (psi)

S = maximum allowable working stress, Pa (psi)

S_f = fatigue endurance limit, in the high-cycle limit, Pa (psi)

ρ_m = mass density of the thermowell material, kg/m³ (lb/in.³)

Material properties do not need to be known to better than 1% of the property value for mass density, 5% for elastic modulus, and 10% for stress and endurance limits. Any interpolation of values in tables or figures should follow the methods recommended by the source of the tables.

Necessary properties of the temperature sensor installed in the thermowell include ρ_s = density of the temperature sensor, kg/m³ (lb/in.³)

The temperature-sensor density enters into the calculations only as a small correction, and the default values of para. 6-5.3 may be used.

7-4.3 Temperature Dependence of Properties

Evaluate the thermowell material density and all of the thermowell dimensions at ambient temperature. Fluid properties, the elastic modulus of the thermowell material, and the stress- and fatigue-amplitude limits shall all be evaluated at the operating temperature.

7-4.4 Installation Details

Necessary details of the installation include K_M = rotational stiffness of thermowell support, (N·m)/rad [(in.-lb)/rad]

See subsection 6-6 for additional information on the factor K_M . Subsection 6-7 discusses determination of values of L , A , and b for various installation types.

7-5 REYNOLDS AND STROUHAL NUMBERS

Calculate the Reynolds number and Strouhal number characterizing the fluid flow (para. 6-4.1). Obtain the coefficients of lift and drag characterizing fluid forces on the thermowell (para. 6-4.2).

7-6 NATURAL FREQUENCY AT OPERATION TEMPERATURE

Calculate the natural frequency of the mounted thermowell at operation temperature (subsection 6-5). This calculation consists of the following steps:

Step 1. Calculate the approximate natural frequency.

Step 2. Use the correlations of subsection 6-5 to correct for deviations from the approximate slender-beam theory.

Step 3. Correct for sensor and fluid mass.

Step 4. Correct for foundation compliance.

7-7 NATURAL FREQUENCY AT EXPECTED MODE OF OPERATION

Determine if the natural frequency of the mounted thermowell is sufficiently high for the expected mode of operation (subsection 6-8). For the general case, this determination will require calculation of the maximum stresses, as described in subsection 7-8.

7-8 STEADY-STATE AND DYNAMIC STRESSES

Calculate the maximum steady-state and dynamic stresses at the support plane of the thermowell (subsections 6-10 and 6-11). For step-shank thermowells, repeat this calculation at the root of the reduced-diameter portion of the shank.

7-9 ALLOWABLE FATIGUE LIMITS

Determine if the stresses exceed allowable fatigue limits (subsection 6-12).

7-10 PRESSURE RATING

Calculate the pressure rating of the thermowell, based on the pressure rating of the tip, thermowell shank, and any flange (subsection 6-13). Determine if the pressure rating exceeds the design pressure.

Section 8 Examples

NOTE: In the following examples, intermediate results are given for the multistep calculations. Although intermediate results are rounded to four significant digits in the text for clarity, numerical calculations were performed for the full chain of calculations without rounding.

8-1 TAPERED, WELDED THERMOWELL FOR A STEAM-HEADER APPLICATION (U.S. CUSTOMARY UNITS)

8-1.1 Application, Properties, Dimensions, and Installation

Consider a thermowell for a steam bypass line, for use under ASME B31.1, *Power Piping*.

8-1.1.1 Steam Properties

- (a) superheated steam pressure: $P = 235$ psig
- (b) operating temperature: $T = 450^\circ\text{F}$
- (c) normal flow condition: $V = 295$ ft/sec
- (d) steam density: $\rho = 0.499$ lb/ft³
- (e) viscosity: $\mu = 0.0171$ cP, or using the conversion factor $1 \text{ cP} = 6.7197 \times 10^{-4} \text{ lb}/(\text{ft} \cdot \text{sec})$, $\mu = 1.149 \times 10^{-5} \text{ lb}/(\text{ft} \cdot \text{sec})$

8-1.1.2 Thermowell Dimensions. The thermowell has a tapered shank, with a machined fillet at the root of the shank, which is also the support plane. For this high-velocity application, the thermowell is welded directly into the process piping, with the support plane in the heat-affected zone of the weld. The nominal insertion of the thermowell into the process stream is 4 in. The unsupported length, L , exceeds this nominal length due to the possible incomplete penetration of the weld [see Fig. 6-6-1, illustration (e)].

- (a) root diameter: $A = 1.5$ in.
- (b) tip diameter: $B = 1.0$ in.
- (c) fillet radius at base: $b = 0.0$ in.
- (d) bore: $d = 0.26$ in.
- (e) unsupported length: $L = 4.06$ in.
- (f) minimum wall thickness: $t = 0.188$ in

8-1.1.3 Materials Properties. The material of construction is ASTM A 105 carbon steel [18], with the following properties:

(a) from ASME B31.1, Table C-1 (interpolated in temperature), modulus of elasticity at service temperature: $E = 27.5 \times 10^6$ psi

(b) from ASME B31.1, Table A-1, maximum allowable working stress: $S = 19,800$ psi

(c) thermowell construction is welded, then machined, so from Table 6-12.3-1 (Class A, welded), fatigue endurance limit, in the high-cycle limit: $S_f = 3,000$ psi

(d) from reference [19], mass density of carbon steel: $\rho_m = 0.284$ lb/in.³

8-1.1.4 Installation Details. For the rotational stiffness of the thermowell support, K_M , we will assume the thermowell is mounted to a thick-wall pipe (subsection 6-6) and will use eq. (6-6-5).

For the average density of the temperature sensor, we will use the default value found in para. 6-5.3, Step 5, $\rho_s = 169$ lb/ft³.

8-1.1.5 Reynolds and Strouhal Numbers. The Reynolds number is calculated [eq. (6-4-3)] as

$$Re = \frac{VB\rho}{\mu} = \frac{(295 \text{ ft/sec})(1.0 \text{ in.})(0.499 \text{ lb/ft}^3)}{[1.149 \times 10^{-5} \text{ lb}/(\text{ft} \cdot \text{sec})](12 \text{ in/ft})} = 1.068 \times 10^6$$

For this example, $Re > 5 \times 10^5$, and either eq. (6-4-2) or (6-4-4) gives the Strouhal number $N_s = 0.22$.

The force coefficients using eq. (6-4-5) are

$$\begin{aligned} C_D &= 1.4 \\ C_d &= 0.1 \\ C_l &= 1.0 \end{aligned}$$

8-1.2 Natural Frequency Calculation

Step 1. Approximate natural frequency [eq. (6-5-1)]:

$$\begin{aligned} I &= \pi(D_a^4 - d^4)/64 \\ &= \pi[(1.25 \text{ in.})^4 - (0.26 \text{ in.})^4]/64 \\ &= 0.1196 \text{ in.}^4 \\ m &= \rho_m \pi(D_a^2 - d^2)/4 \\ &= (0.284 \text{ lb/in.}^3) \pi[(1.25 \text{ in.})^2 - (0.26 \text{ in.})^2]/4 \\ &= 0.3334 \text{ lb/in.} \end{aligned}$$

where

$$D_a = (1.5 \text{ in.} + 1.0 \text{ in.})/2 = 1.25 \text{ in.}$$

Calculate the approximate natural frequency of the thermowell as

$$f_a = \frac{1.875^2}{2\pi} \left(\frac{EI}{m} \right)^{1/2} \frac{1}{L^2}$$

$$= \frac{1.875^2}{2\pi} \left[\frac{(27.5 \times 10^6 \text{ psi})[386.088 \text{ in.-lb}/(\text{lb} \cdot \text{sec}^2)](0.1196 \text{ in.}^4)}{0.3334 \text{ lb/in.}} \right]^{1/2}$$

$$= \frac{1}{(4.06 \text{ in.})^2}$$

$$= 2,095 \text{ Hz}$$

where

E = the elastic modulus at the operating temperature
 $I = \pi(D_a^4 - d^4)/64$, which is the second moment of inertia

L = unsupported length of the thermowell

$m = \rho_m \pi(D_a^2 - d^2)/4$, which is the mass per unit length of the thermowell

The conversion factor $386.088 \text{ in.-lb} = 1 \text{ lb} \cdot \text{sec}^2$ is necessary when E is given in units of pounds per square inch (equivalent to $\text{lb}/\text{in.}^2$). (See para. 6-5.3, Step 2, and Nonmandatory Appendix A.)

Step 2. Use the correlations of subsection 6-5 to correct for deviations from the approximate slender-beam theory:

$$H_f = \frac{0.99[1 + (1 - B/A) + (1 - B/A)^2]}{1 + 1.1(D_a/L)^{3[1 - 0.8(d/D_a)]}}$$

$$= \frac{0.99[1 + (1 - 0.6667) + (1 - 0.6667)^2]}{1 + 1.1(0.3079)^{3[1 - 0.8(0.2080)]}}$$

$$= 1.352$$

where

$$B/A = (1.0 \text{ in.})/(1.5 \text{ in.}) = 0.6667$$

$$D_a/L = (1.25 \text{ in.})/(4.06 \text{ in.}) = 0.3079$$

$$d/D_a = (0.26 \text{ in.})/(1.25 \text{ in.}) = 0.2080$$

Step 3. Correct for the fluid mass:

$$H_{af} = 1 - \frac{\rho}{2\rho_m}$$

$$= 1 - \frac{(0.499 \text{ lb}/\text{ft}^3)}{2(0.284 \text{ lb}/\text{in.}^3)(1,728 \text{ in.}^3/\text{ft}^3)}$$

$$= 0.9995$$

Step 4. Correct for the sensor mass:

$$H_{as} = 1 - \frac{\rho_s}{2\rho_m} \left[\frac{1}{(D_a/d)^2 - 1} \right]$$

$$= 1 - \frac{(169 \text{ lb}/\text{ft}^3)}{2(0.284 \text{ lb}/\text{in.}^3)(1,728 \text{ in.}^3/\text{ft}^3)} \left(\frac{1}{4.808^2 - 1} \right)$$

$$= 0.9922$$

where

$$D_a/d = (1.25 \text{ in.})/(0.26 \text{ in.}) = 4.808$$

Step 5. The lowest-order natural frequency of the thermowell with ideal support [eq. (6-5-6)] is given by

$$f_n = H_f H_{af} H_{as} f_a$$

$$= (1.352)(0.9995)(0.9922)(2,095 \text{ Hz})$$

$$= 2,809 \text{ Hz}$$

Step 6. Correct for foundation compliance [eq. (6-6-5)]:

$$H_c = 1 - (0.61) \frac{(A/L)}{[1 + 1.5(b/A)]^2}$$

$$= 1 - (0.61) \frac{(0.3695)}{[1 + 1.5(0)]^2}$$

$$= 0.7746$$

where

$$A/L = (1.5 \text{ in.})/(4.06 \text{ in.}) = 0.3695$$

$$b/A = (0.0 \text{ in.})/(1.5 \text{ in.}) = 0.0$$

The in situ natural frequency of the mounted thermowell [eq. (6-6-1)] is given as

$$f_n^c = H_c f_n$$

$$= (0.7746)(2,809 \text{ Hz})$$

$$= 2,176 \text{ Hz}$$

8-1.3 Scruton Number Calculation

Because the Reynolds number exceeds 10^5 , the general frequency limits of para. 6-8.3 apply and no calculation of Scruton number is needed. The calculation is included here as an example. We take a conservative value of 0.0005 for the damping factor, ζ , used in eq. (6-8-1):

$$N_{sc} = \pi^2 \zeta (\rho_m / \rho) [1 - (d/B)^2]$$

$$= \pi^2 (0.0005) \left[\frac{0.284 \text{ lb}/\text{in.}^3}{(0.499 \text{ lb}/\text{ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in.}^3)} \right] (1 - 0.2600^2)$$

$$= 4.525$$

where

$$d/B = (0.26 \text{ in.})/(1.0 \text{ in.}) = 0.26$$

Although N_{sc} is greater than 2.5, the Reynolds number exceeds 10^5 , and the in-line resonance cannot be assumed to be suppressed.

8-1.4 Frequency Limit Calculation

Step 1. From eq. (6-4-1), the vortex shedding rate with a Strouhal number of $N_s = 0.22$ and at the normal flow condition is

$$f_s = \frac{N_s V}{B}$$

$$= \frac{(0.22)(295 \text{ ft/sec})(12 \text{ in./ft})}{(1.0 \text{ in.})}$$

$$= 778.8 \text{ Hz}$$

Step 2. Check that the natural frequency of the mounted thermowell is sufficiently high. In the present example, the thermowell passes the most stringent frequency limit [eq. 6-8-7):

$$f_s < 0.4 f_n^c$$

$$778.8 \text{ Hz} < 870.2 \text{ Hz} = 0.4(2,176 \text{ Hz})$$

In this case, no calculation of cyclic stress at in-line resonance is needed, because the forced or Strouhal frequency is less than the in-line resonance frequency. However, for the sake of completeness, calculation of this quantity is included in para. 8-1.5.

8-1.5 Cyclic Stress at the In-Line Resonance

Step 1. Use eqs. (6-8-3) and (6-8-4) to establish the flow velocity corresponding to the in-line resonance:

$$\begin{aligned} V_{IR} &= \frac{Bf_n^c}{2N_s} \\ &= \frac{(1.0 \text{ in.})(12 \text{ in./ft})^{-1} (2,176 \text{ Hz})}{2(0.22)} \\ &= 412.0 \text{ ft/sec} \end{aligned}$$

Step 2. Evaluate cyclic drag stress at the root. The magnification factor, F'_M , for the drag or in-line resonance is set at 1,000 [see paras. 6-8.3, Step 1; and 6-9.2]. Begin by evaluating the value of G_{SP} using eq. (6-10-7):

$$\begin{aligned} G_{SP} &= \frac{16L^2}{3\pi A^2 [1 - (d/A)^4]} [1 + 2(B/A)] \\ &= \frac{16(4.06 \text{ in.})^2}{3\pi(1.5 \text{ in.})^2 (1 - 0.1733^2)} [1 + 2(0.6667)] \\ &= 29.05 \end{aligned}$$

where

$$d/A = (0.26 \text{ in.}) / (1.5 \text{ in.}) = 0.1733$$

From eq. (6-3-3), the force per unit area due to cyclic drag is

$$\begin{aligned} P_d &= \frac{1}{2} \rho C_d V_{IR}^2 \\ &= \frac{1}{2} \frac{(0.499 \text{ lb/ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in.}^3)(0.1)}{[386.088 \text{ in.-lb}/(\text{lb} \cdot \text{sec}^2)]} \\ &\quad [(412.0 \text{ ft/sec})(12 \text{ in./ft})]^2 \\ &= 0.9143 \text{ psi} \end{aligned}$$

where the conversion factor $386.088 \text{ in.-lb} = 1 \text{ lb} \cdot \text{sec}^2$ is included to give a final answer in units of pounds per square inch (psi).

The cyclic stresses due to cyclic drag [eq. (6-10-6)] at the in-line resonance condition are

$$\begin{aligned} S_d &= G_{SP} F'_M P_d \\ &= 29.05(1,000)(0.9143 \text{ psi}) \\ &= 26,560 \text{ psi} \end{aligned}$$

Step 3. Evaluate the stress concentration factor from eq. (6-12-4):

$$K_t = 2.2$$

Step 4. Evaluate combined drag and lift stresses, with lift stress set to zero [eq. (6-12-3)]:

$$\begin{aligned} S_{o,\max} &= K_t (S_d^2 + S_L^2)^{1/2} = K_t S_d \\ &= 58,430 \text{ psi} \end{aligned}$$

Step 5. Evaluate the temperature de-rating factor from eq. (6-12-6):

$$\begin{aligned} F_T &= E(T) / E_{\text{ref}} \\ &= \frac{27.5 \times 10^6 \text{ psi}}{29.3 \times 10^6 \text{ psi}} \\ &= 0.9386 \end{aligned}$$

The environmental de-rating factor, F_E , is taken as unity for steam service.

Step 6. Compare the predicted stress with the fatigue stress limit, given by the right-hand side of eq. (6-12-5):

$$\begin{aligned} F_T F_E S_f &= (0.9386)(1.0)(3,000 \text{ psi}) \\ &= 2,816 \text{ psi} \end{aligned}$$

The fatigue stress limit, 2,816 psi, is less than the combined stress, 58,430 psi. The thermowell would not pass the cyclic stress condition for steady-state operation at the in-line resonance, corresponding to a fluid velocity of 412 ft/sec, if the vortex shedding frequency, f_s , had been greater than $0.4 f_n^c$ (see para. 8-1.4, Step 2).

8-1.6 Steady-State Stress at the Design Velocity

Step 1. Evaluate the radial, tangential, and axial stresses due to the external pressure, at the location of maximum stress [eqs. (6-11-1) through (6-11-3)]:

$$\begin{aligned} S_r &= P = 235 \text{ psi} \\ S_t &= P \frac{1 + (d/A)^2}{1 - (d/A)^2} = (235 \text{ psi}) \frac{1 + (0.1733)^2}{1 - (0.1733)^2} \\ &= 249.6 \text{ psi} \\ S_a &= \frac{P}{1 - (d/A)^2} = (235 \text{ psi}) \frac{1}{1 - (0.1733)^2} \\ &= 242.3 \text{ psi} \end{aligned}$$

Step 2. Evaluate steady-state drag stress at the root. First, evaluate the steady-state drag force per unit area:

$$\begin{aligned} P_D &= \frac{1}{2} \rho C_D V^2 \\ &= \frac{1}{2} \frac{(0.499 \text{ lb/ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in.}^3)(1.4)}{[386.088 \text{ in.-lb}/(\text{lb} \cdot \text{sec}^2)]} \\ &\quad [(295 \text{ ft/sec})(12 \text{ in./ft})]^2 \\ &= 6.561 \text{ psi} \end{aligned}$$

where the conversion factor $386.088 \text{ in.-lb} = 1 \text{ lb} \cdot \text{sec}^2$ is included to give a final answer in units of pounds per square inch (psi).

Step 3. Evaluate the steady-state stress due to the drag force [eq. (6-10-4)]:

$$\begin{aligned} S_D &= G_{SP} P_D \\ &= 29.05(6.561 \text{ psi}) \\ &= 190.6 \text{ psi} \end{aligned}$$

Step 4. Before using the Von Mises criterion to assess the stress limit at the root, compute the maximum stress given by eq. (6-12-1):

$$\begin{aligned} S_{\max} &= S_D + S_a \\ &= 432.9 \text{ psi} \end{aligned}$$

Step 5. Compute the left-hand side (LHS) of the Von Mises criteria [eq. (6-12-2)]:

$$\begin{aligned} \text{LHS} &= \sqrt{\frac{(S_{\max} - S_r)^2 + (S_{\max} - S_t)^2 + (S_t - S_r)^2}{2}} \\ &= 191.0 \text{ psi} \end{aligned}$$

Step 6. Compute the stress limit given by the right-hand side (RHS) of the Von Mises criteria [eq. (6-12-2)]:

$$\begin{aligned}\text{RHS} &= 1.5S \\ &= 1.5(19,800 \text{ psi}) \\ &= 29,700 \text{ psi}\end{aligned}$$

The Von Mises stress, 191 psi, does not exceed the stress limit, 29,700 psi, and the thermowell passes the steady-state stress criterion.

8-1.7 Dynamic Stress at the Design Velocity

Step 1. The magnification factors for the lift (transverse) and drag (in-line) resonances are given by eqs. (6-9-1) and (6-9-2), respectively:

$$\begin{aligned}r &= \frac{f_s}{f_n^c} = \frac{778.8 \text{ Hz}}{2,176 \text{ Hz}} = 0.3580 \\ F_M &= \frac{1}{1-r^2} = \frac{1}{1-0.3580^2} = 1.147 \\ r' &= \frac{2f_s}{f_n^c} = \frac{2(778.8 \text{ Hz})}{2,176 \text{ Hz}} = 0.7159 \\ F'_M &= \frac{1}{1-(0.7159)^2} = 2.052\end{aligned}$$

Step 2. Evaluate the dynamic drag and lift stresses at the root. Using eq. (6-3-3), calculate the force per unit area due to cyclic drag and lift:

$$\begin{aligned}P_d &= \frac{1}{2} \rho C_d V^2 \\ &= \frac{1}{2} \frac{(0.499 \text{ lb/ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in.}^3)(0.1)}{[386.088 \text{ in.-lb/(lb} \cdot \text{sec}^2)]} [(295 \text{ ft/sec})(12 \text{ in./ft})]^2 \\ &= 0.4686 \text{ psi} \\ P_l &= \frac{1}{2} \rho C_l V^2 \\ &= \frac{1}{2} \frac{(0.499 \text{ lb/ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in.}^3)(1.0)}{[386.088 \text{ in.-lb/(lb} \cdot \text{sec}^2)]} [(295 \text{ ft/sec})(12 \text{ in./ft})]^2 \\ &= 4.686 \text{ psi}\end{aligned}$$

The cyclic stresses due to drag and lift [eqs. (6-10-5) and (6-10-6)] are

$$\begin{aligned}S_d &= G_{sp} F'_M P_d \\ &= (29.05)(2.052)(0.4686 \text{ psi}) \\ &= 27.93 \text{ psi} \\ S_l &= G_{sp} F_M P_l \\ &= (29.05)(1.147)(4.686 \text{ psi}) \\ &= 156.1 \text{ psi}\end{aligned}$$

The concentration factor is identical to the value calculated in 8-1.5, Step 3, $K_t = 2.2$.

Step 3. Evaluate combined drag and lift stresses [eq. (6-12-3)]:

$$\begin{aligned}S_{o,\max} &= K_t (S_d^2 + S_l^2)^{1/2} \\ &= 2.2 [(27.93 \text{ psi})^2 + (156.1 \text{ psi})^2]^{1/2} \\ &= 348.9 \text{ psi}\end{aligned}$$

Step 4. The temperature de-rating factor is identical to the value calculated in para. 8-1.5, Step 5, $F_T = 0.9386$. The environmental de-rating factor, F_E , is taken as unity for steam service.

Step 5. Compare the predicted stress with the fatigue stress limit, given by the right-hand side of eq. (6-12-5):

$$\begin{aligned}F_T F_E S_f &= (0.9386)(1.0)(3,000 \text{ psi}) \\ &= 2,816 \text{ psi}\end{aligned}$$

The predicted stress of 348.9 psi is below the fatigue stress limit, and the thermowell passes the dynamic stress criterion.

8-1.8 Pressure Stress

Step 1. Compute the external pressure rating for the shank using eq. (6-13-1):

$$\begin{aligned}P_c &= 0.66S \left[\frac{2.167}{2B/(B-d)} - 0.0833 \right] \\ &= 0.66(19,800 \text{ psi}) \left[\frac{2.167}{2(1.0 \text{ in.})/(1.0 \text{ in.} - 0.26 \text{ in.})} - 0.0833 \right] \\ &= 9,389 \text{ psi}\end{aligned}$$

Step 2. Compute the external pressure rating for the tip using eq. (6-13-2):

$$\begin{aligned}P_t &= \frac{S}{0.13} \left(\frac{t}{d} \right)^2 \\ &= \frac{19,800 \text{ psi}}{0.13} \left(\frac{0.188 \text{ in.}}{0.26 \text{ in.}} \right)^2 \\ &= 79,630 \text{ psi}\end{aligned}$$

The pressure rating for the thermowell is the lesser of P_t and P_c , which is 9,389 psi in the present case. This rating exceeds the 235-psi operating pressure, and the thermowell passes the external pressure criterion.

8-2 STEP-SHANK, THREADED THERMOWELL FOR A HOT WATER APPLICATION (SI UNITS)

8-2.1 Application, Properties, Dimensions, and Installation

Consider a thermowell for a heated-water application, for use under ASME B31.1, *Power Piping*.

8-2.1.1 Fluid Properties

- (a) operating pressure: $P = 0.400 \text{ MPa}$ (gauge pressure)
- (b) operating temperature: $T = 85^\circ\text{C}$
- (c) normal flow condition: $V = 10 \text{ m/s}$
- (d) density: $\rho = 968.8 \text{ kg/m}^3$
- (e) viscosity: $\mu = 3.334 \times 10^{-4} \text{ Pa} \cdot \text{s} = 3.334 \times 10^{-4} \text{ kg/(m} \cdot \text{s)}$

Density and viscosity values were obtained from reference [9], based on the operating pressure and temperature.

The Reynolds number is calculated [eq. (6-4-3)] as

$$Re = \frac{VB\rho}{\mu} = \frac{(10 \text{ m/s})(0.0127 \text{ m})(968.8 \text{ kg/m}^3)}{(3.334 \times 10^{-4} \text{ Pa} \cdot \text{s})} = 3.690 \times 10^5$$

For this example, the Strouhal number is calculated using eq. (6-4-2) as

$$\begin{aligned} N_s &= 0.213 - 0.0248 [\text{Log}_{10}(Re / 1300)]^2 + 0.0095 [\text{Log}_{10}(Re / 1300)]^3 \\ &= 0.213 - 0.0248 [\text{Log}_{10}(3.690 \times 10^5 / 1300)]^2 + 0.0095 \\ &\quad [\text{Log}_{10}(3.690 \times 10^5 / 1300)]^3 \\ &= 0.2040 \end{aligned}$$

and the force coefficients using eq. (6-4-5) are

$$\begin{aligned} C_D &= 1.4 \\ C_d &= 0.1 \\ C_L &= 1.0 \end{aligned}$$

8-2.1.2 Thermowell Dimensions. The thermowell has a step shank with a threaded base, as shown in Fig. 4-1-3, illustration (a).

- (a) root diameter: $A = 0.0222 \text{ m}$
- (b) tip diameter: $B = 0.0127 \text{ m}$
- (c) fillet radius at support plane: $b = 0 \text{ m}$
- (d) fillet radius at base of step: $b_s = 0.0032 \text{ m}$
- (e) bore: $d = 0.0066 \text{ m}$
- (f) unsupported length: $L = 0.19 \text{ m}$
- (g) length of reduced-diameter shank: $L_s = 0.0635 \text{ m}$
- (h) minimum wall thickness: $t = 0.0048 \text{ m}$

8-2.1.3 Materials Properties. The material of construction is ASTM A 182 F316 stainless steel [20], with properties as follows:

- (a) from ASME B31.1, Table C-1 (interpolated in temperature), modulus of elasticity at service temperature: $E = 1.91 \times 10^5 \text{ MPa} = 1.91 \times 10^{11} \text{ Pa}$
- (b) from ASME B31.1, Table C-1 (interpolated in temperature), modulus of elasticity at ambient temperature: $E = 1.95 \times 10^5 \text{ MPa}$
- (c) from ASME B31.1, Table A-3 (interpolated in temperature), maximum allowable working stress: $S = 122 \text{ MPa}$
- (d) thermowell construction is threaded base, so from Table 6-12.3-1 (Class B, threaded), fatigue stress amplitude limit: $S_s = 37.2 \text{ MPa}$
- (e) from reference [19], mass density of F316 steel at ambient temperature: $\rho_m = 8000 \text{ kg/m}^3$

8-2.1.4 Installation Details. For the rotational stiffness of the thermowell support, K_M , we will assume the thermowell is mounted to a rigid flange (see subsection 6-6) and will use eq. (6-7-1) to evaluate the correction factor on the natural frequency.

For the average density of the temperature sensor, we will use the default value from para. 6-5.3, Step 5, $\rho_s = 2700 \text{ kg/m}^3$.

8-2.2 Natural Frequency Calculation

Step 1. Approximate natural frequency [eq. (6-5-1)]:

$$\begin{aligned} I &= \pi (D_a^4 - d^4) / 64 \\ &= \pi [(0.0222 \text{ m})^4 - (0.0066 \text{ m})^4] / 64 \\ &= 1.183 \times 10^{-8} \text{ m}^4 \\ m &= \rho_m \pi (D_a^2 - d^2) / 4 \\ &= (8000 \text{ kg/m}^3) \pi [(0.0222 \text{ m})^2 - (0.0066 \text{ m})^2] / 4 \\ &= 2.823 \text{ kg/m} \end{aligned}$$

where

$$D_a = A = 0.0222 \text{ m}$$

Calculate the approximate natural frequency of the thermowell as

$$\begin{aligned} f_a &= \frac{1.875^2}{2\pi} \left(\frac{EI}{m} \right)^{1/2} \frac{1}{L^2} \\ &= \frac{1.875^2}{2\pi} \left[\frac{(1.91 \times 10^{11} \text{ Pa})(1.183 \times 10^{-8} \text{ m}^4)}{2.823 \text{ kg/m}} \right]^{1/2} \frac{1}{(0.19 \text{ m})^2} \\ &= 438.5 \text{ Hz} \end{aligned}$$

where

E = the elastic modulus at the operating temperature
 $I = \pi (D_a^4 - d^4) / 64$, which is the second moment of inertia

L = unsupported length of the thermowell

$m = \rho_m \pi (D_a^2 - d^2) / 4$, which is the mass per unit length of the thermowell

Step 2. Use the correlations of subsection 6-5 to correct for deviations from the approximate slender-beam theory:

$$\begin{aligned} y_1 &= [c_1(A/B) + c_2](L_s/L) + [c_3(A/B) + c_4] \\ &= [1.407(1.748) - 0.839]0.3342 + [-0.022(1.748) + 1.022] \\ &= 1.525 \\ y_2 &= [c_5(A/B) + c_6](L_s/L) + [c_7(A/B) + c_8] \\ &= [-2.228(1.748) + 1.594]0.3342 + [1.313(1.748) + 0.362] \\ &= 1.888 \\ \beta &= [c_9(A/B) + c_{10}] \\ &= [8.299(1.748) - 5.376] \\ &= 9.131 \\ H_f &= (y_1^{-\beta} + y_2^{-\beta})^{-1/\beta} \\ &= (1.525^{-9.131} + 1.888^{-9.131})^{-1/9.131} \\ &= 1.503 \end{aligned}$$

where

$$A/B = (0.0222 \text{ m}) / (0.0127 \text{ m}) = 1.748$$

$$L_s/L = (0.0635 \text{ m}) / (0.190 \text{ m}) = 0.3342$$

Step 3. Correct for the fluid mass:

$$\begin{aligned} H_{af} &= 1 - \frac{\rho}{2\rho_m} \\ &= 1 - \frac{(968.8 \text{ kg/m}^3)}{2(8000 \text{ kg/m}^3)} \\ &= 0.9395 \end{aligned}$$