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ASME Boiler and Pressure Vessel Code, Section VIII, Division 3, Example Problem Manual

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SETTING THE STANDARD

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ASME BOILER AND PRESSURE VESSEL CODE, SECTION VIII, DIVISION 3, EXAMPLE PROBLEM MANUAL

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During the 1980s, ASME's Special Working Group on High Pressure Vessels was established for the purpose of creating a standard dealing with the construction of high-pressure vessels, which generally operate at a pressure above 10,000 pounds per square inch ("psi"). This was done on the basis of recommendations made by the Operations, Applications, and Components Technical Committee of the ASME Pressure Vessel and Piping Division (the "Committee"). The ASME Boiler and Pressure Vessel Code ("BPVC"), Section VIII, Division 3, titled "Alternative Rules for Construction of High Pressure Vessels" was first published in 1997; the Committee continues to refine and develop this code.

Some of the innovative concepts which began with BPVC Section VIII, Division 3 include:

- Use of elastic-plastic finite element analysis in design of pressure equipment;
- One of the lowest design margins (originally published at 2.0 and then lowered to 1.8);
- Use of high strength materials in manufacturing of high-pressure equipment;
- Stringent requirements on fracture toughness for materials used in construction;
- Complete volumetric and surface examination after hydrotest;
- The use of fracture mechanics for evaluation of design life assessment in all cases where "leak-before-burst" cannot be shown; and
- Consideration of beneficial residual stresses in the evaluation of the design life of vessels.

This publication is provided to illustrate some of the design calculations and methodologies used in the BPVC Section VIII, Division 3. It is recognized that many high-pressure designs are unique and quite innovative and therefore, this example problem manual cannot cover all design aspects within the scope of BPVC Section VIII, Division 3. This is an attempt at covering some of the most common ones.

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To develop this Example Problem Manual, STLLC contracted with Structural Integrity Associates, Inc. to perform the work.

EXECUTIVE SUMMARY

An introduction to the example problems in this publication is described in Part 2 of this publication. The remaining parts of this publication contain the example problems. The remaining Parts of this publication coincide with the following sections of BPVC Section VIII Division 3:

- Part 2 Parts KM - Materials,
- Part 3 - KD-2 General Design Issues,
- Part 4 - KD-3 Fatigue Assessment,
- Part 5 - KD-4 Life Assessment Using Fracture Mechanics,
- Part 6 - KD-5 – Residual Stresses using Autofrettage,
- Part 7 KD-6 – Closures and Connections,
- Part 8 – KD-8 Residual Stresses in Multiwall Vessels,
- Part 9 – KD-9 Wire Wound Vessels,
- Part 10 – KT-3 - Determination of Hydrostatic Test Pressure, and
- Part 11 – Appendix E – Determination of Blind End Dimensions and Thread Load Distribution.

The example problems in this manual follow the calculation procedures in BPVC Section VIII, Division 3. It is suggested that users of this manual obtain a copy of "Criteria of the ASME Boiler and Pressure Vessel Code Section VIII, Division 3" [2], which contains the original criteria on its use when it was first published.

It should be noted that BPVC Section VIII, Division 3 requires the use of API 579-1/ASME FFS-1 [3] for some calculation procedures. When reviewing certain example problems in this manual, it is recommended that a copy be obtained of this standard for this purpose.

It is noted that many analysis techniques are covered in this manual as examples. Some of these modelling techniques are problem specific and are not requirements of the BPVC Section VIII, Division 3, but are provided herein as examples of methods of compliance with the code requirements. Alternative techniques may be used, as appropriate for certain problems, where complete guidance is not mandated in the BPVC Section VIII, Division 3.

ABBREVIATIONS AND ACRONYMS

Abbreviation | Acronym	Description
$2c/a$	Assumed Crack Aspect Ratio
A_B	Cross-Sectional Area of Vessel Normal to Vessel Axis through Internal Threads
A_C	Cross-Sectional Area of Vessel Normal to Vessel Axis through External Threads
API	American Petroleum Institute
ASME	American Society of Mechanical Engineers
b	Assumed Seal Width
BPVC	ASME Boiler and Pressure Vessel Code
C_M	Combined Flexibility Factor of Body and Closure
C_T	Flexibility Factor of the Threads
CTOD	Crack Tip Opening Displacement
CVN	Charpy V-Notch
D	Diameter
D	Dead Weight Applied as Acceleration $1g$ in Finite Element Model
D_i	Inside Diameter
D_{if}	Diameter Interface between Cylinder and Winding
D_{if}	Interface Diameter Between Cylinder Layers
D_o	Outside Diameter
$D_{o.1}$	Liner Outside Diameter
$D_{o.2}$	Outer Body Outside Diameter
D_p	Pitch Diameter of the Threads
D_s	Opening Seal Diameter
D_w	Instantaneous Applied Outside Diameter of Winding
E	Elastic Modulus
El	Elongation
E_y	Modulus of Elasticity
FFS	Fitness-for-Service (ASME)
ft-lbf	Foot-Pounds, Force
FTT	Fracture Toughness Testing
in	Inch(es)
in^2	Square Inches
in^3	Cubic Inches
ISO	International Organization for Standardization
J_{Ic}	Fracture Toughness Testing
K_{Ic}	Fracture Toughness Testing
K_{Ic}	Critical Stress Intensity Factor
K_{Ic}	Material Fracture Toughness
K_r	Surface Roughness Factor
ksi	Kilopounds per Square Inch
L	Length
lb	Pound(s)
m	Seal Factor
MAWP	Maximum Allowable Working Pressure
Min.	Minimum
Mo	Molybdenum
n	Number of Threads
NDE	Non-Destructive Examination
Ni	Nickel
OD	Outside Diameter
P	Pressure Load
P_A	Autofrettage Pressure
P_b	Primary Bending Stress Intensity
P_D	Design Pressure
$P_{D.Dual}$	Design Pressure for Dual Wall

Abbreviation | Acronym	Description
P_{if}	Interface Pressure
P_L	Local Primary Membrane Stress Intensity
P_m	General Primary Membrane Stress Intensity
psi	Pounds per Square Inch
P_T	Hydrostatic Test Pressure
P_T	Thread Pitch
PTCS	ASME Pressure Technology Codes and Standards
Q	Secondary Stress Intensity
RA	Reduction in Area
R_c	Inside Corner Radius
$S`_a$	Alternating Stress Component
$S_{alt\ i,j}$	Alternating Stress Intensities
S_{ij}	Stress Intensities
STLLC	ASME Standards Technology, LLC
S_u	Tensile Strength
S_y	Yield Strength
t_w	Wall Thickness
σ_{uts}	Tensile Strength
V	Vanadium
W	Weight
x_1	Any Diameter of the Cylinder
x_2	Any Diameter of the Winding
Y	Diameter Ratio, D_o / D_i
Y_i	Liner Wall Ratio
Y_o	Outer Body Ratio
ε_{ys}	Offset Strain
$\Delta\Lambda L$	Elongation in Each Yoke Plate
δ	Diametral Interference
ε	Longitudinal Strain in Each Side of Yoke
ν	Poisson's Ratio
ν_l	Poisson's Ratio for Liner
ν_o	Poisson's Ratio for Body
ρ	Density
$\sigma_{n\ i,j}$	Associated Mean Stress
$\sigma_{n\ i,j}$	Stress Normal to the Plane of Maximum Shear
$\sigma_{nm\ i,j}$	Associated Mean Stress
σ	Longitudinal Stress on Each Side of Yoke
$S\_(eq\ i,j$	Equivalent Alternating Stress Intensity
ϵ_L	Limiting Triaxial Strain
ϵ_cf	Cold Forming Strain

PURPOSE AND USE

The BPVC Section VIII, Division 3 [1] contains mandatory requirements, specific prohibitions, and non-mandatory guidance for the design, materials, fabrication, examination, inspection, testing, and certification of high-pressure vessels and their associated pressure relief devices. This Example Problem Manual is based on the 2019 edition of the BPVC Section VIII, Division 3, and all paragraph references herein are to this same edition.

Scope

Example problems illustrating the use of the analysis methods in BPVC Section VIII, Division 3 are provided in this publication.

PART 1

Example Problem Descriptions

1 EXAMPLE PROBLEM DESCRIPTIONS

1.1 General

Example problems are provided for the following parts of the BPVC Section VIII Division 3:

- Part KM – Materials Requirements
- Part KD-2 – Design by Rule Requirements
- Part KD-2 – Elastic-Plastic Analysis
- Part KD-2 – Elastic Analysis Requirements
- Part KD-3 – Life Assessment using Fatigue
- Part KD-4 – Life Assessment using Fracture Mechanics
- Part KD-5 – Evaluation of Residual Stress due to Autofrettage
- Part KD-6 – Design Assessment of Heads and Connections
- Part KD-8 – Evaluation of Residual Stress Due to Shrink Fitting
- Part KD-9 – Special Design Requirements for Wire-Wound Vessels and Frames
- Part KT – Determination of Hydrostatic Test Range
- Appendix E – Special Design by Rules for Closed Ends and Threads

A summary of the example problems provided are shown in Table 1 as follows.

Table 1 – Summary of Example Problems

Part	Example	Description
2	E-KM-2.1.1	Evaluation of Testing for Cylindrical Forgings in Accordance with KM-2
2	E-KM-2.1.2	Calculation of Fracture Toughness based on Charpy Impact Tests (KM-251)
2	E-KM-2.2.1	Generate a Stress-Strain Curve for Use in Elastic-Plastic Finite Element Analysis
3	E-KD-2.1.1	Determination of Design Pressure in Cylindrical Vessel – Monobloc Vessel
3	E-KD-2.1.2	Determination of Design Pressure in Cylindrical Vessel – Dual Layered Vessel
3	E-KD-2.2.1	Elastic Plastic Analysis
3	E-KD-2.2.2	Protection Against Local Failure (Elastic-Plastic Analysis)
3	E-KD-2.2.3	Ratcheting Assessment Elastic-Plastic Stress Analysis
3	E-KD-2.2.4	Protection Against Local Failure for Series of Applied Loads
3	E-KD-2.3.1	Linear Elastic Stress Analysis
3	E-KD-2.3.2	Elastic Stress Analysis Protection Against Local Failure Mandatory Appendix 9-280
4	E-KD-3.1.1	Evaluation of Leak-Before-Burst in Cylindrical Vessel – Monobloc Vessel
4	E-KD-3.1.2	Evaluation of Leak-Before-Burst in Cylindrical Vessel – Dual Layered Vessel
4	E-KD-3.1.3	Fatigue Assessment of Welds – Elastic Analysis and Structural Stress
4	E-KD-3.1.4	Non-Welded Vessel Using Design Fatigue Curves
4	E-KD-3.1.5	Autofrettaged, Non-Welded Vessel using Design Fatigue Curves
5	E-KD-4.1.1	Determine the Design Life of a Vessel from E-KD-2.1.1
6	E-KD-5.1.1	Determine Residual Stresses in Autofrettaged Cylinder Wall with known Autofrettage Pressure
6	E-KD-5.1.2	Determine Autofrettage Pressure in a Cylinder Wall with known Residual ID Tangential Strain
7	E-KD-6.1.1	Evaluation of a Connection in a 60 ksi Pressure Vessel at 100°F
7	E-KD-6.1.2	Alternative Evaluation of Stresses in Threaded End Closures
7	E-KD-6.1.3	Evaluation of Yoke Misalignment KD-652.2
8	E-KD-8.1.1	Dual Wall Cylindrical Vessel Stress Distribution
9	E-KD-9.1.1	Stress Evaluation in Wire Wound Cylinder
10	E-KT-3.1.1	Determination of Hydrostatic Test Pressure in Cylindrical Vessel
11	E-AE-2.1.1	Blind End Dimensions and Corner Stresses in a Vessel without Detailed Stress Analysis – Thick Wall Pressure Vessel
11	E-AE-2.1.2	Blind End Dimensions and Corner Stresses in a Vessel without Detailed Stress Analysis – Thin Wall Pressure Vessel
11	E-AE-2.2.1	Thread Load Distribution

1.2 Calculation Precision

The calculation precision used in the example problems is intended for demonstration proposes only; an intended precision is not implied. In general, the calculation precision should be equivalent to that obtained by computer implementation, rounding of calculations should only be performed on the final results.

PART 2

Example Problems: Materials

2 EXAMPLE PROBLEMS, MATERIALS

2.1 Example Problem E-KM-2.1.1 – Evaluation of Testing for Cylindrical Forgings in Accordance with KM-2

This problem presents an evaluation of requirements for the minimum number, location, and type of tests required for the following two cases.

Material dimensions

Case 1 – 12 inches diameter x 18 inches long (15 inches diameter x 27 inches long at heat treatment)

Case 2 – 12 inches diameter x 15 feet long [180 inches] (15 inches diameter x 200 inches long at heat treatment)

In both cases, the diameter of the forging is 15 inches at the time of heat treatment, and 12 inches is the final machined diameter. It is critical to know the dimensions of the forgings in the heat-treated condition to evaluate this problem.

The forgings are ASME SA-723, Grade 2, Class 2 material with a minimum specified yield strength of 120,000 psi and a minimum specified tensile strength of 135,000 psi (BPVC Section II, Part A). The forgings are both solid cylinders and all tests are to be taken from test material at the end of the cylinders, which when removed, will leave the size of material listed in each of the cases.

STEP 1 – Determine the thicknesses of the forgings at the time of heat treatment

KM-201.2(e) applies. The thicknesses of the forging are defined as:

Case 1 – This is a cylindrical forging in which the thickness is equal to the diameter of the forging of 15 inches (T = 15 inches) at the time of heat treatment.

Case 2 – This is a cylindrical forging in which the thickness is equal to the diameter of the forging of 15 inches (T = 15 inches) at the time of heat treatment.

STEP 2 – Determine the location of the datum point for the forgings

KM-211.2(b) applies. The datum point is either the mid-point of the tension test specimen or the area under the notch of the impact test specimens. This datum will be used for all the specimens including both the tension and Charpy V-notch specimens.

Case 1 – The datum points are located at a position of T/4 or 3 ¾ inches from the OD of the cylinder and 2T/3 or 10 inches from the end of the cylinder. Tensile samples shall be longitudinal and Charpy V-notch (CVN) samples shall be transverse.

Case 2 – The datum points are located at a position of T/4 or 3 ¾ inches from the OD of the cylinder and 2T/3 or 10 inches from the end of the cylinder.

STEP 3 – Determine the minimum number of test specimens required

Case 1 – The overall dimensions of this forging at the time of heat treatment are:

Diameter (D) = 15 inches

Length (L) = 27 inches

The weight (W) of the forging can be calculated using the following equation:

$$W = \frac{\pi}{4} * D^2 * L * \rho = \frac{\pi}{4} * 15\ in^2 * 27\ in * 0.280 \frac{lb}{in^3} = 1336\ lb$$

The density of steel (ρ) in accordance with Table PRD of the BPVC Section II, Part D is 0.280 lb/in^3.

KM-231(b) therefore applies, and the piece will require at least one tension test and one set of three Charpy V-notch test specimens per component.

Case 2 – The overall dimensions of this forging at the time of heat treatment are:

Diameter (D) = 15 inches

Length (L) = 200 inches

Using the same equation as Case 1 for determining the weight:

$$W = \frac{\pi}{4} * 15\, in^2 * 200\, in * 0.280 \frac{lb}{in^3} = 9896\, lb$$

Therefore, KM-231(c) applies, and two tension tests and two sets of Charpy V-notch impact tests shall be taken at a datum point from each end of the forging 180° apart. Therefore, for each forging, four tension tests (two from each end) and four sets of three impact specimens (two sets from each end) shall be taken from the forging. The tests at one end shall be offset from the tests at the other end by 90°.

STEP 4 – Supplementary Fracture Toughness Testing (KM-250)

It is the responsibility of the designer to specify to the material supplier if supplementary toughness testing is required. Material toughness determination can be accomplished in using several methods including:

Charpy V-Notch Impact Testing

Crack Tip Opening Displacement (*CTOD*) Fracture Toughness Testing

J_{Ic} Fracture Toughness Testing

K_{Ic} Fracture Toughness Testing

It is noted that if Charpy V-notch impact testing, *CTOD*, or J_{Ic} testing data is used for determination of the K_{Ic} value for use in fracture mechanics calculations, the Manufacturer is required to determine the appropriate conversion correlation to determine K_{Ic}. Also note that the orientation of the direction of crack propagation for all test coupons shall be the same as the direction of crack propagation expected in the fracture mechanics analysis conducted in accordance with Article KD-4.

2.2 Example Problem E-KM-2.1.2 – Calculation of Fracture Toughness based on Charpy Impact Tests (KM-251)

Determine the K_{Ic} fracture toughness of a vessel made from ASME SA-723, Grade 2, Class 2 alloy steel forgings using Charpy V-notch impact strength.

Vessel Data

- Material – All Components = ASME SA-723, Grade 2, Class 2 (S_y = 120 ksi; refer to BPVC Section II, Part D, Table Y-1)
- Charpy Impact value used in the calculation is assumed to be the minimum required for a single specimen in KM-234.2(a):

Specimen Orientation	Number of Specimens	Energy (CVN), ft-lbf for Specified Min. Yield Strength up to 135 ksi
Transverse	Average for 3	30 ft-lbf
	Minimum for 1	24 ft-lbf

The fracture toughness K_{Ic} is then found using the first equation in Appendix D-600 which has been re-written here as:

$$K_{Ic} = S_y * \sqrt{5 * \left(\frac{CVN}{S_y} - 0.05\right)} = 120 * \sqrt{5 * \left(\frac{24}{120} - 0.05\right)} = 104 ksi\sqrt{in}$$

The equation yields K_{Ic} = 104 ksi-in$^{0.5}$ in this case, conservatively based on the minimum value for a single specimen from the table above. The units of S_y in the above equation are ksi. Note that in accordance with KM-251, the designer could require that the pressure-retaining component meet minimum Charpy V-notch absorbed energy values greater than those specified in KM-234.2.

2.3 Example Problem E-KM-2.2.1 –Generate a Stress-Strain Curve for Use in Elastic-Plastic Finite Element Analysis

Generate a true stress – true strain curve for use in elastic-plastic finite element analysis. Generate this curve for SA-723, Grade 2, Class 2 material at 150°F.

Material Data:

- Engineering Yield Strength (σ_{ys}) = 117,000 psi @ 150°F; refer to Table Y-1 of BPVC Section II, Part D
- Engineering 0.2% Offset Strain (ε_{ys}) = 0.002
- Engineering Tensile Strength (σ_{uts}) = 135,000 psi @ 150°F; refer to Table U of BPVC Section II, Part D
- Modulus of Elasticity (E_y) = 27,371 ksi; refer to linear interpolation of Table TM-1 of BPVC Section II, Part D
- Material Parameter (ε_p) = 2 x 10^{-5}; refer to Table KM-620

STEP 1 – Constants Generation

The first step in the evaluation is to determine the constants required from Table KM-620 and paragraph KM-620. SA-723 is a ferritic steel. The following equations calculate the constants of the problem:

$$R = \frac{\sigma_{ys}}{\sigma_{uts}} = \frac{117{,}000\ psi}{135{,}000\ psi} = 0.867 \qquad KM-620.10$$

$$m_1 = \frac{ln(R) + (\epsilon_p - \epsilon_{ys})}{ln\left(\frac{ln(1+\epsilon_p)}{ln(1+\epsilon_{ys})}\right)} = \frac{ln(0.867) + (0.00002 - 0.002)}{ln\left(\frac{ln(1+0.00002)}{ln(1+0.002)}\right)} = 0.0315 \qquad KM-620.6$$

$$A_1 = \frac{\sigma_{ys} * (1+\epsilon_{ys})}{(ln(1+\epsilon_{ys}))^{m_1}} = \frac{117{,}000 psi * (1+0.002)}{(l\,n(1+0.002))^{0.032}} = 142{,}598\ psi \qquad KM-620.5$$

$$m_2 = 0.6 * (1-R) = 0.6 * (1-0.867) = 0.08 \qquad Table\ KM-620$$

$$A_2 = \frac{\sigma_{uts} * e^{m_2}}{m_2^{m_2}} = \frac{135{,}000\ psi * e^{0.08}}{0.08^{0.08}} = 178{,}991\ psi \qquad KM-620.8$$

$$K = 1.5 * R^{1.5} - 0.5 * R^{2.5} - R^{3.5} \qquad KM-620.12$$

$$K = 1.5 * 0.867^{1.5} - 0.5 * 0.867^{2.5} - 0.867^{3.5} = 0.255$$

$$\sigma_{uts,t} = \sigma_{uts} * e^{m_2} = 135{,}000\, psi * e^{0.08} = 146{,}244\, psi \qquad KM-620.13$$

STEP 2 – Curve Generation

The second step is to create the true stress-strain curve. The following table uses the equations below to determine the true stress and true strain values. Values in the equations correspond to the second row of Table 2.

$$H = \frac{2 * \left(\sigma_t - \left(\sigma_{ys} + K * \left(\sigma_{uts} - \sigma_{ys}\right)\right)\right)}{K * \left(\sigma_{uts} - \sigma_{ys}\right)} \qquad KM-620.9$$

$$H = \frac{2 * \left(5{,}850 psi - \left(117{,}000\, psi + 0.255 * (135{,}000\, psi - 117{,}000\, psi)\right)\right)}{0.255 * (135{,}000\, psi - 117{,}000\, psi)} = -50.5081$$

$$\epsilon_1 = \left(\frac{\sigma_t}{A_1}\right)^{\frac{1}{m_1}} = \left(\frac{5{,}850 psi}{142{,}598 psi}\right)^{\frac{1}{0.032}} = 0 \qquad KM-620.4$$

$$\epsilon_2 = \left(\frac{\sigma_t}{A_2}\right)^{\frac{1}{m_2}} = \left(\frac{5{,}850 psi}{178{,}991 psi}\right)^{\frac{1}{0.08}} = 0 \qquad KM-620.7$$

$$\gamma_1 = \frac{\epsilon_1}{2} * (1.0 - tanh(H)) = \frac{0}{2} * (1.0 - tanh(-50.5081)) = 0 \qquad KM-620.2$$

$$\gamma_2 = \frac{\epsilon_2}{2} * (1.0 + tanh(H)) = \frac{0}{2} * (1.0 + tanh(-50.5081)) = 0 \qquad KM-620.3$$

$$\epsilon_{ts} = \frac{\sigma_t}{E_y} + \gamma_1 + \gamma_2 = \frac{5{,}850 psi}{27{,}371{,}000 psi} + 0 + 0 = 0.0002 \qquad KM-620.1$$

The equations used are defined in paragraph KM-620. The table uses 26 equally spaced values of the true stress from 0 to the ultimate true stress calculated in Step 1, which is 146,244 psi. A larger number of equally spaced true stress values can be used to improve the resolution in certain areas of the curve such as in the area of the yield point. The final column calculates the plastic strain, which is the remainder of the total strain minus the elastic portion. Simply, the plastic strain is the addition of γ_1 and γ_2.

Table 2 – Tabulated Values for Generation of a True Stress – True Strain Curve

σ_t	H	ε_1	ε_2	γ_1	γ_2	ε_{ts}	$\gamma_1 + \gamma_2$
0	-53.0610	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5,850	-50.5081	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000
11,700	-47.9551	0.0000	0.0000	0.0000	0.0000	0.0004	0.0000
17,549	-45.4022	0.0000	0.0000	0.0000	0.0000	0.0006	0.0000
23,399	-42.8492	0.0000	0.0000	0.0000	0.0000	0.0009	0.0000
29,249	-40.2963	0.0000	0.0000	0.0000	0.0000	0.0011	0.0000
35,099	-37.7434	0.0000	0.0000	0.0000	0.0000	0.0013	0.0000
40,948	-35.1904	0.0000	0.0000	0.0000	0.0000	0.0015	0.0000
46,798	-32.6375	0.0000	0.0000	0.0000	0.0000	0.0017	0.0000
52,648	-30.0845	0.0000	0.0000	0.0000	0.0000	0.0019	0.0000
58,498	-27.5316	0.0000	0.0000	0.0000	0.0000	0.0021	0.0000
64,347	-24.9787	0.0000	0.0000	0.0000	0.0000	0.0024	0.0000
70,197	-22.4257	0.0000	0.0000	0.0000	0.0000	0.0026	0.0000
76,047	-19.8728	0.0000	0.0000	0.0000	0.0000	0.0028	0.0000
80,000	-18.1475	0.0000	0.0000	0.0000	0.0000	0.0029	0.0000
81,897	-17.3198	0.0000	0.0001	0.0000	0.0000	0.0030	0.0000
87,746	-14.7669	0.0000	0.0001	0.0000	0.0000	0.0032	0.0000
93,596	-12.2139	0.0000	0.0003	0.0000	0.0000	0.0034	0.0000
99,446	-9.6610	0.0000	0.0006	0.0000	0.0000	0.0036	0.0000
105,296	-7.1081	0.0001	0.0013	0.0001	0.0000	0.0039	0.0001
111,145	-4.5551	0.0004	0.0026	0.0004	0.0000	0.0044	0.0004
116,995	-2.0022	0.0019	0.0049	0.0018	0.0001	0.0062	0.0019
122,845	0.5508	0.0088	0.0090	0.0022	0.0068	0.0135	0.0090
128,695	3.1037	0.0386	0.0162	0.0001	0.0162	0.0209	0.0162
134,544	5.6566	0.1580	0.0282	0.0000	0.0282	0.0331	0.0282
140,394	8.2096	0.6100	0.0480	0.0000	0.0480	0.0532	0.0480
146,244	10.7625	2.2281	0.0800	0.0000	0.0800	0.0853	0.0800

<u>**STEP 3**</u> <u>– Plot the True Stress – True Strain Curve</u>

The true stress – true strain curve for SA-723, Grade 2, Class 2 is shown in Figure E-KM-2.2.1-1. This curve is generated by plotting the first column (true stress) (σ_t) verse the total strain (ε_{ts}) column shown in Table 2 in Step 2.

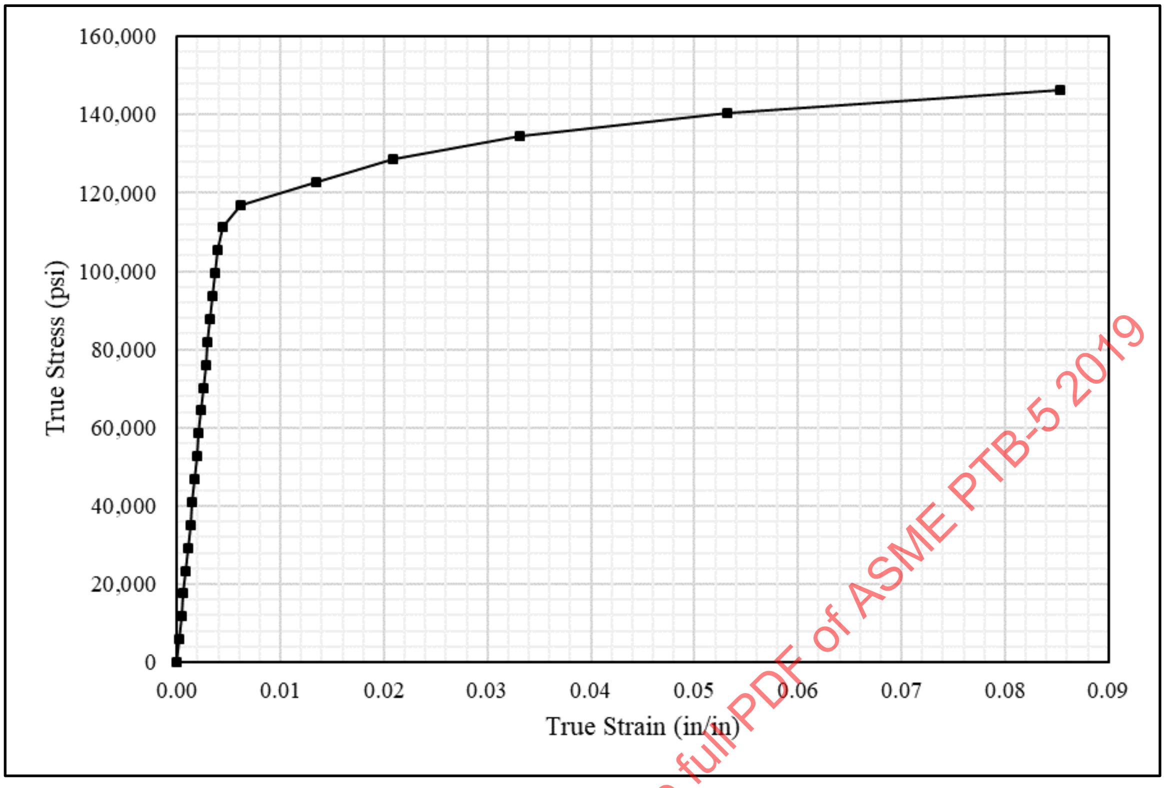

Figure 1 – E-KM-2.2.1-1 – True Stress – True Strain Curve for SA-723 Grade 2 Class 2

STEP 4 – Determination of the Proportional Limit

Many FEA programs accept tabular input of true stress vs. true plastic strain. This input generally starts at the point where the equivalent plastic strain equals zero, also known as the proportional limit.

This point is determined with a combination of engineering judgement and through an iterative procedure. Note that the total true stress (σ_{ts}) is equal to the true elastic stress (σ_{es}) plus the true plastic stress (σ_{ps}), i.e.,

$$\sigma_{ts} = \sigma_{es} + \sigma_{ps}$$

Also, at the proportional limit, there is no plasticity, therefore,

$$\varepsilon_{ts} = \varepsilon_{es} = \frac{\sigma_t}{E_y}$$

And comparing this with equation KM-620.1, at the proportional limit it can be observed that:

$$\gamma_1 + \gamma_2 = 0$$

An iterative procedure is used to determine the value of σ_t = 80 ksi, ε_{ts} = 0.002923, and ($\gamma_1 + \gamma_2$) = 1.08e^{-8}, which is approximately zero.

PART 3

Example Problems: General Design Issues

3 EXAMPLE PROBLEMS, GENERAL DESIGN ISSUES

3.1 Example Problem E-KD-2.1.1 – Determination of Design Pressure in Cylindrical Vessel – Monobloc Vessel

Determine the design pressure for a monobloc cylindrical vessel and associated stress distribution given the following data. Perform calculations for both open and closed-end vessels.

Vessel Data:

- Material = SA-705 Gr. XM-12 Condition H1100
- Design Temperature = 70°F
- Inside Diameter (D_i) = 6.0 inches
- Outside Diameter (D_o) = 12.0 inches
- Diameter Ratio (Y) = 2.0 (KD-221)
- Min Specified Yield Strength (S_y) = 115,000 psi @ 100°F; refer to Table Y-1 of BPVC Section II, Part D
- Min Specified Tensile Strength (S_u) = 140,000 psi @ 100°F; refer to Table U of BPVC Section II, Part D

It should be noted that this example is limited to the application of the equations in KD-220. An actual vessel requires evaluation in accordance with all the rules in Part KD.

It should be noted that all design pressure equations require the use of a parameter K_{ut}, which is defined in KT-312 as:

$$K_{ut} = 0.95 \; for \; \frac{S_y}{S_u} \leq 0.7$$

$$K_{ut} = 1.244 - 0.42\left(\frac{S_y}{S_u}\right) \; for \; 0.7 < \frac{S_y}{S_u} \leq 0.9$$

$$K_{ut} = 0.866 \; for \; \frac{S_y}{S_u} > 0.9$$

$$\frac{S_y}{S_u} = \frac{115{,}000\;psi}{140{,}000\;psi} = 0.821$$

$$K_{ut} = 1.244 - 0.42 * \left(\frac{115{,}000\;psi}{140{,}000\;psi}\right) = 0.899$$

$$Y = \frac{D_o}{D_i} = \frac{12.0\;in}{6.0\;in} = 2.0$$

Evaluate design pressure per KD-220 for an open-end cylindrical shell for $Y \leq 2.85$.

$$P_D = min\left(2.986 * K_{ut} * \left(S_y\right) * \left(Y^{0.268} - 1\right), 1.0773 * \left(S_y + S_u\right) * \left(Y^{0.268} - 1\right)\right) \qquad KD-221.1$$

$$P_D = min\left(2.986 * 0.899 * (115{,}000psi)(2.0^{0.268} - 1), 1.0773 * (115{,}000psi + 140{,}000psi) * (2.0^{0.268} - 1)\right)$$

$$P_D = min(63{,}019\ psi, 56{,}079\ psi) = 56{,}079\ psi$$

Note that this calculation does not account for any loading in addition to internal pressure. If shell is subject to additional loading, the design shall be modified per KD-221.5.

Evaluate the stress distribution for the open-end cylinder (Mandatory Appendix 9)

The stresses for the cylinder can then be determined by using equations 9-300.1 and 9-300.2. The equations are evaluated at the inner diameter (D is equal to D_i).

$$\sigma_t = \frac{P}{Y^2 - 1} * (1 + Z^2) = \frac{56{,}079\ psi}{2^2 - 1} * (1 + 2^2) = 93{,}465\ psi$$

$$\sigma_r = \frac{P}{Y^2 - 1} * (1 - Z^2) = \frac{56{,}079\ psi}{2^2 - 1} * (1 - 2^2) = -56{,}079\ psi$$

$$Z = \frac{D_o}{D} = \frac{12\ in}{6\ in} = 2.0$$

It is noted that the longitudinal stress in an open-end cylinder is zero ($\sigma_l = 0$).

The stress distribution is shown in Figure E-KD-2.1.1-1.

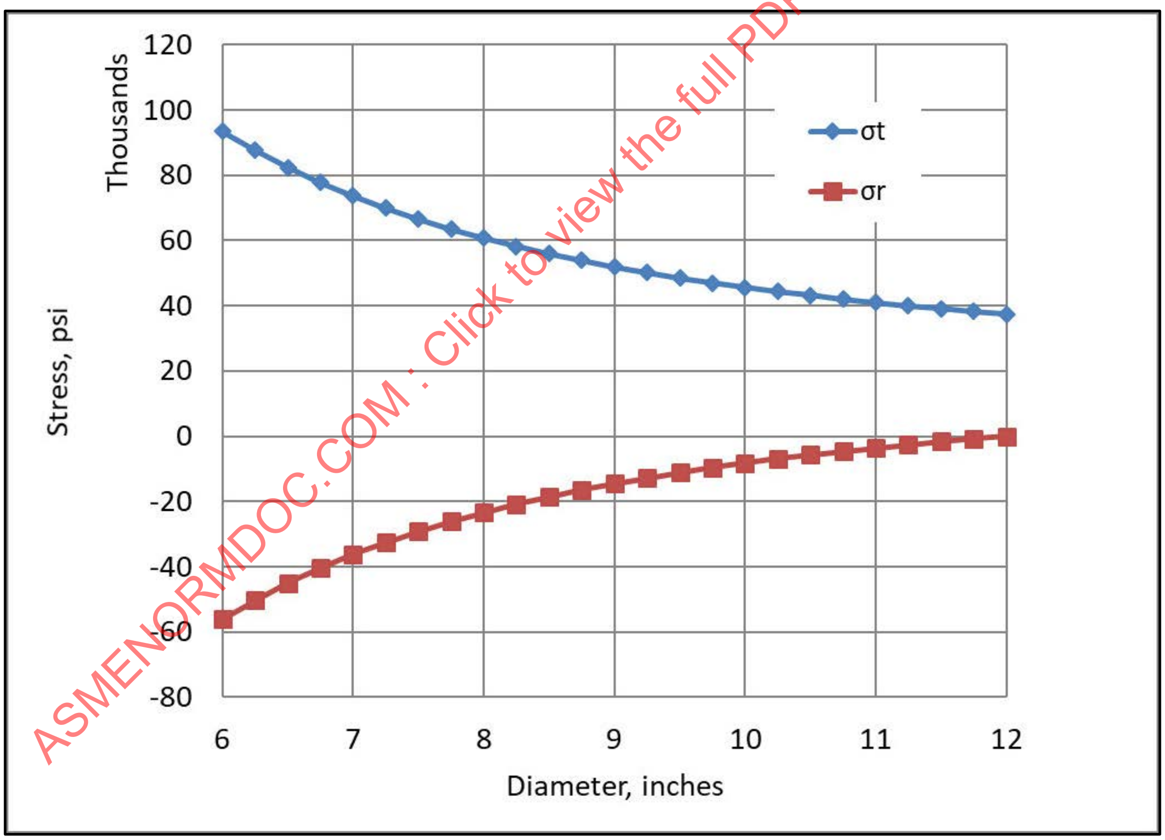

Figure 2 – E-KD-2.1.1-1 – Stress Distribution in Monoblock Open End Cylindrical Shell

Evaluate design pressure per KD-220 for a closed-end cylindrical shell.

$$P_D = min\left(0.924 * K_{ut} * S_y * ln(Y), \frac{1}{3} * (S_y + S_u) * ln(Y)\right) \qquad KD - 221.2$$

$$P_D = min\left(0.924 * 0.899 * 115{,}000\ psi * ln(2), \frac{1}{3} * (115{,}000\ psi + 140{,}000\ psi) * ln(2)\right)$$

$$P_D = min(66{,}215\ psi, 58918\ psi) = 58{,}918\ psi$$

Evaluate the stress distribution for the closed ended cylinder (KD-221)

The stresses for the cylinder can then be determined by using KD-221 equations (1) and (2), as discussed previously. The primary difference is that for the closed ended cylinder, the longitudinal stress is calculated using equation 9-300.3:

$$\sigma_l = \frac{P}{Y^2 - 1} = \frac{58{,}918\ psi}{2^2 - 1} = 19{,}639\ psi$$

For the case of the closed end cylinder, at the inside diameter at the design pressure, σ_t = 98,196 psi, σ_r = -58,918 psi, and σ_l = 19,639 psi. The stress distribution is shown in Figure E-KD-2.1.1-2.

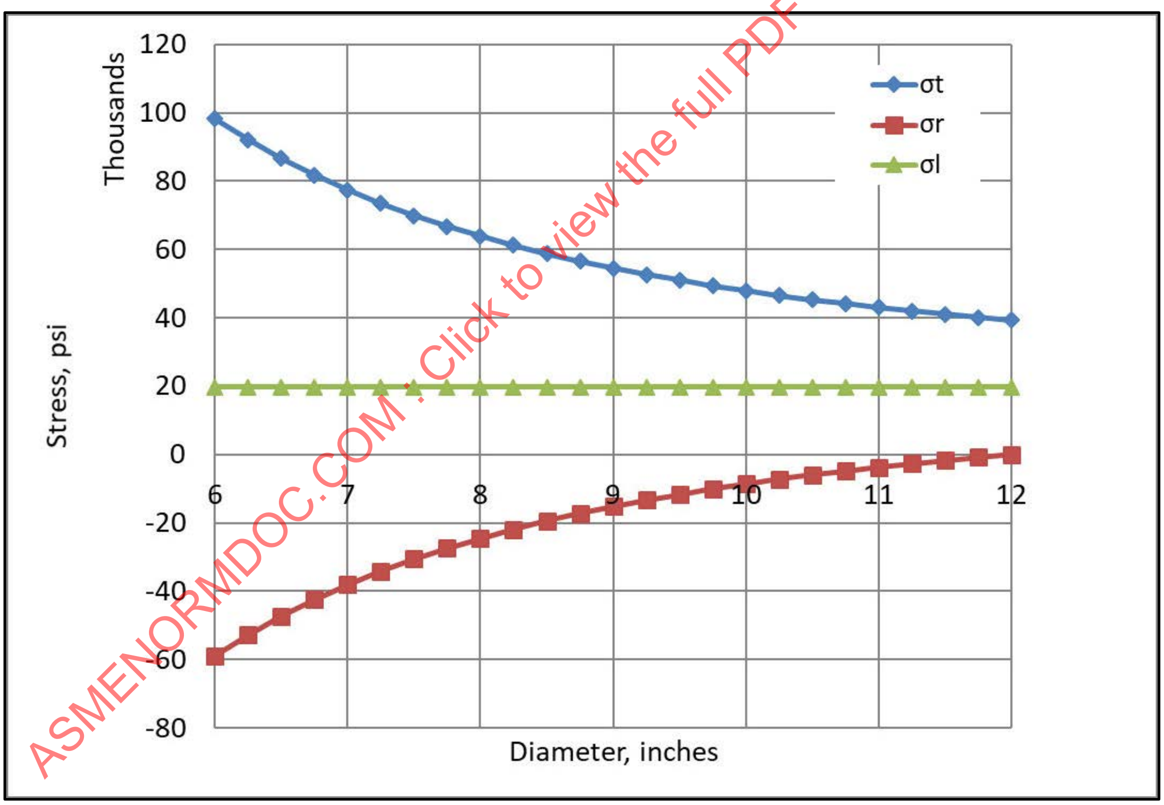

Figure 3 – E-KD-2.1.1-2 – Stress Distribution in Monoblock Closed End Cylindrical Shell

Table 3 – E-KD-2.1.1-1 – Tabulated Stresses from Figures E-KD-2.1.1-1 and E-KD-2.1.2-2 at Corresponding Design Pressure

	Figure E-KD-2.1.1-1 Open Ended		Figure E-KD-2.1.1-1 Closed Ended		
Do/D (Z)	σ_t	σ_r	σ_t	σ_r	σ_l
2.000	93,465	-56,079	98,197	-58,918	19,639
1.920	87,603	-50,217	92,038	-52,759	19,639
1.846	82,404	-45,018	86,576	-47,297	19,639
1.778	77,772	-40,386	81,709	-42,431	19,639
1.714	73,628	-36,242	77,355	-38,076	19,639
1.655	69,904	-32,518	73,443	-34,165	19,639
1.600	66,547	-29,161	69,916	-30,637	19,639
1.548	63,510	-26,124	66,725	-27,446	19,639
1.500	60,752	-23,366	63,828	-24,549	19,639
1.455	58,242	-20,856	61,190	-21,912	19,639
1.412	55,950	-18,564	58,782	-19,503	19,639
1.371	53,851	-16,465	56,577	-17,299	19,639
1.333	51,925	-14,539	54,554	-15,275	19,639
1.297	50,153	-12,767	52,692	-13,413	19,639
1.263	48,519	-11,133	50,975	-11,697	19,639
1.231	47,009	-9,623	49,389	-10,110	19,639
1.200	45,611	-8,225	47,920	-8,641	19,639
1.171	44,314	-6,928	46,557	-7,279	19,639
1.143	43,108	-5,722	45,291	-6,012	19,639
1.116	41,986	-4,600	44,111	-4,833	19,639
1.091	40,939	-3,553	43,012	-3,733	19,639
1.067	39,961	-2,575	41,985	-2,706	19,639
1.043	39,047	-1,661	41,024	-1,745	19,639
1.021	38,190	-804	40,123	-845	19,639
1.000	37,386	0	39,279	0	19,639

3.2 Example Problem E-KD-2.1.2 – Determination of Design Pressure in Cylindrical Vessel – Dual Layered Vessel

Determine the design pressure for a dual wall cylindrical vessel given the following data. Perform calculations for both open and closed-end vessels.

Vessel Data:

- Liner Material = SA-705 Gr. XM-12 Condition H1100 [8]
  - Yield Strength ($S_{y.1}$) = 115,000 psi @ 70°F per Table Y-1 of BPVC Section II, Part D II-D
  - Tensile Strength ($S_{u.1}$) = 140,000 psi @ 70°F per Table U of BPVC Section II, Part D
- Body Material = SA-723 Gr. 2 Class 2
  - Yield Strength ($S_{y.2}$) = 120,000 psi @ 70°F per Table Y-1 of BPVC Section II, Part D
  - Tensile Strength ($S_{u.2}$) = 135,000 psi @ 70°F per Table U of BPVC Section II, Part D
- Design Temperature = 70°F
- Liner Inside Diameter ($D_{i.1}$) = 16.00 inches
- Liner Outside Diameter ($D_{o.1}$) = 24.00 inches
- Outer Body Inside Diameter ($D_{i.2}$) = 23.95 inches
- Outer Body Outside Diameter ($D_{o.2}$) = 50.00 inches
- Overall Diameter Ratio (Y) = 50/16 = 3.125

It should be noted that all design pressure equations require the use of a parameter K_{ut} which is defined in KT-312 as:

$$K_{ut} = 0.95 \; for \; \frac{S_y}{S_u} \leq 0.7$$

$$K_{ut} = 1.244 - 0.42\left(\frac{S_y}{S_u}\right) \; for \; 0.7 < \frac{S_y}{S_u} \leq 0.9$$

$$K_{ut} = 0.866 \; for \; \frac{S_y}{S_u} > 0.9$$

$$\frac{S_{y.1}}{S_{u.1}} = \frac{115{,}000 \; psi}{140{,}000 \; psi} = 0.821$$

$$K_{ut.1} = 1.244 - 0.42 * \left(\frac{115{,}000 \; psi}{140{,}000 \; psi}\right) = 0.899$$

$$Y_1 = \frac{D_{o.1}}{D_{i.1}} = \frac{24.00 \; in}{16.00 \; in} = 1.5$$

$$\frac{S_{y.2}}{S_{u.2}} = \frac{120{,}000 \; psi}{135{,}000 \; psi} = 0.888$$

$$K_{ut.2} = 1.244 - 0.42 * \left(\frac{120{,}000 \; psi}{135{,}000 \; psi}\right) = 0.871$$

$$Y_2 = \frac{D_{o.2}}{D_{i.2}} = \frac{50.00\ in}{23.95\ in} = 2.088$$

Evaluate design pressure per KD-221.4 for an open-ended cylindrical shell with $Y > 2.85$ and for a closed-end cylindrical shell (all Y values).

$$P_D = min\left(\sum_{j=1}^{n}\left(0.924 * K_{ut.j} * S_{y.j} * ln(Y_j)\right), \sum_{j=1}^{n}\left(\frac{1}{3} * \left(S_{y.j} + S_{u.j}\right) * ln(Y_j)\right)\right) \qquad KD-221.4$$

$$P_D = min\left(0.924 * 0.899 * 115{,}000\ psi * ln(1.5) + 0.924 * 0.871 * 120{,}000\ psi * ln(2.088), \frac{1}{3} * (115{,}000\ psi + 140{,}000\ psi) * ln(1.5) + \frac{1}{3} * (120{,}000\ psi + 135{,}000\ psi) * ln(2.088)\right)$$

$$P_D = min(109{,}792\ psi, 97{,}029\ psi) = 97{,}029\ psi$$

The design pressure for both open and closed-end dual walled vessel is 97,029 psi.

Note that this calculation does not account for any loading in addition to internal pressure. If shell is subject to additional loading, the design shall be modified per KD-221.5.

3.3 Example Problem E-KD-2.2.1 – Elastic Plastic Analysis

Evaluate a monobloc vessel for compliance with respect to the elastic-plastic analysis criteria for plastic collapse provided in paragraph KD-231.

STEP 1 – Develop a numerical model using finite element analysis of the component including all relevant geometry characteristics. The model used for the analysis shall accurately represent the component geometry, boundary conditions, and applied loads. In addition, the model shall be refined around areas of stress and strain concentrations to accurately assess local areas for the criteria to be satisfied in KD-230. The analysis of one or more numerical models may be required to ensure that an accurate description of the stresses and strains in each component is achieved.

The model geometry is depicted in Figure E-KD-2.2.1-1. The monobloc vessel model with the finite element mesh is shown in Figure E-KD-2.2.1-2.

Vessel Data

- Material – All Components = SA-723 Grade 2 Class 2 (2 ¾Ni-1 1/2Cr-1/2Mo-V) [8][12]
- Design Pressure (P_D) = 45,000 psi at 150˚F
- Operating Pressure = 40,000 psi at 100˚F
- Elastic Modulus (E) = 27.37×10^6 ksi at 150˚F (design condition), BPVC Section II, Part D, Table TM
  = 27.64×10^6 ksi at 100˚F (operating condition), BPVC Section II, Part D, Table TM
- Yield Strength (S_y) = 117 ksi at 150˚F, BPVC Section II, Part D, Table Y-1
  = 120 ksi at 100˚F, BPVC Section II, Part D, Table Y-1
- Ultimate Strength (S_u) = 135 ksi at both 100˚F and 150˚F,

BPVC Section II, Part D, Table U

- Density (ρ) = 0.280 lbf/in^3, BPVC Section II, Part D, Table PRD
- Poisson's Ratio (ν) = 0.3, BPVC Section II, Part D, Table PRD
- Minimum Specified Elongation (El) = 14%
- Minimum Specified Reduction in Area (RA) = 45%
- Heat treatment is performed in accordance with Part KF so the forming strain may be assumed to be zero

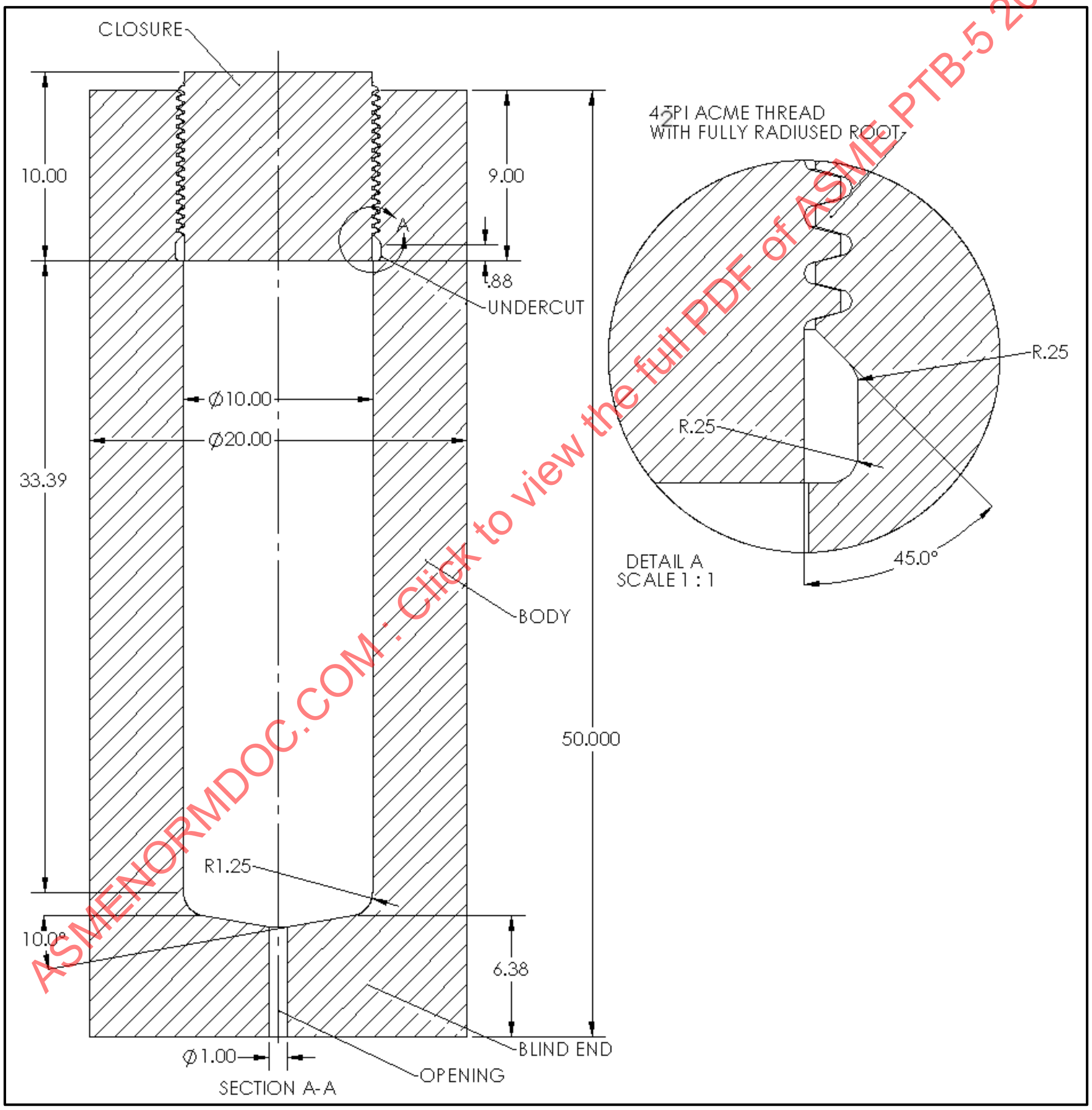

Figure 4 – E-KD-2.2.1-1 – VIII-3 Monobloc Vessel Configuration (Y = 2.0) with 2 TPI ACME thread with full radius root

Note: Dimensions are in inches unless otherwise specified.

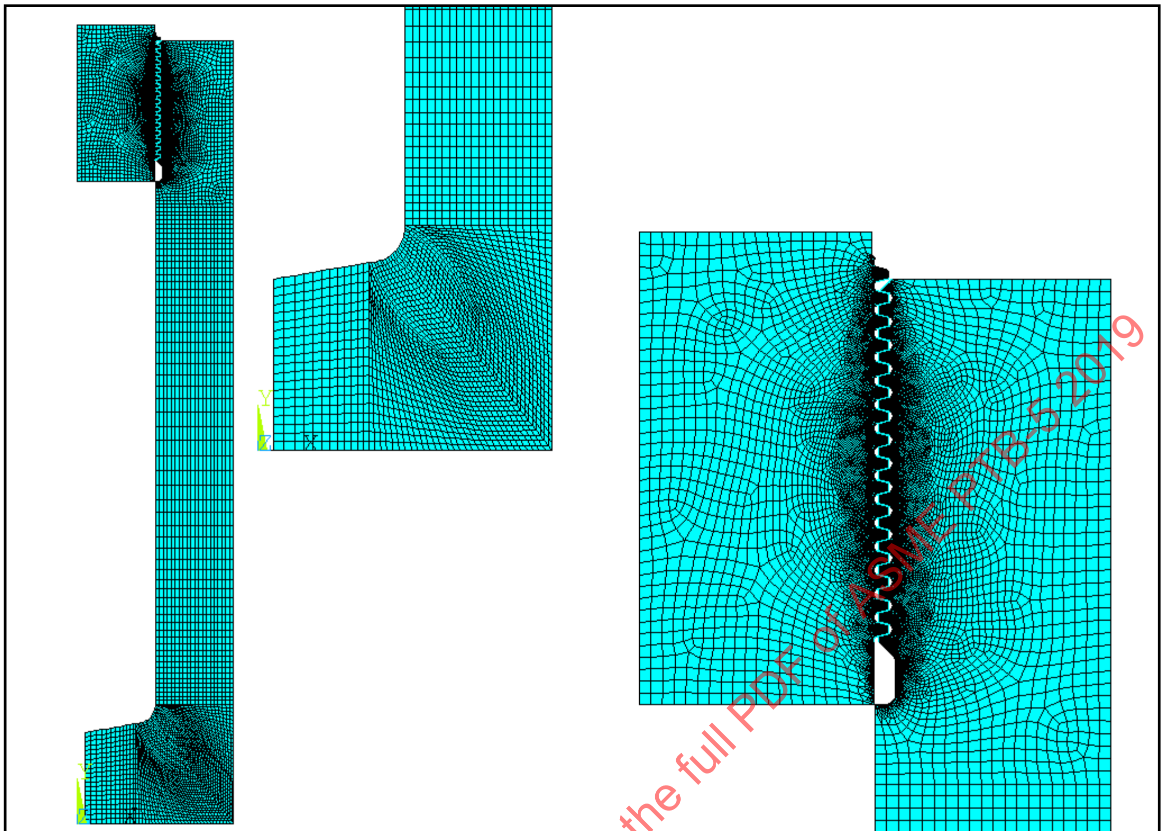

Figure 5 – E-KD-2.2.1-2 – Mesh of the Monobloc Vessel with Detailed Views of the Blind End, Closure and Body Threaded Connection

STEP 2 – Define all relevant loads and applicable load cases. The loads to be considered in the analysis shall include, but not be limited to, those given in Table KD-230.1.

The primary loads to be considered are internal pressure and dead weight factored according to Table KD-230.4, in this example two load cases are analyzed that are as shown below:

Load Case	Criteria	Load Combination [1, 2]
LC # 1	Global Design Condition	$1.8\,(P_D + D)$
LC # 2	Global Hydrostatic Test Condition	$P_t + D$ [3]

Notes:

1. P_D refers to design pressure of 45,000 psi, P_T refers to hydrostatic test pressure of 1.25*P_D multiplied by ratio of yield strength at test temperature to the yield strength at design temperature (120 ksi / 117 ksi) which is equal to 1.28*P_D (57,600 psi), and D refers to dead weight applied as acceleration (*1g*) in the finite element model.
2. The static head in the vessel is negligible compared to the pressure of the vessel.
3. Note that the S_y/S_u for this case is in excess of 0.72, so per KD-231.2(d) the hydrostatic testing criterion is non-mandatory; however, it was evaluated for demonstration purposes of this problem. It did not need to be completed.

The boundary conditions and loading applied on the model are as shown in Figure E-KD-2.2.1-3. It should be noted that the edge of the blind end at the opening is fixed vertically through a distance of 1 inch, as shown in the figure below, to prevent rigid body motion. The one-inch region in this vessel simulates the application of load due to a threaded connection. Note that some analysts prefer to restrain a single node to avoid affecting the stress distribution in the threaded region. However, the stresses in that region are not a particular concern for this example. Frictionless contact was applied between the cover and body threads

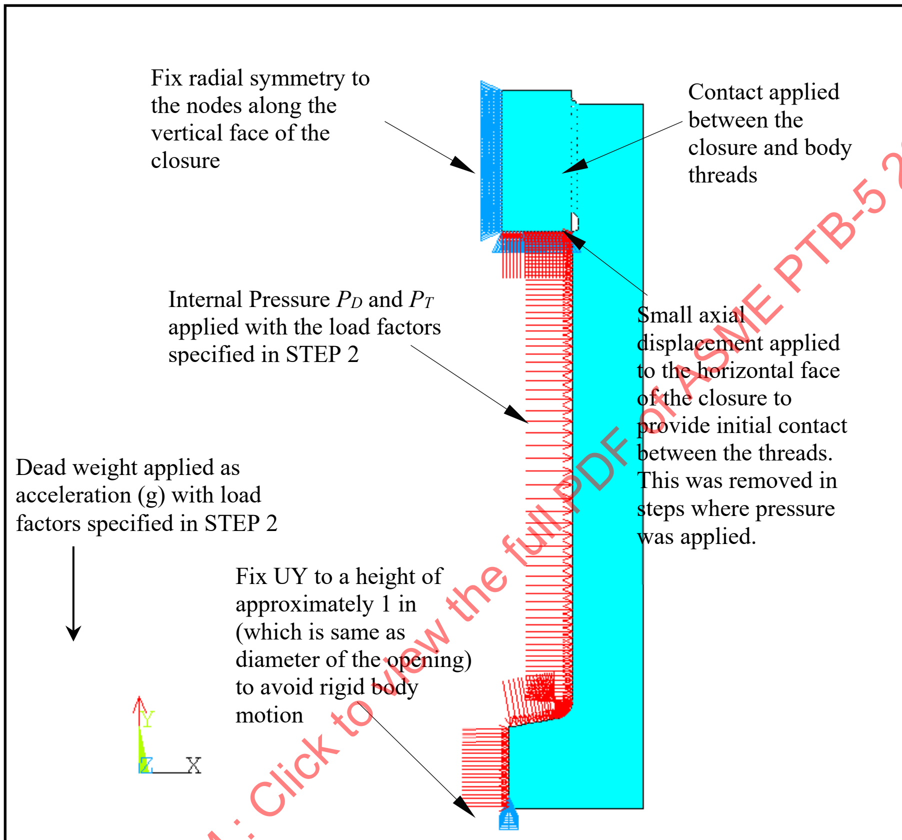

Figure 6 – E-KD-2.2.1-3 – Load and Boundary Conditions on the Monobloc Model

STEP 3 – An elastic-plastic material model shall be used in the analysis for LC # 1 global design condition. The von Mises yield function and associated flow rule should be utilized if plasticity is anticipated. A material model that includes hardening or softening, or an elastic perfectly plastic model may be utilized. A true stress-strain curve model that includes temperature dependent hardening behavior is provided in KM-620 (refer to Example Problem E-KM-2.2.1). When using this material model, the hardening behavior shall be included up to the true ultimate stress and perfect plasticity behavior (i.e., the slope of the stress-strain curves is zero) beyond this limit. The effects of nonlinear geometry shall be considered in the analysis.

The material model for the hydrostatic test pressure case was an elastic-perfectly plastic model (refer to KM-610).

The true stress-strain curve from KM-620 was used for the analysis. The material keywords used in the ANSYS input file are shown below. Refer to problem E-KM-2.2.1 for an example of the generation of a typical stress-strain curve using this method.

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/COM, Material Properties
/COM,*************************************************************************
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MP, EX,   1,     27.37e6
MP, DENS, 1,     0.280
MP, NUXY, 1,     0.3

/COM,*************************************************************************
****
/COM, True Stress-True Strain Data using KD-231.4 Elastic-Plastic Stress-Strain
Curve Model
/COM,*************************************************************************
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TBPT,,1.0,120000
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STEP 4 – Perform an elastic-plastic analysis for each of the load cases defined in STEP 2. If a converged solution is achieved with the application of the full load, the component is acceptable for a given load case. If convergence is not achieved, the model of the vessel should be investigated to determine the cause of the non-convergence, and the design (i.e., thickness) should be modified or applied loads reduced and the analysis repeated until convergence is achieved.

The von Mises and equivalent plastic strain results for the two load cases evaluated are as shown in Figures E-KD-2.2.1-4 through E-KD-2.2.1-7, convergence was achieved therefore the vessel satisfies the global criteria for these load cases.

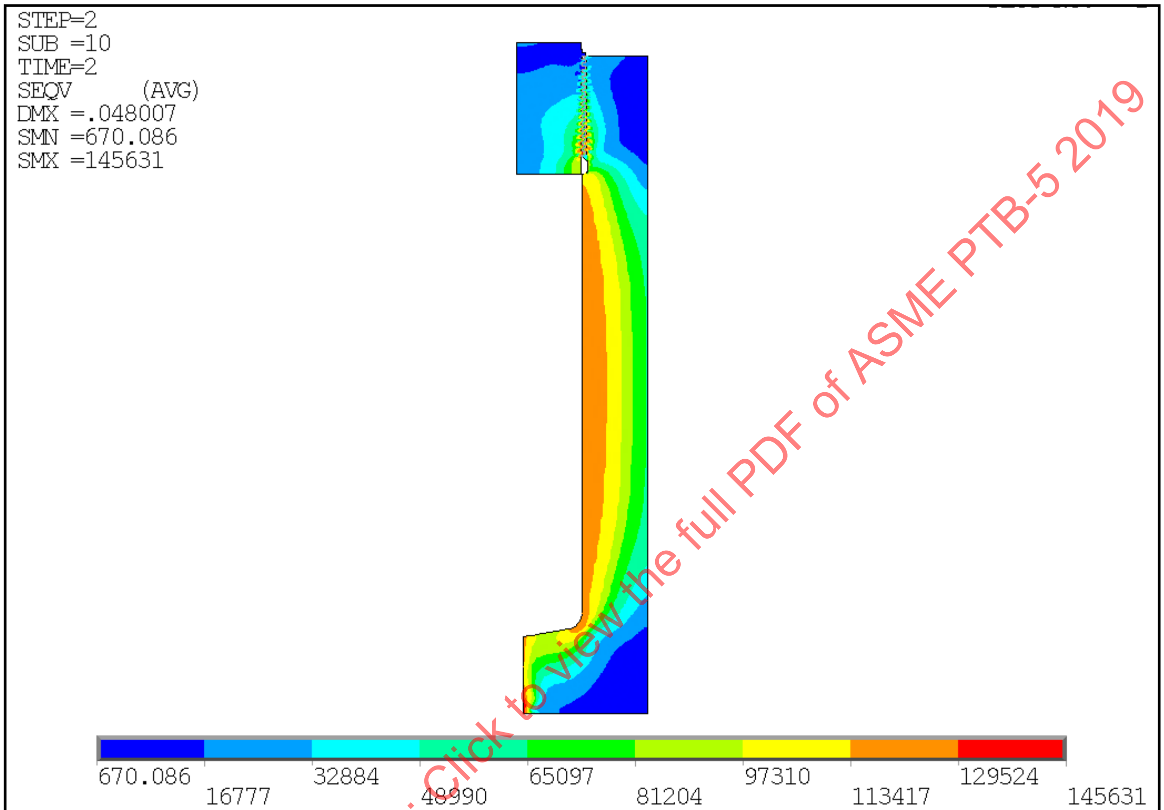

Figure 7 – E-KD-2.2.1-4 – Results of the Elastic-Plastic Analysis for LC #1 at a Factored Load of 81000 psi and acceleration of 1.8g; von Mises Stress

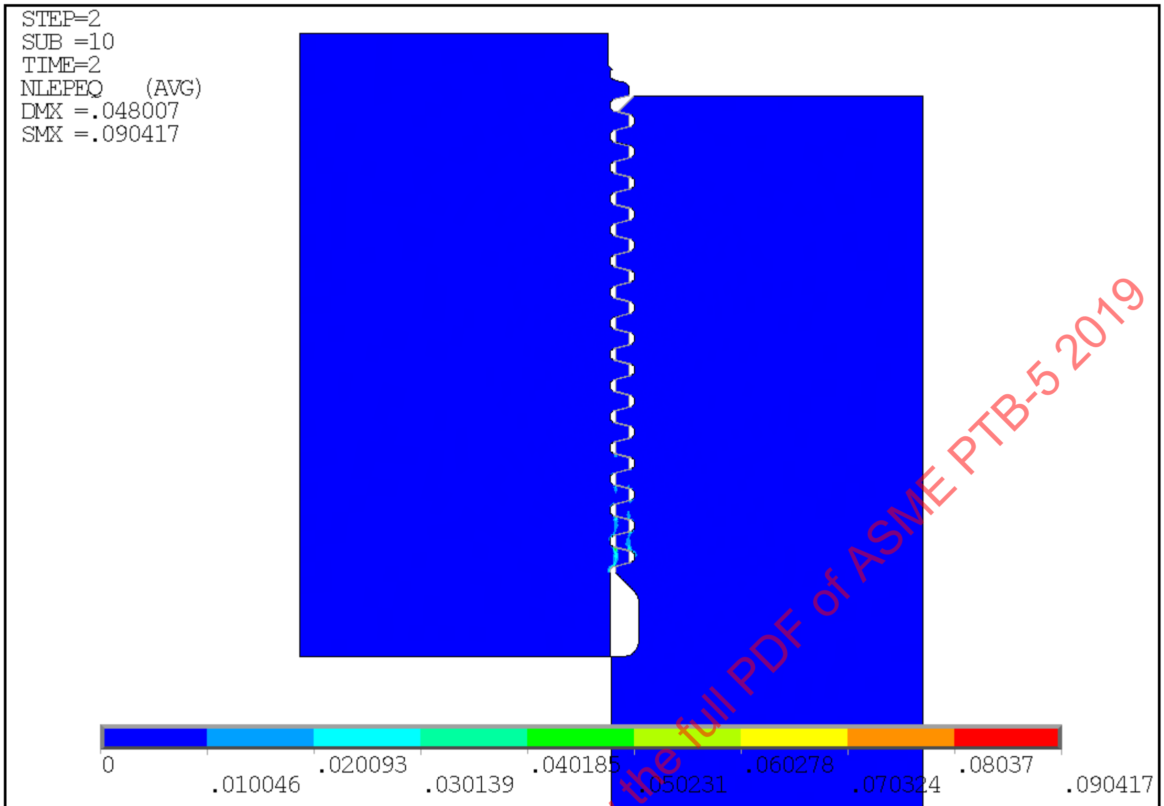

Figure 8 – E-KD-2.2.1-5 – Results of the Elastic-Plastic Analysis for LC #1 at a Factored Load of 81000 psi and acceleration of 1.8g; Equivalent Plastic Strain

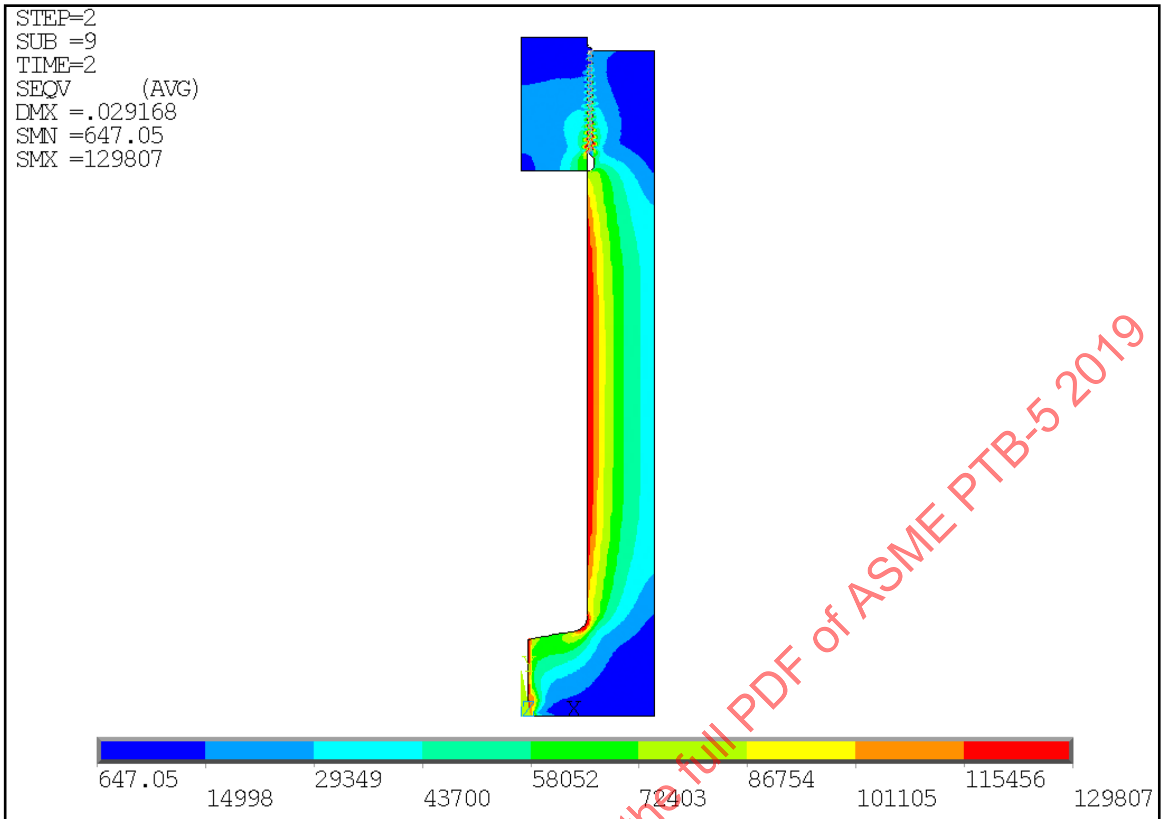

Figure 9 – E-KD-2.2.1-6 – Results of the Elastic-Plastic Analysis for LC #2 at a Factored Load of 57,600 psi and gravitational load of 1.0g; von Mises Stress

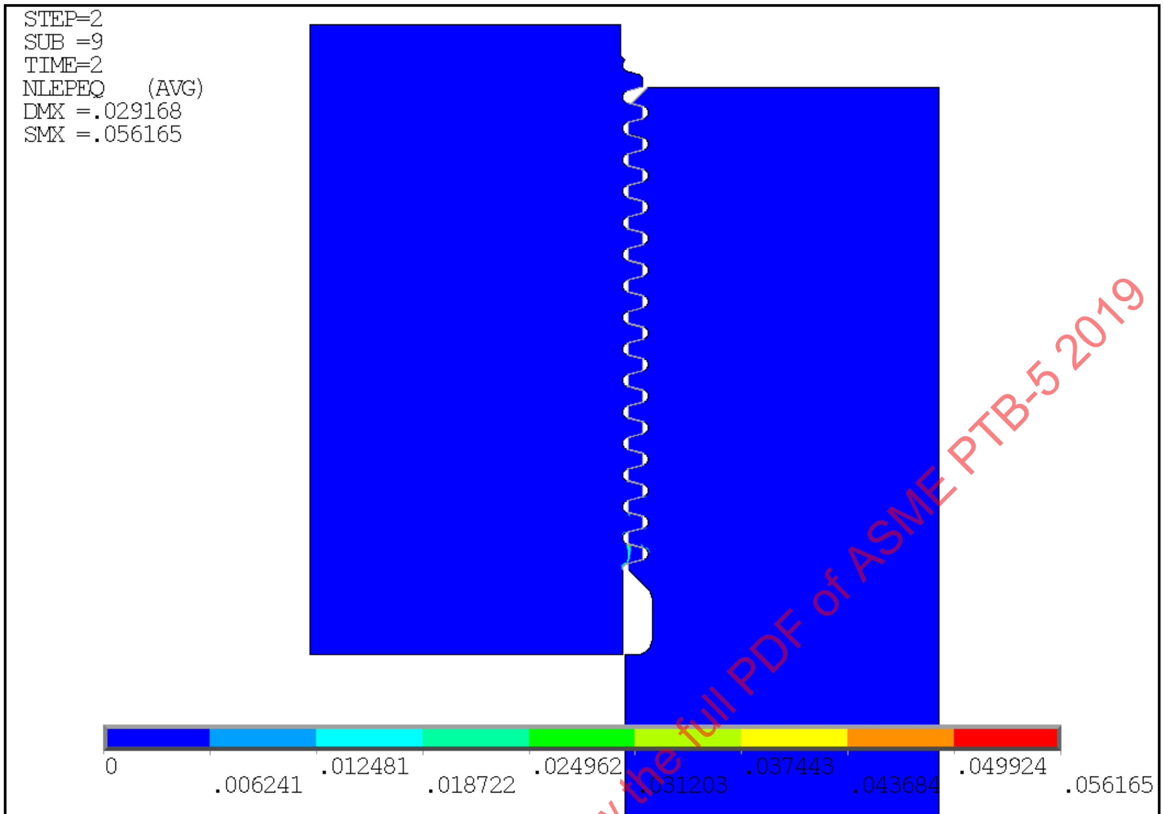

Figure 10 – E-KD-2.2.1-7 – Results of the Elastic-Plastic Analysis for LC #2 at a Factored Load of 57,600 psi and gravitational load of 1.0g; Equivalent Plastic Strain

3.4 Example Problem E-KD-2.2.2 – Protection Against Local Failure (Elastic-Plastic Analysis)

Paragraph KD-232.1 states that a strain limit evaluation shall be performed using two independent elastic-plastic analyses:

- The local criteria in Table KD-230.4
- A series of applied loads as described in KD-234 for ratcheting.

Each analysis used with respect to KD-232.1 shall use the elastic-plastic stress strain model in KM-620. The following procedure shall be used to evaluate protection against local failure for the local criteria loading specified in Table KD-230.4. The evaluation for a series of applied loads as described in KD-234 will be performed in Example problem E-KD-2.2.4.

STEP 1 – Perform an elastic-plastic stress analysis based on the load case combinations for the local criterion given in Table KD-230.4. The effects of non-linear geometry shall be considered in the analysis.

The same model and material conditions were used in Example Problem E-KD-2.2.1. The only load to be considered is internal pressure factored according to Table KD-230.4 for the local criterion, i.e., $1.28(P_D + D)$ where P_D equals 45,000 psi and D is dead weight applied as acceleration due to gravity (1g).

STEP 2 – The method requires determination of the principal stresses, σ_1, σ_2, σ_3, and the equivalent stress, σ_e, using Equation (13) of paragraph KD-232.1 and the total equivalent plastic strain ε_{peq}.

Values for the principal stresses, equivalent stress and total equivalent plastic strain for each point in the model were extracted from the finite element results file. The full model will be evaluated using custom automated postprocessing of the finite element output.

The principal stresses to be evaluated are shown below. A single point under consideration to demonstrate the calculation procedure in detail is the point in the root of the first body thread as shown in Figure 9:

σ_1 = 155,508 psi

σ_2 = 83,575 psi

σ_3 = 12,512 psi

σ_e = 123,493 psi

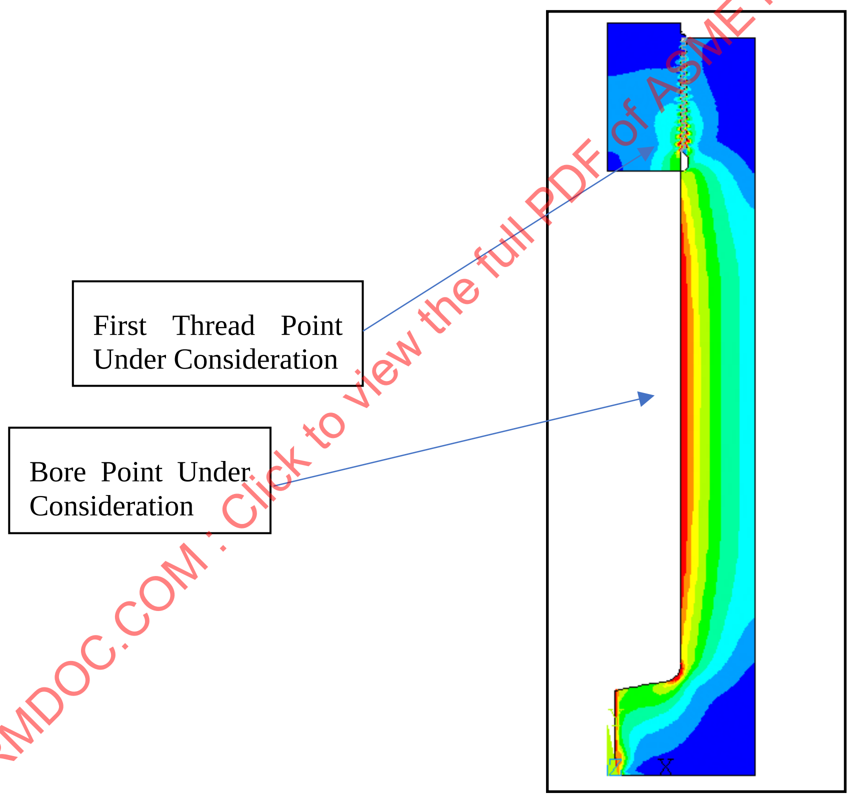

Figure 11 – Point Under Consideration from the Model

STEP 3 – Determine the limiting triaxial strain:

$$\epsilon_L = \epsilon_{Lu} * e^{\frac{-m_5}{1+m_2}*\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3*\sigma_e}-\frac{1}{3}\right)} \qquad KD-232.2$$

The strain limit parameters are shown below (Table KD-230.5):

$$R = \frac{S_y}{S_u} = \frac{117{,}000psi}{135{,}000psi} = 0.867$$

$$m_2 = 0.60 * (1.00 - R) = 0.60 * (1.00 - 0.867) = 0.08$$

$$m_3 = 2 * ln\left(1 + \frac{El}{100}\right) = 2 * ln\left(1 + \frac{14}{100}\right) = 0.262$$

$$m_4 = ln\left(\frac{100}{100 - RA}\right) = ln\left(\frac{100}{100 - 45}\right) = 0.598$$

$$m_5 = 2.2$$

$$\epsilon_{Lu} = max(m_2, m_3, m_4) = max(0.08, 0.262, 0.598) = 0.598$$

The computed limit strain is:

$$\epsilon_L = 0.598 * e^{\frac{-2.2}{1+0.08} * \left(\frac{155{,}508\ psi + 83{,}575\ psi + 12{,}512\ psi}{3 * 123{,}493\ psi} - \frac{1}{3}\right)} = 0.296$$

STEP 4 – Determine the forming strain (ε_{cf}) based on the material and fabrication method in accordance with Part KF. If heat treatment is performed in accordance with Part KF, the forming strain may be assumed to be zero. It should be noted that the vessel is fully machined with no forming processes used in the manufacture. The forming strain is:

$$\epsilon_{cf} = 0$$

STEP 5 – Determine if the strain limit is satisfied. The location in the component is acceptable for the specified load case if equation below is satisfied.

$$\epsilon_{peq} + \epsilon_{cf} \le \epsilon_L$$

The total equivalent plastic strain taken from the model is:

$$\epsilon_{peq} = 0.014$$

$$\epsilon_{peq} + \epsilon_{cf} = 0.014 + 0 \le \epsilon_L = 0.296$$

This can also be expressed using equation KD-232.4:

$$D_\varepsilon = {}^{\Delta\varepsilon_{peq}}/_{\varepsilon_L} = {}^{0.014}/_{0.296} = 0.0474$$

The strain at this point passes the Elastic-Plastic criterion.

A second point at the vessel bore is also evaluated to demonstrate the process for comparison. Note that R, m_2, m_3, m_4, m_5, ε_{Lu}:

Table 4 – Comparison of Strain Limit Calculations at Two Points

	Bore	**First Thread**
σ_1	52,209 psi	155,508 psi
σ_2	888 psi	83,575 psi
σ_3	-56,584 psi	12,512 psi
σ_e	94,269 psi	123,493 psi
ε_L	1.209	0.296
ε_{peq}	0.00345	0.140
D_ε	0.0029	0.0474

The same process needs to be completed for the entire model. A full model contour plot of the strain limit is shown in Figure E-KD-2.2.2-1.

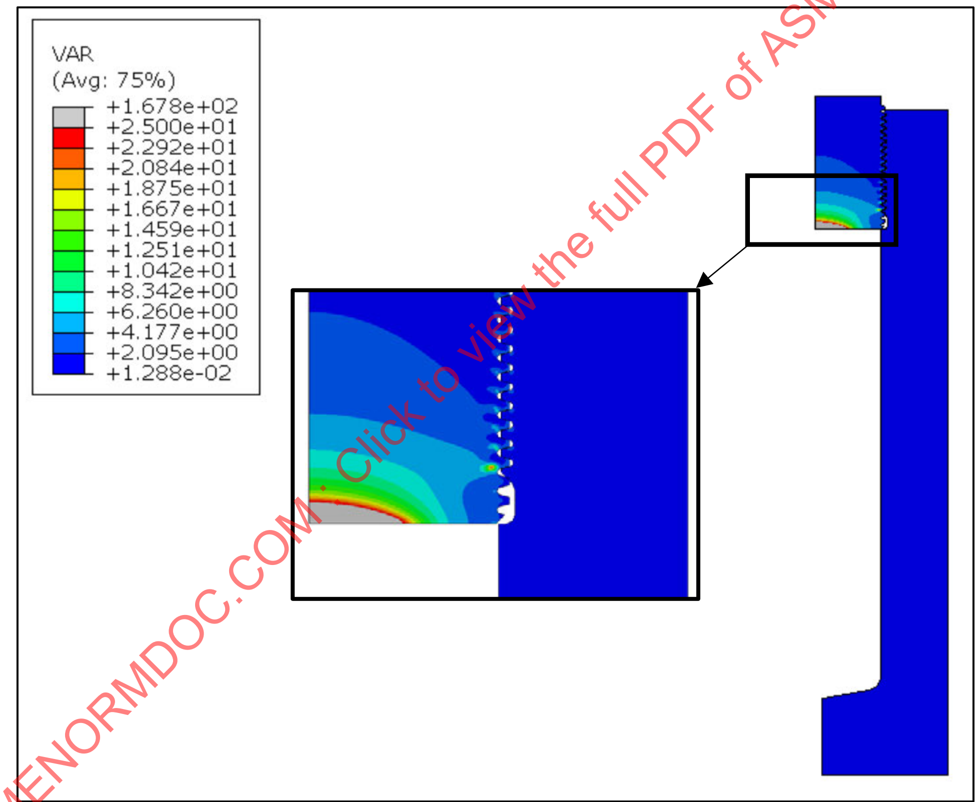

Figure 12 – E-KD-2.2.2-1 – Contour Plot of the Strain Limit, (VAR = ε_L)

A full model contour plot of the equivalent plastic strain is shown in Figure E-KD-2.2.2-2.

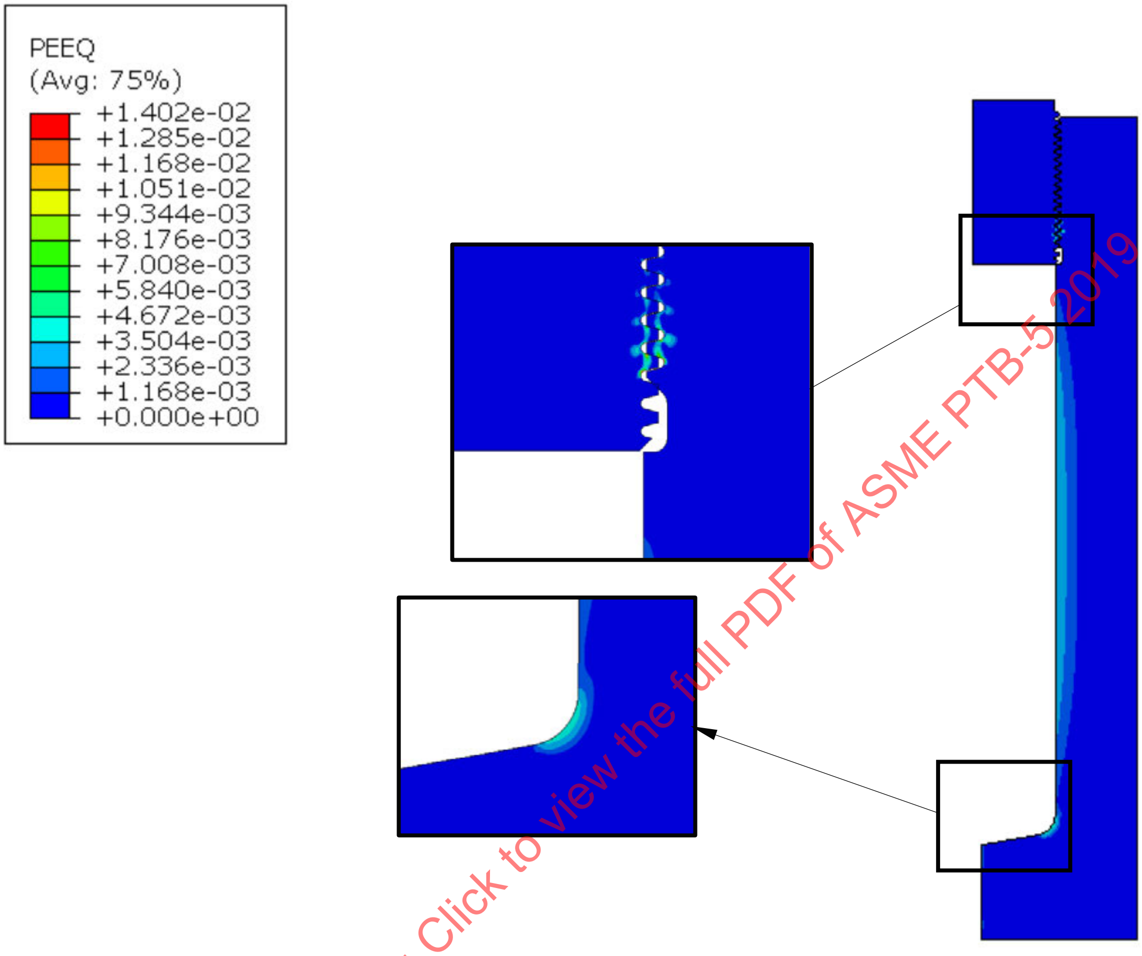

Figure 13 – E-KD-2.2.2-2 – Contour Plot of Equivalent Plastic Strain, ε_{peq} - Local Criteria

Full model evaluation of the Elastic-Plastic criterion is shown in Figure E-KD-2.2.2-3.

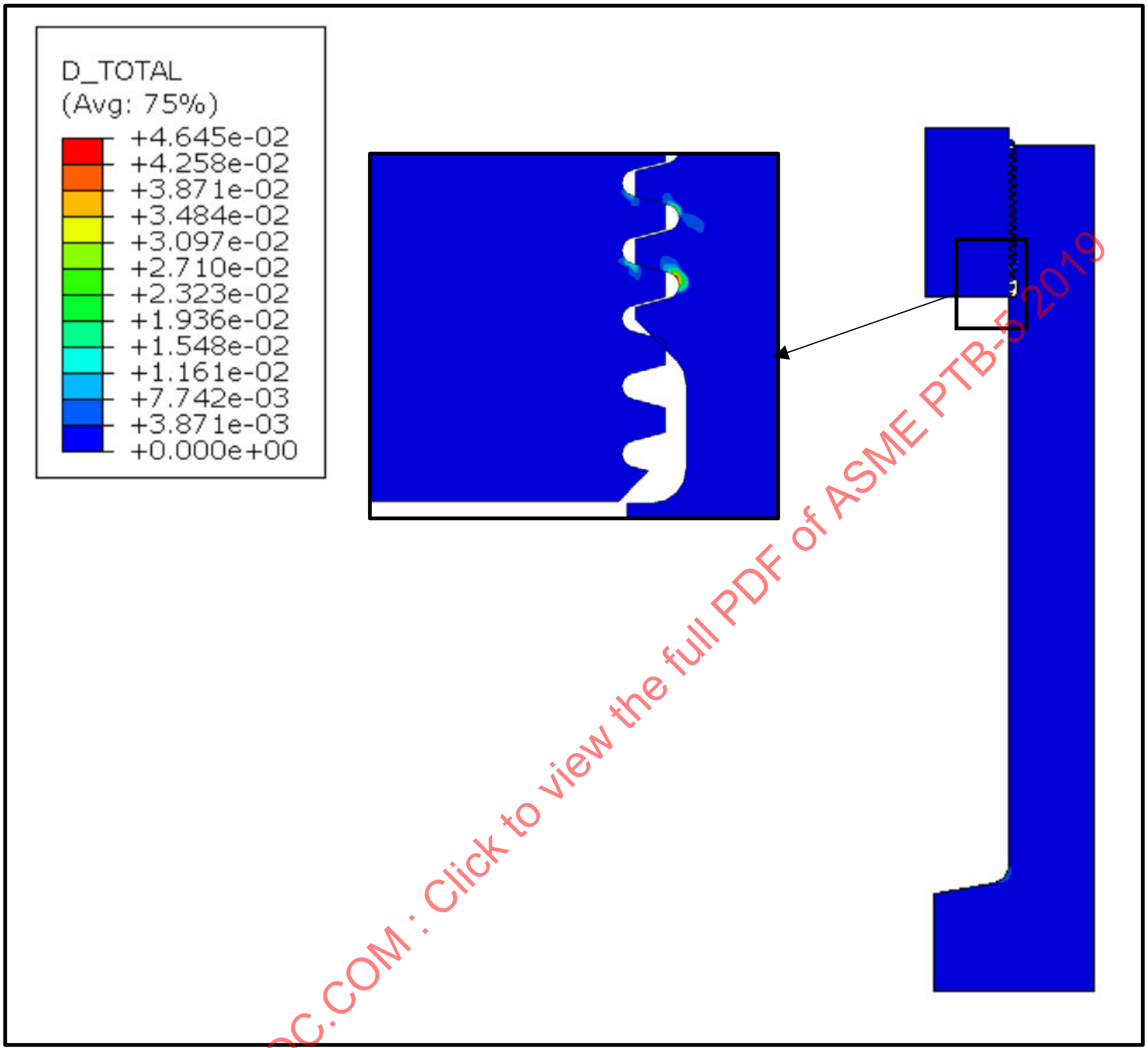

Figure 14 – E-KD-2.2.2-3 – Elastic-Plastic Strain Limit Ratio Results for Local Failure Analysis Results at 57600 psi

Full model evaluation indicates that the entire vessel meets the criterion of:

$$\epsilon_{peq} + \epsilon_{cf} \le \epsilon_L$$

The maximum strain limit ratio for this model as indicated in Figure E-KD-2.2.2-3 is:

$$\frac{\epsilon_{peq} + \epsilon_{cf}}{\epsilon_L} = 0.04645$$

Since this value is less than 1.0, the model passes the elastic-plastic local strain analysis.

3.5 Example Problem E-KD-2.2.3 – Ratcheting Assessment Elastic-Plastic Stress Analysis

Evaluate the monobloc vessel shown in Example Problem E-KD-2.2.1 for compliance with respect to the elastic-plastic ratcheting criteria provided in paragraph KD-234.

STEP 1 – Develop a numerical model of the vessel components. The axisymmetric finite element model geometry was taken from Example E-KD-2.2.1 (refer to Figures E-KD-2.2.1-1 and E-KD-2.2.1-2). The boundary conditions and relevant internal pressure load applied as shown in Figure E-KD-2.2.3-1.

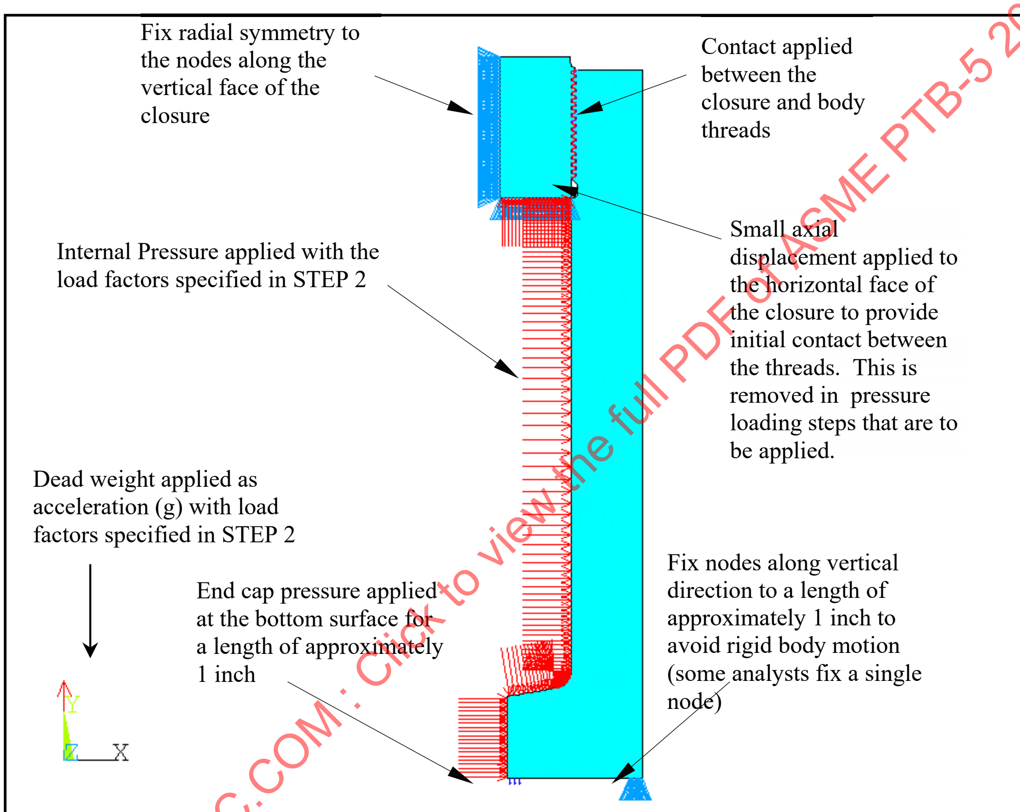

Figure 15 – E-KD-2.2.3-1 – Loads and Boundary Conditions on the Monobloc Model for Ratcheting Assessment

STEP 2 – Define all relevant loads and applicable load cases. The loads considered in accordance with Table KD-230.1 are internal pressure and dead weight. First the vessel is ramped up to hydrostatic test pressure ($1.28P_D = 57{,}600$ psi) and then cycled with internal pressure between 0 psi and operating pressure of 40,000 psi for three cycles.

STEP 3 –The material model used here is an elastic-perfectly plastic material model. The effects of nonlinear geometry shall be considered. The engineering stress strain data from Table Y-1 is converted to true stress-strain data for SA-723, Grade 2, Class 2 material at operating temperature of 100°F in accordance with paragraph KM-620 and the material keywords used in ANSYS are as shown below:

```
/COM,*************************************************************************
/COM, Material Properties
/COM,*************************************************************************

MPTEMP,   1,     100
MP, EX,   1,     27.64e6
MP, DENS, 1,     0.280
MP, NUXY, 1,     0.3

/COM,****************************************************************
/COM, True Stress-True Strain Data Elastic-Perfectly Plastic Model
/COM,****************************************************************

! SA-723 Gr 2 Class 2

TB,KINH,1, ,3                ! Activate a data table
TBTEMP,100             ! Temperature = 20.0
TBPT,,0.0,0.0                ! Strain = 0.0, Stress = 0.0
TBPT,,0.00434153,120000       ! Strain = 0.00434153, Stress = 120000
TBPT,,1.0,120000             ! Strain = 1.0, Stress = 120000
```

STEP 4 – Perform an elastic-plastic analysis using the applicable loading from STEP 2.

The elastic-plastic analysis was performed using a series of load steps including the 57,600 psi hydrostatic test pressure and three 40,000 psi operating pressure steps and the elastic-perfectly plastic material model from STEP 3. A plot of the pressure loading sequence is shown in Figure E-KD-2.2.3-2

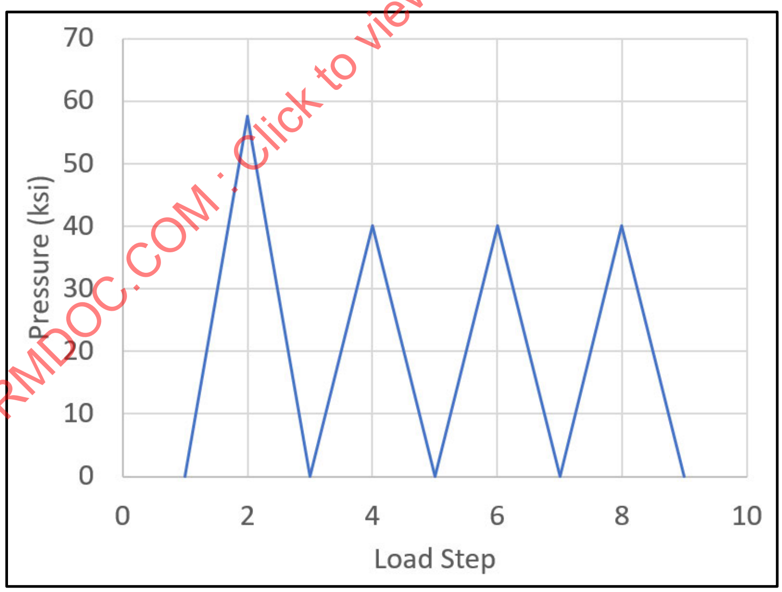

Figure 16 – E-KD-2.2.3-2 – Pressure Loading Sequence on the Monobloc Vessel

A plot of the von Mises stress and equivalent plastic strain under these loads are shown in Figures E-KD-2.2.3-3 through E-KD-2.2.3-6.

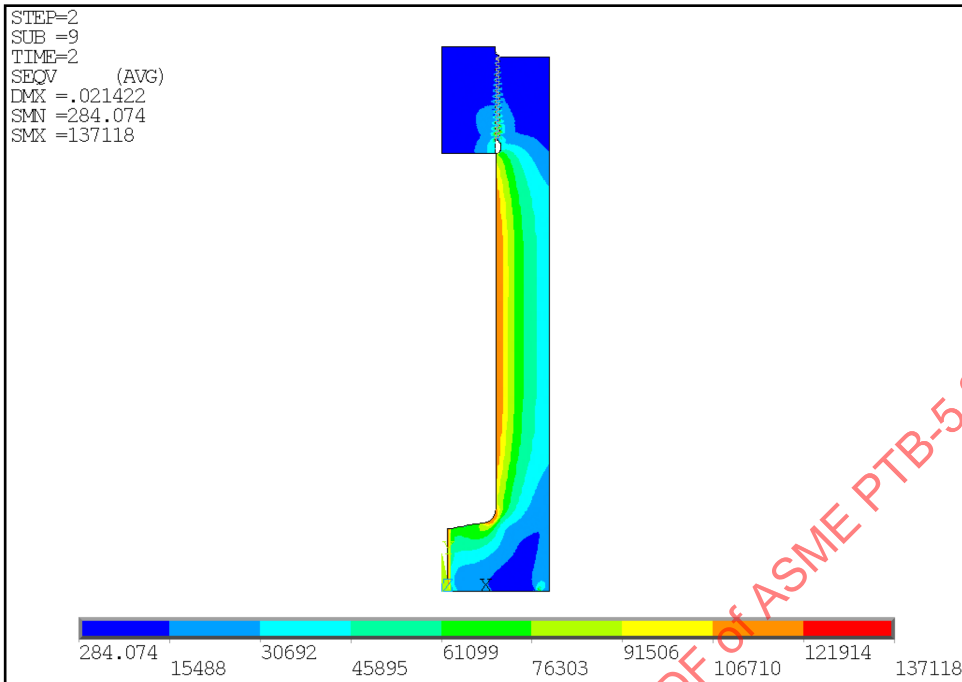

Figure 17 – E-KD-2.2.3-3 – von Mises Stress Plot for Hydrostatic Test Pressure of 57,600 psi

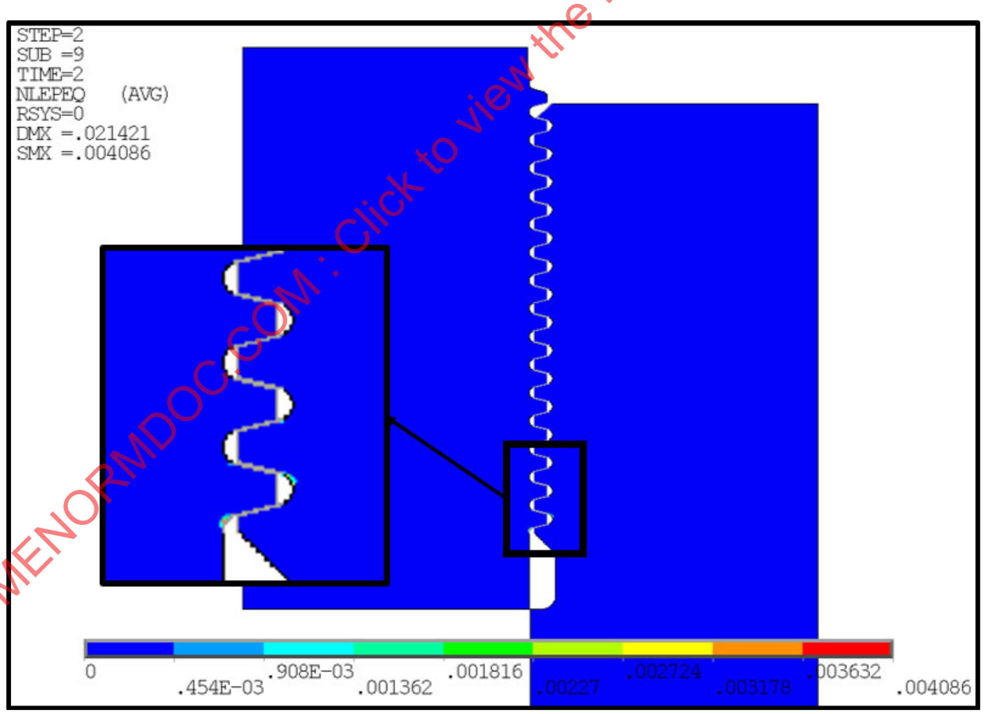

Figure 18 – E-KD-2.2.3-4 – Equivalent Plastic Strain for Hydrostatic Test Pressure of 57,600 psi

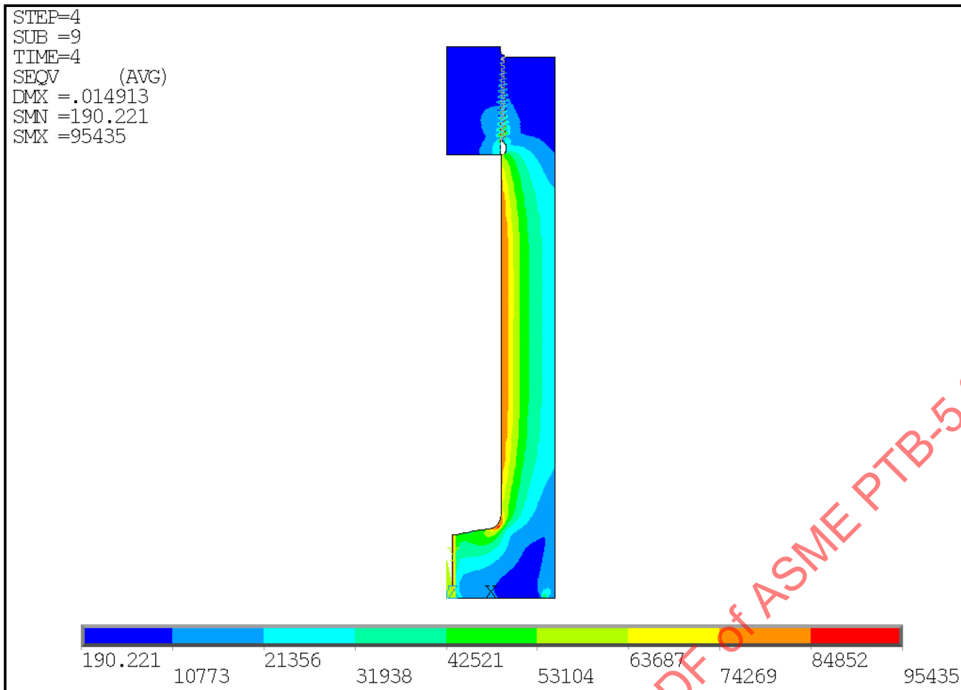

Figure 19 – E-KD-2.2.3-5 – von Mises Stress Plot for Operating Pressure of 40,000 psi, 1st cycle

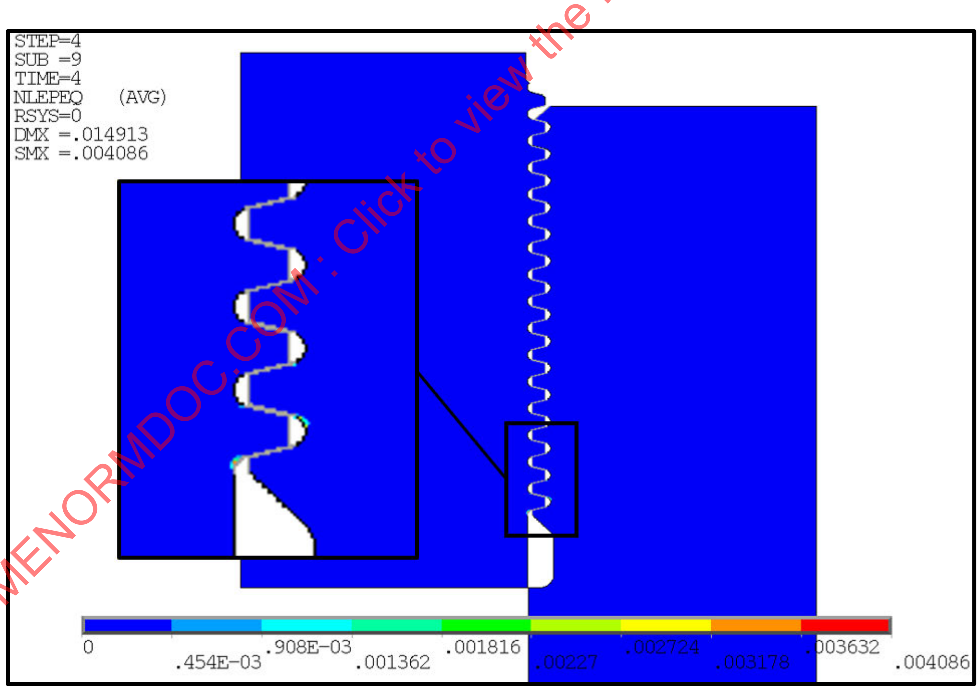

Figure 20 – E-KD-2.2.3-6 – Equivalent Plastic Strain for Operating Pressure of 40,000 psi, 1st cycle

STEP 5 – Evaluate the ratcheting criteria in paragraph KD-234.1 STEP 5 at the end of the third cycle. Figures E-KD-2.2.3-6 and E-KD-2.2.3-7 show the von Mises stress and equivalent plastic strain in the model following the completion of the third cycle.

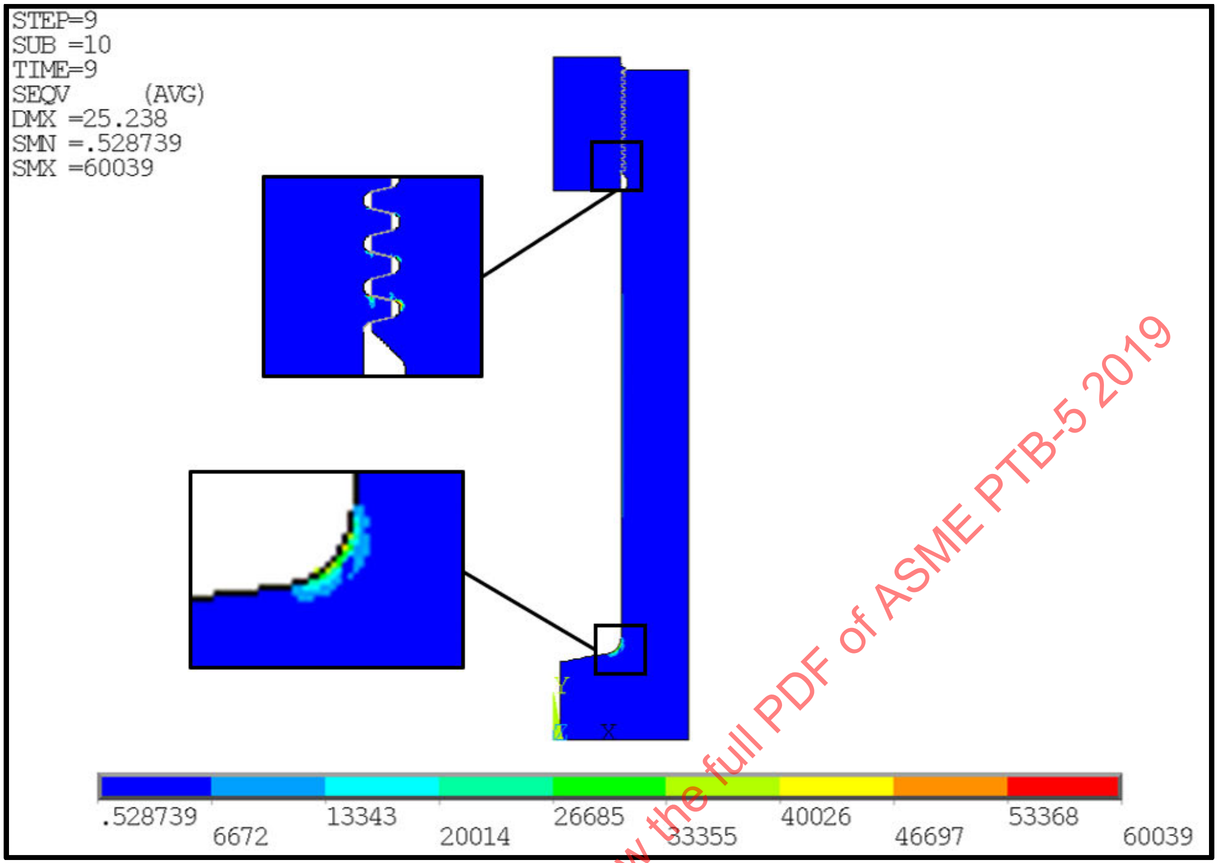

Figure 21 – E-KD-2.2.3-6 – von Mises Stress Plot for Operating Pressure of 40,000 psi, End of the 3rd cycle

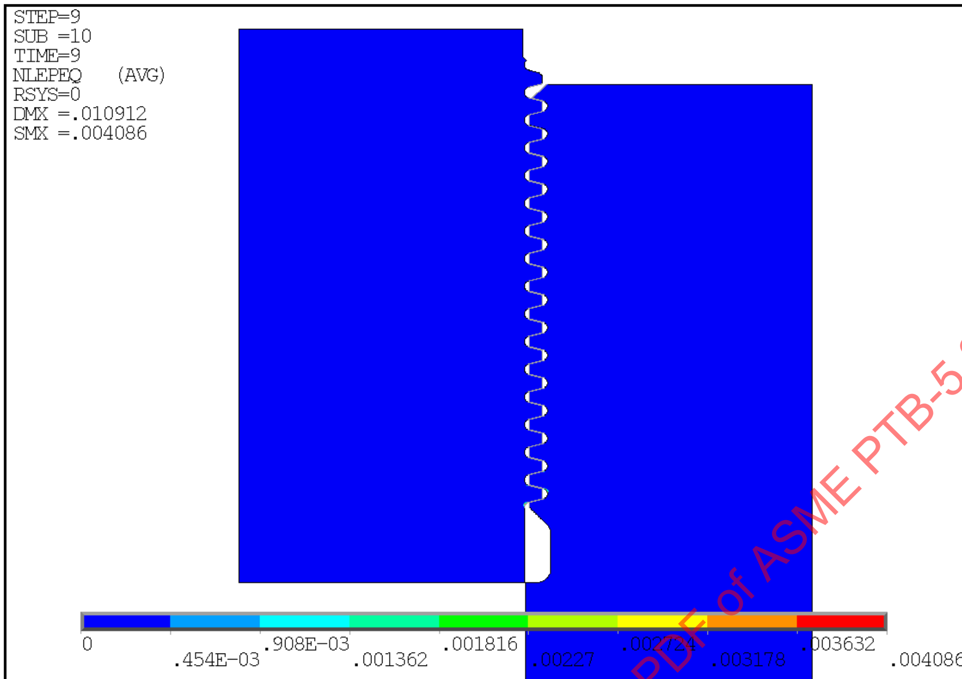

Figure 22 – E-KD-2.2.3-7 – Equivalent Plastic Strain for Operating Pressure of 40,000 psi, End of the 3rd cycle

It can be seen from Figures E-KD-2.2.3-3, E-KD-2.2.3-5, and E-KD-2.2.3-7 that the equivalent plastic strain does not change among these cases. In other words, zero plastic strains have been incurred in the closure, body, and blind end of the monobloc vessel from the first cycle to the end of third cycle. Thus, these components meet the condition detailed in KD-234.1 STEP 5 (a) and the ratcheting criteria are satisfied. The vessel components, therefore, are acceptable per the elastic-plastic ratcheting criteria for an operating pressure cycle between 0 and 40,000 psi.

3.6 Example Problem E-KD-2.2.4 – Protection Against Local Failure for a Series of Applied Loads

Paragraph KD-232.1 states that a strain limit evaluation shall be performed using two independent elastic-plastic analyses with loads from the following:

- The local criteria in Table KD-230.4
- A series of applied loads as described in KD-234 for ratcheting.

Example problem E-KD-2.2.2 demonstrated the evaluation of local failure for the local criteria loading from Table KD-230.4. The following evaluation is for a series of applied loads as described in KD-234. This example utilizes Abaqus finite element software to complete this problem. The analysis used with respect to KD-232.1 shall use the elastic-plastic stress strain model in KM-620.

This evaluation followed a procedure similar to what is described in E-KD-2.2.2, with the exception that the strain damage is evaluated at each loading increment throughout the series of ratcheting steps (“k” load steps) as described in E-KD-2.2.3.

The strain damage is then calculated using a summation of damage over time for the load steps, accumulating a total damage as:

$$D_{\epsilon,k} = \frac{\Delta\epsilon_{peq,k}}{\epsilon_{L,k}} \qquad KD-232.4$$

$$D_{\epsilon t} = \sum_{1}^{k} D_{\epsilon,k}$$

where $D_{\epsilon t}$ is the total accumulated damage. The total accumulated damage at the end of the third operating cycle is shown in Figure E-KD-2.2.4-1. In the whole model the accumulated damage is less than 1.0, therefore satisfying the KD-232.1(i) criteria.

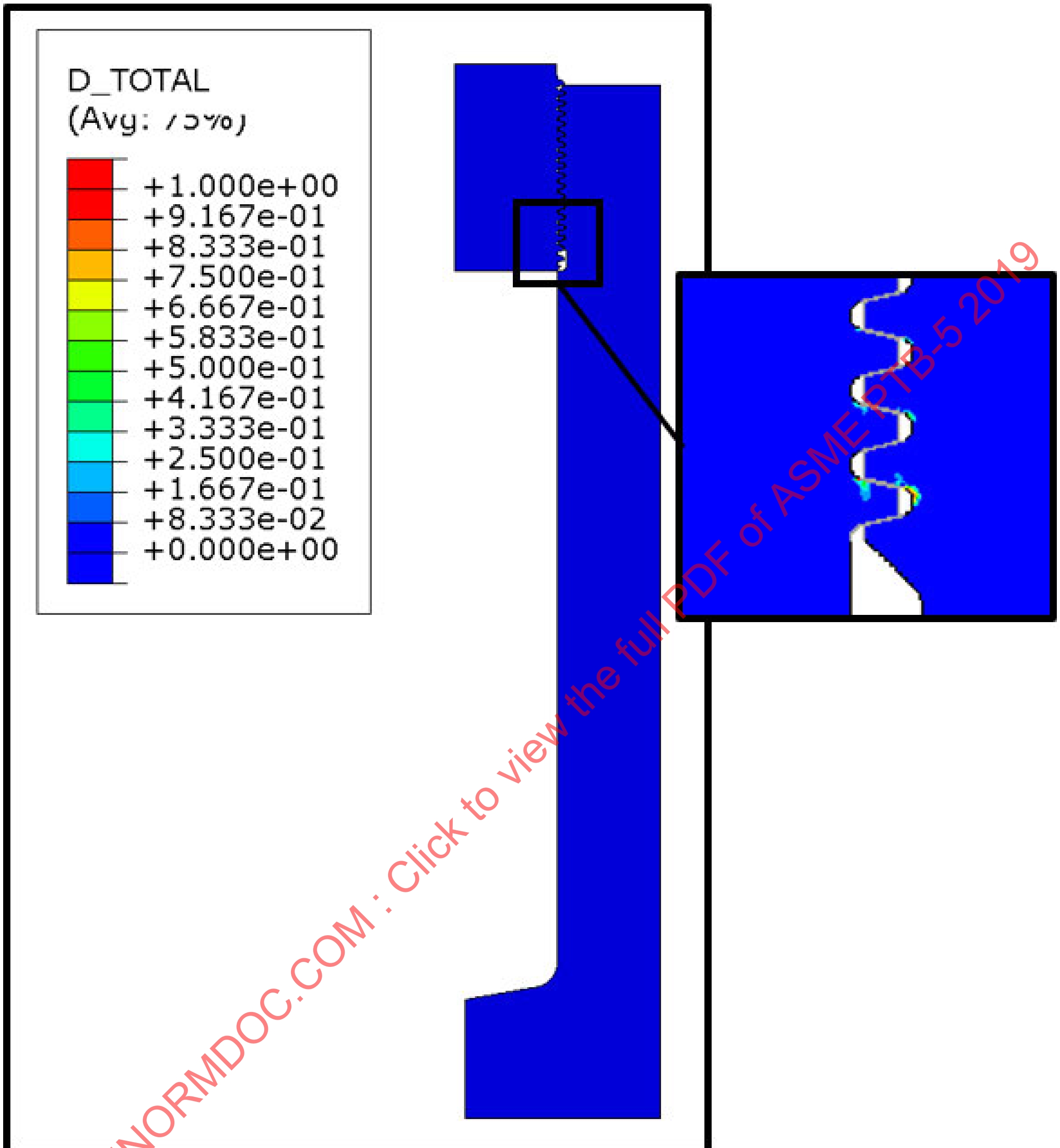

Figure 23 – E-KD-2.2.4-1 – Contour Plot of the Total Accumulated Damage, $D_{\varepsilon t}$ – End of 3rd Operating Cycle

3.7 Example Problem E-KD-2.3.1 – Linear Elastic Stress Analysis

Evaluate a monobloc vessel body for compliance with BPVC Section VIII, Division 3 according to the elastic stress analysis criteria provided in Appendix 9. Load due to internal pressure and the load acting on the body threads due to internal pressure applied on the closure inside surface are the loads that need to be considered. Relevant design data, geometry, and nomenclature of the vessel body are provided below in Figure E-KD-2.3.1-1.

Vessel Data

- Material – All Components = SA-723 Grade 2 Class 2 [8][12]
- Design Pressure (P_D) = 11,000 psi at 150°F
- Operating Pressure = 90% of Design Pressure
  = 9,900 psi at 150°F
- Elastic Modulus (E) = 27.37×10^6 ksi at 150°F, BPVC Section II, Part D, Table TM-1, Group B
- Density (ρ) = 0.280, BPVC Section II, Part D, Table PRD
- Poisson's Ratio (ν) = 0.3, BPVC Section II, Part D, Table PRD

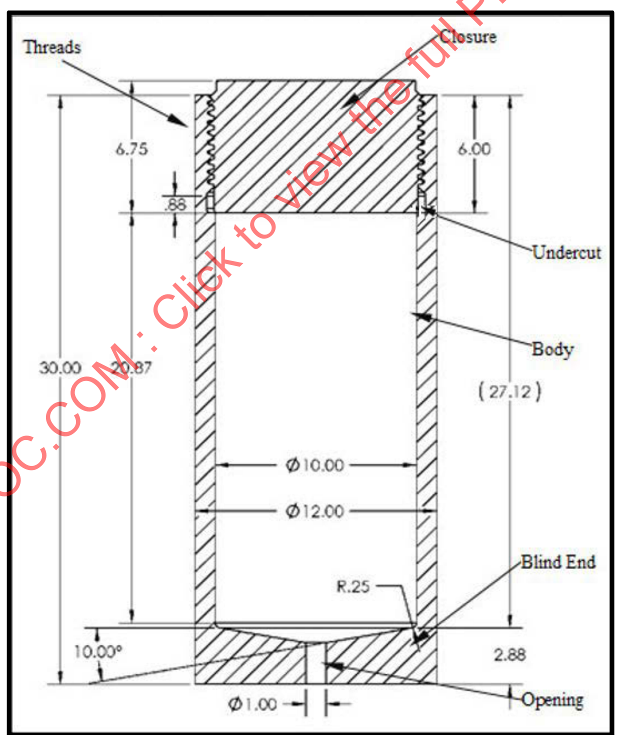

Figure 24 – E-KD-2.3.1-1 – VIII-3 Monobloc Vessel Configuration with 2 TPI ACME thread with Full Radius Root

Note: Dimensions are in inches unless otherwise specified.

STEP 1 – Determine that the vessel being analyzed has appropriate wall ratio for Linear Elastic Analysis per KD-200. KD-200(d) states that the wall ratio must be less than 1.25 to use the Linear Elastic Analysis method. The wall ratio of this vessel is:

$$Y = \frac{OD}{ID} = \frac{12in}{10in} = 1.2 < 1.25$$

STEP 2 – Determine the types of loads acting on the component. In general, separate load cases are analyzed to evaluate “load-controlled” loads such as pressure and “strain-controlled” loads resulting from imposed displacements. The load analyzed is internal design/operating pressure. The resulting load on the body threads due to the internal pressure acting on the closure is also considered. Since distribution of the load is not uniform on all the threads, the load distribution on each thread is calculated per Appendix E-221 continuous load distribution equations as shown in example problem E-AE-2.2.1.

STEP 3 – Develop the finite element model.

1) Due to symmetry in geometry and loading, an axisymmetric solid model is generated. The axisymmetric model consists of the body shell, including the blind end with a centrally located opening and body threads. The closure component is not modeled and the pressure load acting on the closure is modeled by transferring the load on to the body threads. The FE model is illustrated in Figure E-KD-2.3.1-2.

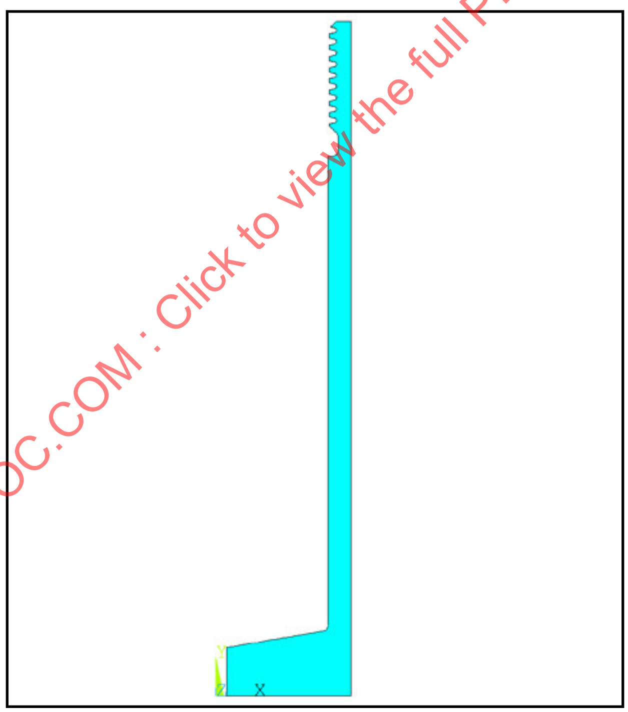

Figure 25 – E-KD-2.3.1-2 – Axisymmetric FE Model

2) Generate mesh. ANSYS [9] 8-noded structural solid element (Plane 82) with axisymmetric key option is specified for the analysis. The mesh is illustrated in Figure E-KD-2.3.1-3.

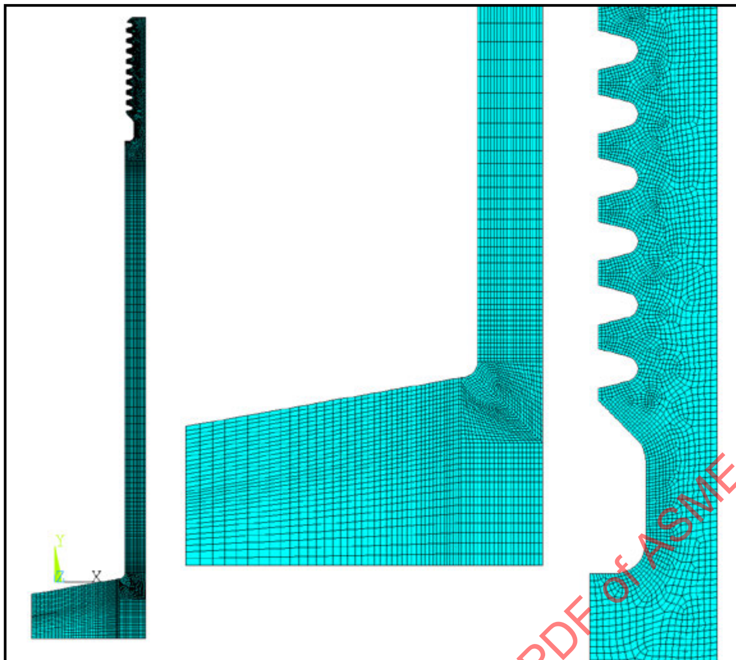

Figure 26 – E-KD-2.3.1-3 – Mesh of the Monobloc Vessel with Detailed Views of the Blind End and Body Thread Components

3) Apply the material properties given below to all the components in the monobloc vessel.

Component	Material	Modulus of Elasticity (psi)	Poisson Ratio
All	SA-723 Grade 2 Class 2	27.37E+06	0.3

4) Linear geometry (small displacement) should be used in the analysis.
5) Apply the internal pressure load to the pressure boundaries of the body shell and the blind end. Also, transfer the internal pressure acting on the closure on to the body threads by applying the loads on the body threads. The load applied on the threads is not equally distributed among all the threads. The first thread takes most of the load while the last thread takes a small portion of the total load applied. The percentage of the total load applied on individual threads is calculated using the equations given in E-221 and Table E-222.1. The actual percentage load applied on the individual treads is calculated in example problem E-AE-2.2.1 and shown in Figure E-KD-2.3.1-4. Apply the appropriate boundary conditions to the body as per the figure. The edge of blind end at the opening is fixed vertically through 1 inch, as shown in the figure below, to simulate a threaded connection in that region.

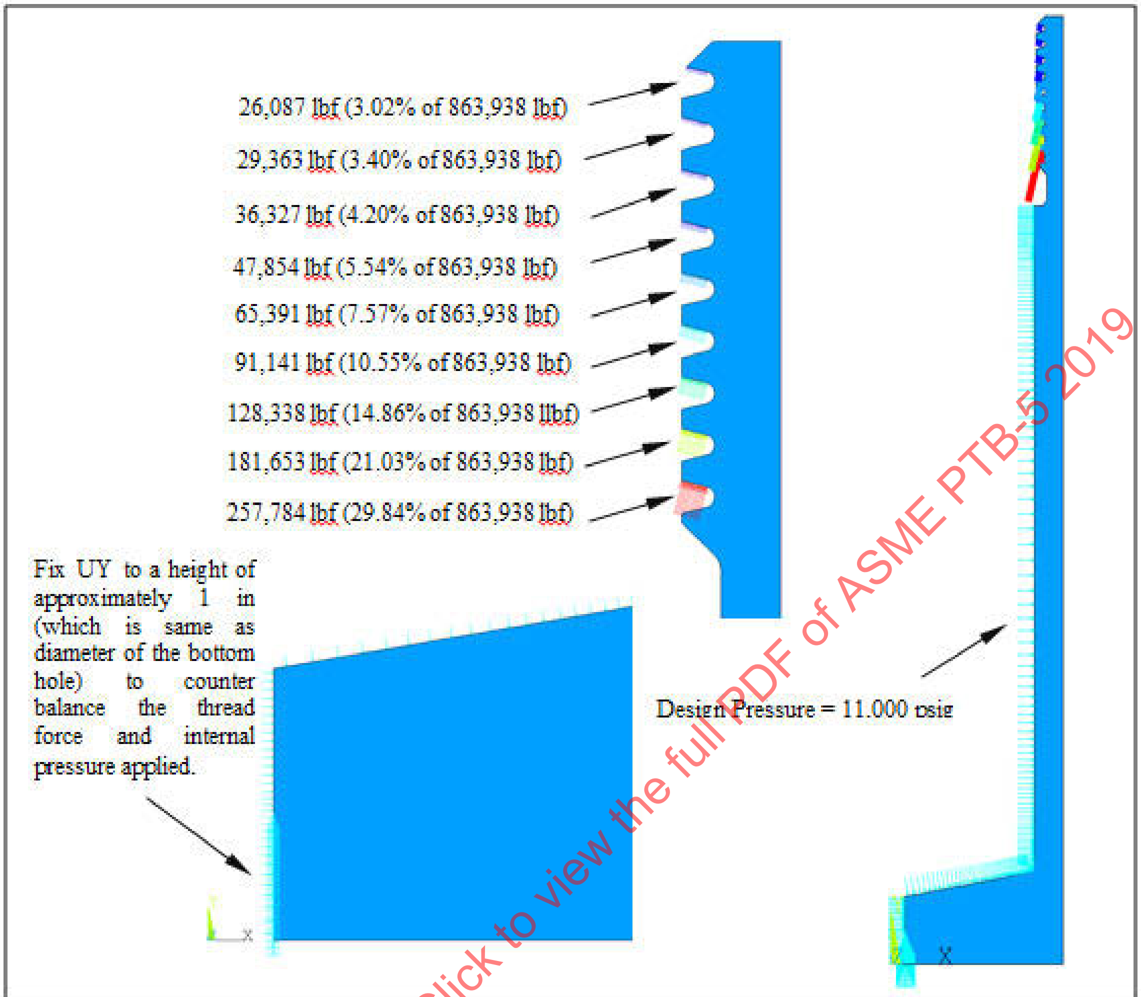

Figure 27 – E-KD-2.3.1-4 – Load and Boundary Conditions for the FE Model

Note: Thread load distribution on each individual body thread is calculated as shown in example problem E-AE-2.2.1

STEP 4 – Run analysis and review results. Evaluate the displacements and compare calculated reaction force values to hand calculated values.

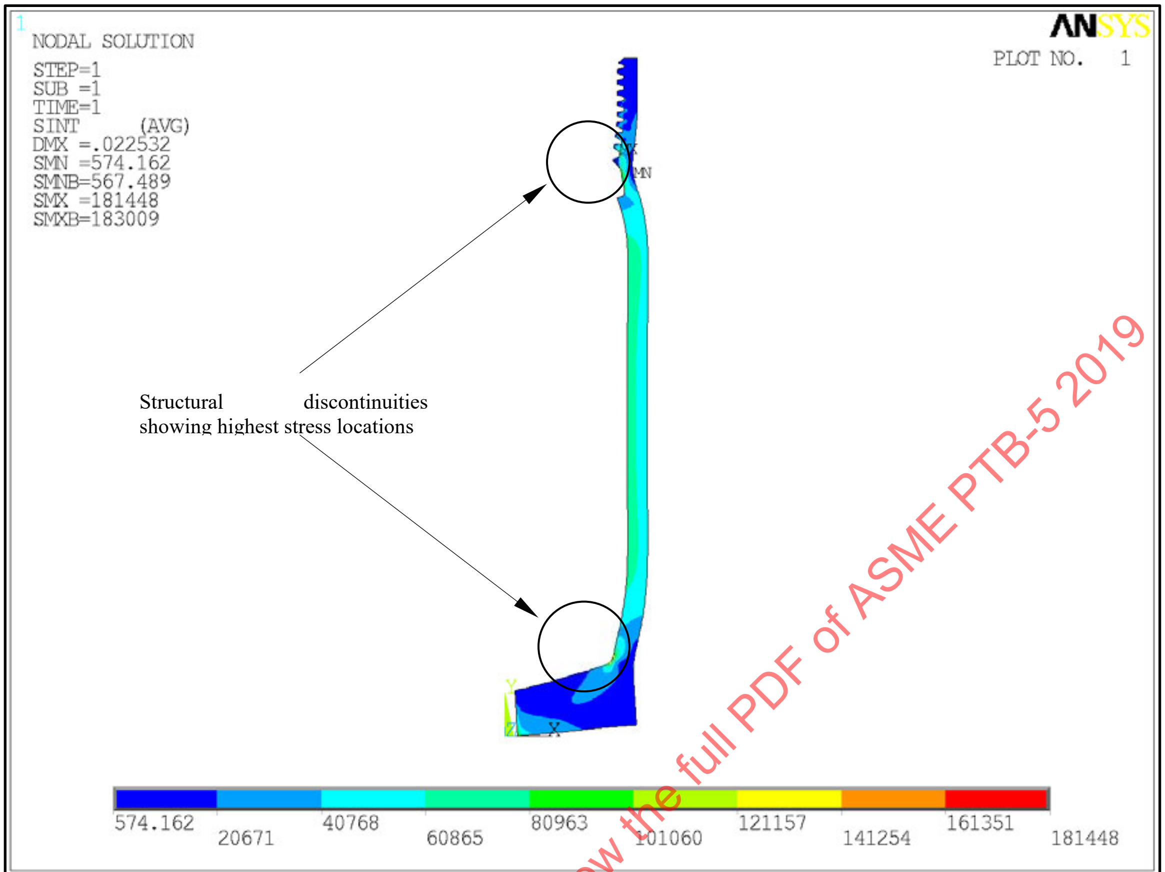

Figure 28 – E-KD-2.3.1-5 – Results of Elastic Analysis, Stress Intensity in Deformed State for Design Pressure and the Critical Locations through the Vessel Requiring Stress Evaluation

Reaction Force (y-direction)

Calculated (ANSYS)	8,640.5 lbf
Hand Calculations	8,639.4 lbf

Note: Results for Steps 4, 5, and 6 were calculated automatically by analysis routines contained in the FEA program. Through-wall stress linearization was conducted at critical areas around the pressure boundary to provide data for the routines. The resultant stress intensities for P_m, P_L, and P_b stress categories are summarized in Tables E-KD-2.3.1-1 and E-KD-2.3.1-2 for design and operating pressures, respectively.

Note that per L-311 Step 2 (a), bending stresses are calculated only for the local hoop and meridional (normal) component stresses, and not for the local component stress parallel to the SCL or in-plane shear stress.

STEP 5 – At the point on the vessel that is being investigated, calculate the stress tensor (six unique components of stress) for each type of load. Assign each of the computed stress tensors to one or to a group of the categories defined below. Assistance in assigning each stress tensor to an appropriate category for a component can be obtained by using Figure KD-240. Note that the stress intensities Q and F do not need to be determined to evaluate protection against plastic collapse; however, these components are needed for fatigue and shakedown/ratcheting assessment of the structure based on elastic stress analysis. Note that the $2*S_y$ limit placed on sum of Primary Local Membrane (P_m) plus Primary Bending (P_L) plus Secondary

Membrane Plus Bending (Q) has been placed at a level to ensure shakedown to elastic action after a few repetitions of the stress cycle. See paragraph KD-3 for the evaluation of fatigue analysis.

General primary membrane stress intensity – P_m

Local primary membrane stress intensity – P_L

Primary bending stress intensity – P_b

Secondary stress intensity – Q

Additional equivalent stress produced by a stress concentration or a thermal stress over and above the nominal ($P + Q$) stress level – F

The stress intensity categories are determined for the SCLs depicted in Figures E-KD-2.3.1-6 through E-KD-2.3.1-8.

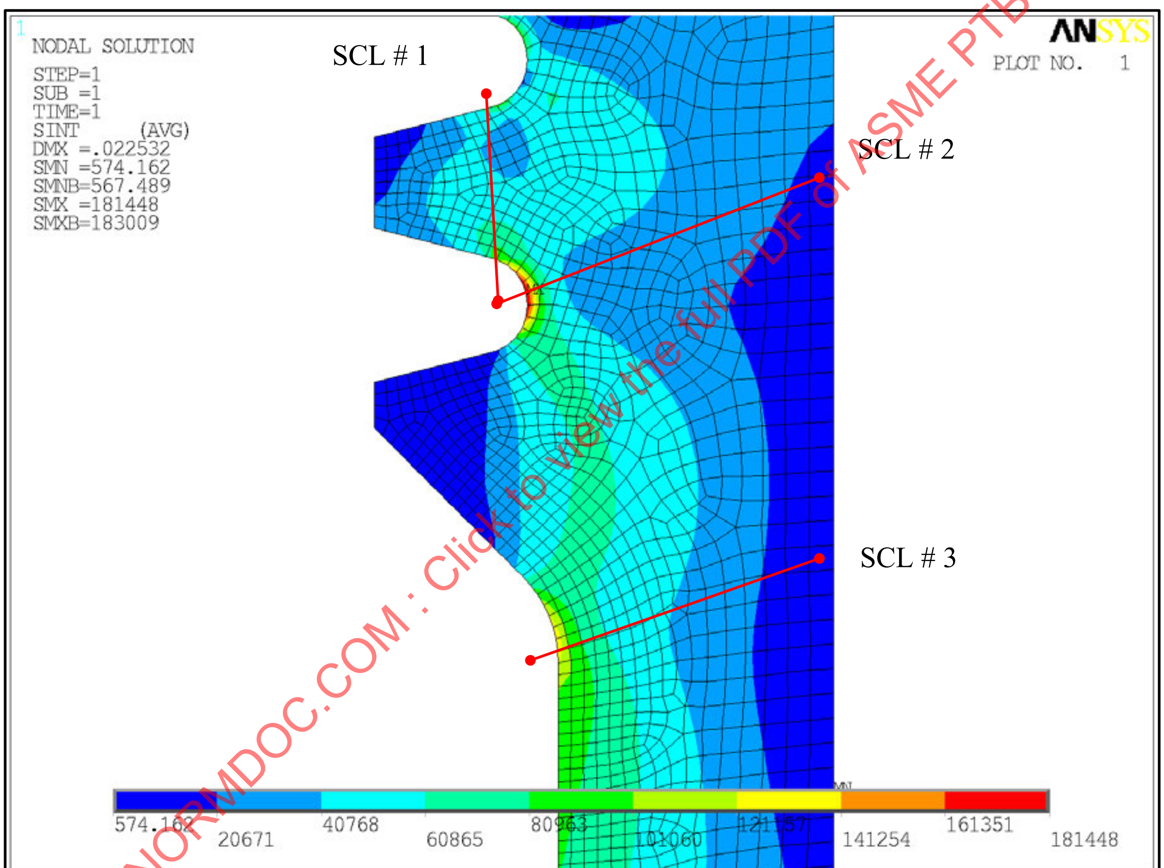

Figure 29 – E-KD-2.3.1-6 –Stress Classification Lines (SCLs) in the First Thread and Undercut Regions – Stress Intensity (psi)

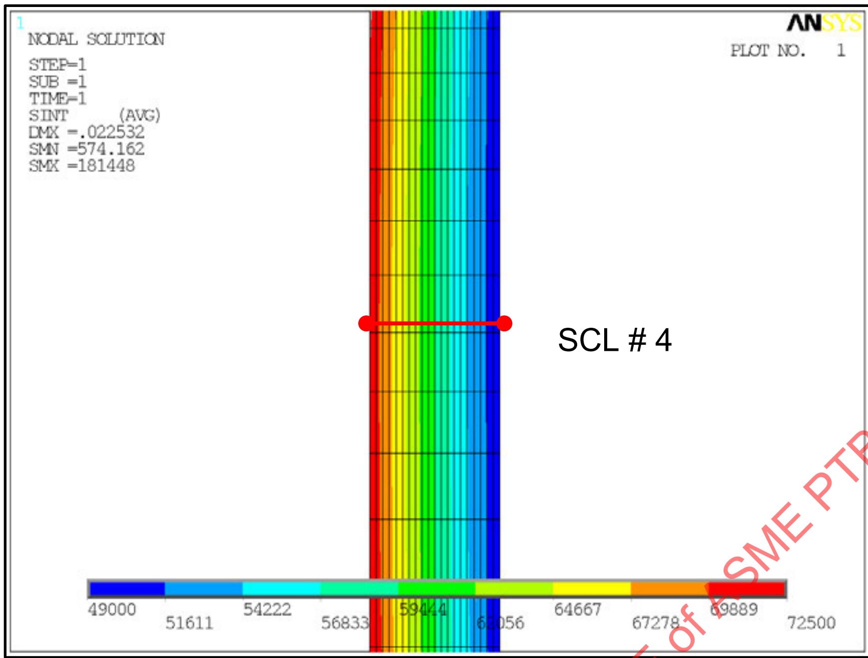

Figure 30 – E-KD-2.3.1-7 –Stress Classification Lines (SCLs) in the Body Shell Region Away from Discontinuities – Stress Intensity (psi)

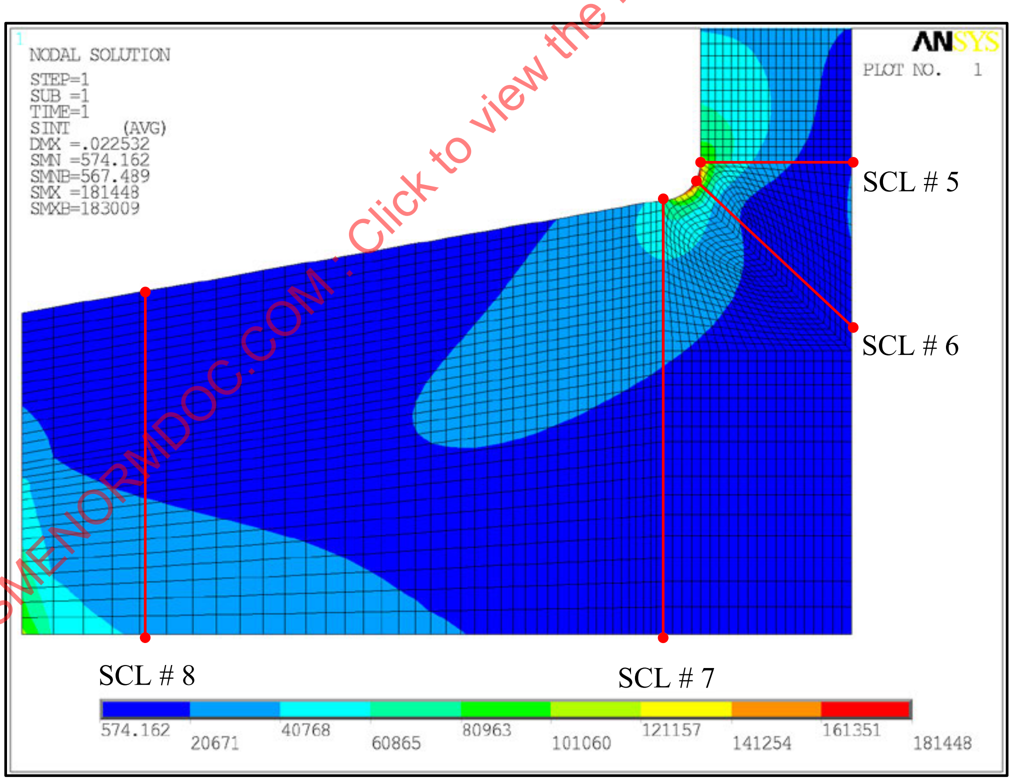

Figure 31 – E-KD-2.3.1-8 –Stress Classification Lines (SCLs) in the Blind End Region – Stress Intensity (psi)

STEP 6 – Sum the stress tensors (stresses are added on a component basis) assigned to each stress intensity category. The final result is a stress tensor representing the effects of all the loads assigned to each stress intensity category. A detailed stress analysis performed using a numerical method such as finite element analysis typically provides a combination of $P_L + P_b$ and $P_L + P_b + Q + F$ directly.

1) If a load case is analyzed that includes only “load-controlled” loads (e.g. pressure and weight effects), the computed stress intensity shall be used to directly represent the P_m, $P_L + P_b$, and $P_L + P_b + Q$. For example, for a vessel subjected to internal pressure with an elliptical head; P_m stress intensity occurs away from the head to shell junction, P_L and $P_L + P_b + Q$ stress intensities occur at the junction.
2) If a load case is analyzed that includes only “strain-controlled” loads (e.g. thermal gradients), the computed stress intensity represents Q alone; the combination $P_L + P_b + Q$ shall be derived from load cases developed from both “load-controlled” and “strain-controlled” loads.
3) If the stress in category F is produced by a stress concentration, the quantity F is the additional stress produced by the stress concentration in excess of the nominal membrane plus bending stress. For example, if a plate has a nominal stress intensity of S_{int}, and has a stress concentration factor K, then: $P_m = S_{int}$, $P_b = 0$, $Q = 0$, and $F = P_m (K - 1)$. The total stress intensity equals $P_m + P_m (K - 1)$.

STEP 7 – Determine the principal stresses of the sum of the stress tensors assigned to the stress intensity categories and compute the stress intensity using Appendix 9, equations 9-200.1 through 9-200.3.

STEP 8 – To evaluate protection against plastic collapse compare the computed stress intensities to their corresponding allowable values (refer to paragraph KD-242). Refer to Tables E-KD-2.3.1-1 and E-KD-2.3.1-2 below for evaluation results.

$$P_m \le S_y/1.5$$

$$P_L \le S_y$$

$P_L + P_b \le \alpha S_y/1.5$, where α is the shape factor equal to 1.5 (refer to KD-210 (o))

STEP 9 – To evaluate shakedown/ratcheting, compare the computed equivalent stresses to their corresponding allowable values (refer to paragraphs 9-210 through 9-250). Refer to Tables E-KD-2.3.1-1 and E-KD-2.3.1-2 below for evaluation results.

$$P_L + P_b + Q \le 2S_y$$

Table 5 – E-KD-2.3.1-1 – Results of the Elastic Analysis Using Criterion from Figure KD-240 of the 2019 Edition of BPVC Section VIII, Division 3, KD-240 – Design Pressure

SCL No.	Location Note (1)	Linearized Stress Intensities					Stress Evaluation			
		P_m	P_L	P_b	Q	F	$P_m \le S_y/1.5$ (78,000 psi)	$P_L \le S_y$ (117,000 psi)	$P_L + P_b \le \alpha S_y/1.5$ (117,000 psi) Note (3)	$P_L + P_b + Q \le 2S_y$ (234,000 psi)
1	First Thread	N/A	63,190	49,360	N/A	75,930	N/A	PASS	PASS	N/A
2	First thread notch section	N/A	31,650	27,070	N/A	129,600	N/A	PASS	PASS	N/A
3	Tapered thread to undercut transition	N/A	38,820	44,100	N/A	57,440	N/A	PASS	PASS	N/A
4	Body shell (away from discontinuities)	59,910	N/A	N/A	N/A	1079	PASS	N/A	N/A	N/A
5	Body shell to blind end transition	N/A	32,460	N/A	53510	66,800	N/A	PASS	PASS	PASS
6	Through blind end radius	N/A	16150	N/A	44220	120,900	N/A	PASS	PASS	PASS
7	Blind end to body shell transition	18270	N/A	23730	N/A	61,650	PASS	N/A	PASS	PASS
8	Blind end close to the opening	16780	N/A	20730	N/A	7422	PASS	N/A	PASS	PASS

Notes:
1) The linearized stress intensities are determined at operating conditions by scaling the linearized stresses computed at design temperature in the above table with a multiplication factor of 0.9.
2) The material at all the locations is SA-723 Gr.2 CL.2 and yield strength at 150°F is 117,000 psi, BPVC Section II, Part D, 2019.
3) α is the shape factor equal to 1.5 (refer to KD-210 (o)).

3.8 Example Problem E-KD-2.3.2 – Elastic Stress Analysis Protection Against Local Failure Mandatory Appendix 9-280

Evaluate the triaxial stress criteria of Mandatory Appendix 9-280 for protection against local failure for the pressure vessel in problem E-KD-2.3.1. The procedure for this is to determine the algebraic sum of the three principal stresses for the eight paths given in Figures E-KD-2.3.1-6 through E-KD-2.3.1-8 and compare to the triaxial stress criteria given in Mandatory Appendix 9-280 ($\sigma_1 + \sigma_2 + \sigma_3 \leq 2.5S_y$).

The sum of the principal stresses was evaluated along each of the eight critical stress classification lines from problem E-KD-2.3.1. The peak value of the sum of the principal stresses along the SCL's are reported here.

Table 6 – E-KD-2.3.1-2 – Mandatory Appendix 9-280 Triaxial Stress Criteria

Path Numbers	σ_1	σ_2	σ_3	Summation of the Principal Stresses	Criteria Evaluation
1	184,000	53,100	2,444	239,544	PASS
2	184,000	53,100	2,444	239,544	PASS
3	137,000	38,500	521	176,021	PASS
4	61,000	25,400	-11,000	75,400	PASS
5	118,000	35,300	-11,000	142,300	PASS
6	171,000	49,000	-11,000	209,000	PASS
7	47,500	6907	-11,000	43,407	PASS
8	39,100	22,500	2.869	61,603	PASS

Notes:
1) The material at all the locations is SA-723 Gr.2 CL.2 and Yield strength Sy at 150°F is 117,000 psi, BPVC Section II, Part D, 2019.
2) σ_1, σ_2, σ_3 are the three principal stresses.
3) All stresses are shown in psi.
4) 2.5 Sy = 292,500 psi

PART 4

Example Problems: Fatigue Assessment

4 EXAMPLE PROBLEMS FATIGUE ASSESSMENT

4.1 Example Problem E-KD-3.1.1 – Evaluation of Leak-Before-Burst in Cylindrical Vessel – Monobloc Vessel

Determine if the mode of failure for a crack in the wall of a pressure vessel is "Leak-Before-Burst" for the case of the open-ended pressure vessel found in problem E-KD-2.1.1. This evaluation is to be in accordance with paragraph KD-141(a) criteria. This evaluation is necessary for determination if KD-3 fatigue assessment or KD-4 fracture mechanics assessment is to be used for the failure of the vessel wall due to a longitudinal crack.

This problem assumes that this is a design without documented experience within industry.

The failure mode to be analyzed is a semi-elliptical surface breaking flaw in the ID of the wall in the radial-axial plane.

Vessel Data:

- Material = SA-705 Gr. XM-12 Condition H1100
- Design Temperature = 70°F
- Critical Stress Intensity Factor (K_{Ic}) = 104 ksi-in$^{0.5}$

  Based on minimum fracture toughness and specification minimum yield strength (refer to methodology in problem E-KM-2.1.2)
- Inside Diameter (D_i) = 6.0 inches
- Outside Diameter (D_o) = 12.0 inches
- Diameter Ratio (Y) = 2.0 (KD-221)
- Design Pressure (P_D) = 56,079 psi (problem E-KD-2.1.1)
- Yield Strength (S_y) = 115,000 psi @ 70°F per Table Y-1 of BPVC Section II, Part D
- Tensile Strength (S_u) = 140,000 psi @ 70°F per Table U of BPVC Section II, Part D
- Assumed Crack Aspect Ratio ($2c/a$) = 3:1 (KD-411)

The stress in the wall of this pressure vessel is a combination of the pressure stress and the residual stresses induced during autofrettage. The residual stresses were calculated in E-KD-5.1.1. The pressure stress distribution was also calculated here using the methods of BPVC Section VIII Division 3 Appendix 9. The principle of superposition was used to combine the two for the total stress at design conditions. Figure E-KD-3.1.1-1 shows a plot of these stresses at the design condition.

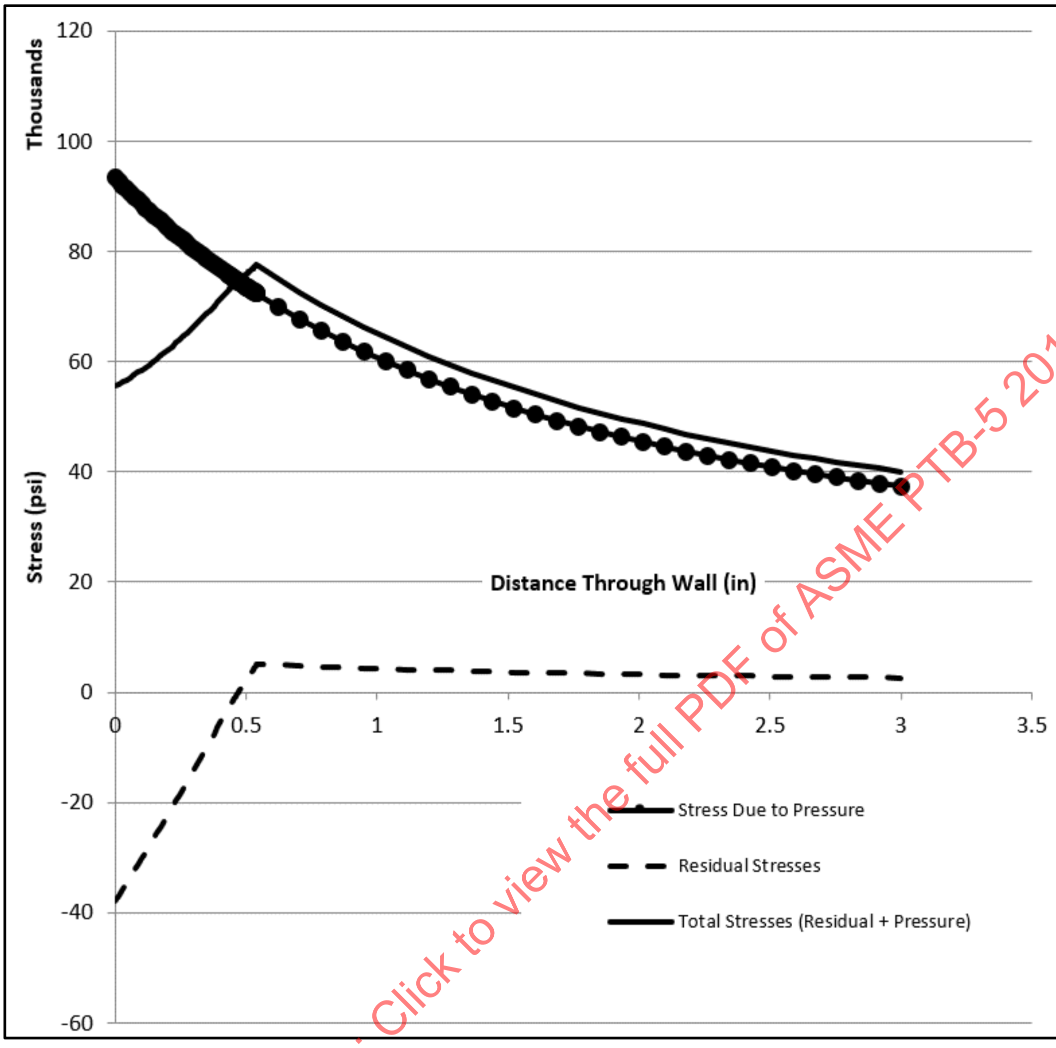

Figure 32 – E-KD-3.1.1-1 – Stress Distribution in Vessel Wall

STEP 1 – Determine if the stress intensity factor for a crack at 80% of the wall thickness will result in brittle failure

Many of the available methods for calculating stress intensity factors are not accurate beyond 80% of the wall.

$$a = 0.8 * \frac{D_O - D_I}{2} = 0.8 * \frac{12in - 6\ in}{2} = 2.4\ in$$

$$\frac{2c}{a} = 3 \xrightarrow[therefore]{} c = \frac{3 * 2.4\ in}{2} = 3.6\ in$$

The stress intensity factor at this depth must be less than K_{Ic}.

The stress intensity factor is to be calculated in accordance with the methods found in API 579-1 / ASME FFS-1 per KD-420(a). The stress intensity factor solutions are found in Annex 9B. Specifically, 9B.5.10 has a solution for "Cylinder – Surface Crack, Longitudinal Direction – Semi-Elliptical Shape, Internal Pressure (KCSCLE1)". Figure E-KD-3.1.1-2 shows the crack being analyzed.

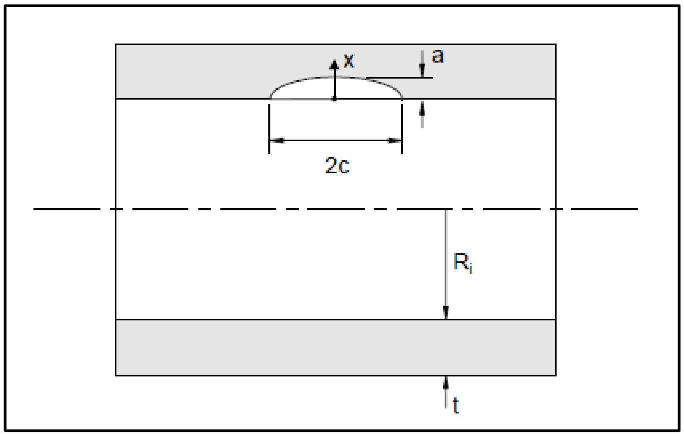

Figure 33 – E-KD-3.1.1-2 – Cylinder – Surface Crack, Longitudinal Direction Semi-Elliptical Shape (API 579-1 / ASME FFS-1 Figure 9B.15)

Paragraph 9B.5.10.1 is for a Mode I Stress Intensity Factor for an inside surface , including pressure in the crack face. Equation C.186 gives:

$$K_I = \frac{P * R_O^2}{R_O^2 - R_I^2} * \left(2G_0 - 2G_1\left(\frac{a}{R_i}\right) + 3G_2\left(\frac{a}{R_i}\right)^2 - 4G_3\left(\frac{a}{R_i}\right)^3 + 5G_4\left(\frac{a}{R_i}\right)^4\right) * \sqrt{\frac{\pi a}{Q}}$$

Constants needed to determine various coefficients are given by:

$$R_o = 0.5 * D_o = 0.5 * 12in = 6in$$

$$R_i = 0.5 * D_i = 0.5 * 6in = 3in$$

$$t = 0.5 * (D_o - D_i) = 0.5 * (12in - 6in) = 3in$$

$$\frac{a}{c} = \frac{2.4in}{3.6in} = 0.667$$

$$\frac{t}{R_i} = \frac{3in}{3in} = 1$$

$$\frac{a}{t} = \frac{2.4in}{3in} = 0.8$$

Where the influence coefficients, G_0 and G_1 are given by:

$$G_0 = A_{0,0} + A_{1,0}\beta + A_{2,0}\beta^2 + A_{3,0}\beta^3 + A_{4,0}\beta^4 + A_{5,0}\beta^5 + A_{6,0}\beta^6$$

$$G_0 = 1.187 - 1.493 * 1 + 6.646 * 1^2 - 13.038 * 1^3 + 15.466 * 1^4 - 9.918 * 1^5 + 2.506 * 1^6 = 1.356$$

$$G_1 = A_{0,1} + A_{1,1}\beta + A_{2,1}\beta^2 + A_{3,1}\beta^3 + A_{4,1}\beta^4 + A_{5,1}\beta^5 + A_{6,1}\beta^6$$

$$G_1 = 0.232 + 0.089 * 1 + 2.428 * 1^2 - 4.133 * 1^3 + 4.457 * 1^4 - 2.984 * 1^5 + 0.756 * 1^6 = 0.845$$

Where Table 9B.12 provides the $A_{i,j}$ coefficients and equation 9B.95 is used for the value of β as:

$$\beta = \frac{2 * \phi}{\pi} = \frac{2 * \frac{\pi}{2}}{\pi} = 1$$

Note ϕ is the elliptical angle where (from Figure 9B.2(a)):

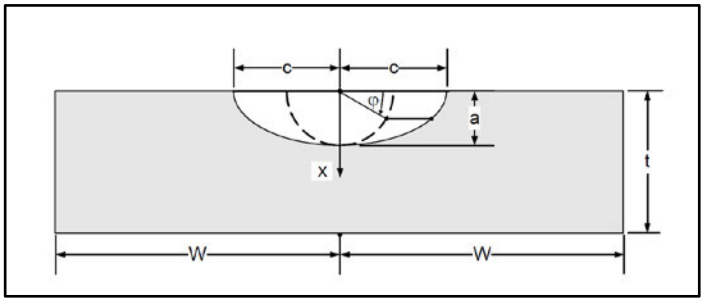

Figure 34 – E-KD-3.1.1-3 – Definition of ϕ from API 579/ASME FFS-1

It should be noted that the $A_{i,j}$ values can require up to triple linear interpolation from Table 9B.12. For this problem, linear interpolation was only required for the a/c column. An example of this interpolation is shown for $A_{0,0}$ below (the intermediate values have been truncated for illustration purposes only):

$$A_{0,0} = 1.142 + (0.667 - 0.5) * \frac{1.277 - 1.142}{(1 - 0.5)} = 1.187$$

Influence coefficients G_2, G_3, and G_4 are then determined by the methods found in paragraph 9B.14.3 or 9B.14.4, typically using the weight function approach. The value of Q is determined with equation 9B.14:

$$Q = 1.0 + 1.464 * \left(\frac{a}{c}\right)^{1.65} = 1.0 + 1.464 * (0.667)^{1.65} = 1.750$$

The influence coefficients also require the calculation of additional coefficients, M_1, M_2, and M_3, which are Equations 9B.266 through 9B.268 and calculated as follows:

$$M_1 = \frac{2 * \pi}{\sqrt{2 * Q}} * (3 * G_1 - G_0) - \frac{24}{5} = \frac{2 * \pi}{\sqrt{2 * 1.750}} * (3 * 1.356 - 0.845) - \frac{24}{5} = -0.841$$

$$M_2 = 3$$

$$M_3 = \frac{6 * \pi}{\sqrt{2 * Q}} * (G_0 - 2 * G_1) + \frac{8}{5} = -1.765$$

Therefore:

$$G_2 = \frac{\sqrt{2 * Q}}{\pi} * \left(\frac{16}{15} + \frac{1}{3} * M_1 + \frac{16}{105} * M_2 + \frac{1}{12} * M_3\right)$$

$$G_2 = \frac{\sqrt{2 * 1.750}}{\pi} * \left(\frac{16}{15} + \frac{1}{3} * (-0.841) + \frac{16}{105} * 3 + \frac{1}{12} * (-1.765)\right) = 0.653$$

$$G_3 = \frac{\sqrt{2 * Q}}{\pi} * \left(\frac{32}{35} + \frac{1}{4} * M_1 + \frac{32}{315} * M_2 + \frac{1}{20} * M_3\right)$$

$$G_3 = \frac{\sqrt{2*1.750}}{\pi}*\left(\frac{32}{35}+\frac{1}{4}*(-0.841)+\frac{32}{315}*3+\frac{1}{20}*(-1.765)\right) = 0.548$$

$$G_4 = \frac{\sqrt{2*Q}}{\pi}*\left(\frac{256}{315}+\frac{1}{5}*M_1+\frac{256}{3465}*M_2+\frac{1}{30}*M_3\right)$$

$$G_4 = \frac{\sqrt{2*1.750}}{\pi}*\left(\frac{256}{315}+\frac{1}{5}*(-0.841)+\frac{256}{3465}*3+\frac{1}{30}*(-1.765)\right) = 0.481$$

Using this methodology, the stress intensity factor for a crack with a depth of 2.4 inches is as follows.

$$K_I = \frac{56{,}079psi*(3in)^2}{(6in)^2-(3in)^2}*\left(2*1.356-2*0.845*\left(\frac{2.4in}{3in}\right)+3*0.653*\left(\frac{2.4in}{3in}\right)^2-4*0.548*\left(\frac{2.4in}{3in}\right)^3+5*0.481*\left(\frac{2.4in}{3in}\right)^4\right)$$
$$*\sqrt{\frac{\pi*2.4in}{1.750}} = 384ksi\sqrt{in}$$

Therefore, the criterion is not satisfied. It should be noted that the stress intensity calculated above is solely based off the internal pressure and conservatively does not account for the residual stresses shown in Figure 32. If the residual stresses are desired to be included, the arbitrary stress distribution methodology of section 9B.5.9 may be used.

STEP 2 – Evaluate if the remaining ligament (distance from the crack tip at the deepest point to the free surface)

The limiting distance is (KD-141(a)(2):

$$\left(\frac{K_{Ic}}{S_y}\right)^2 = \left(\frac{104ksi\sqrt{in}}{115ksi}\right)^2 = 0.818\ inches$$

The remaining distance is:

$$t-a = 3\ inches - 2.4\ inches = 0.6in < 0.818\ inches$$

Therefore, this criterion is satisfied.

The requirement is for both criteria to be satisfied. In this case, the first criterion is not satisfied, but the second one is satisfied; therefore, the vessel is not Leak-Before-Burst.

4.2 Example Problem E-KD-3.1.2 – Evaluation of Leak-Before-Burst in Cylindrical Vessel – Dual Layered Vessel

Determine if the mode of failure for a crack in the wall of a pressure vessel is “Leak-Before-Burst” for the case of the open-end pressure vessel found in problem E-KD-2.1.2. This evaluation is to be in accordance with paragraph KD-141(c) and KD-810(f) criteria. This evaluation is necessary to determine whether the KD-3 fatigue assessment or KD-4 fracture mechanics assessment is applicable.

This problem assumes:

The design is without documented experience within industry.

The closures will remain in place and not be ejected in the event of a failure.

The fast fracture of either of the inner layer will not result in ejection of parts or fragments and the outer layer will remain intact.

The vessel does not contain lethal substances.

The failure mode to be analyzed is a semi-elliptical surface connected flaw in the ID of the wall of each of the layers in the axial-radial plane.

Each of the materials meet the Charpy impact requirements from KM-234.2(a).

Vessel Data:

Liner

- Material = SA-705 Gr. XM-12 Condition H1100
- Design Temperature = 70°F
- Critical Stress Intensity Factor (K_{Ic}) = 104 ksi-in$^{0.5}$

  Based on minimum fracture toughness and specification minimum yield strength (refer to methodology in problem E-KM-2.1.3)
- Wall Ratio (Y) = 1.50

Outer Body

- Material = SA-723 Gr. 2 Class 2
- Design Temperature = 70°F
- Critical Stress Intensity Factor (K_{Ic}) = 104 ksi-in$^{0.5}$ (problem E-KM-2.1.3)
- Design Pressure for Dual Wall ($P_{D.Dual}$) = 97,029 psi (problem E-KD-2.1.2)

The vessel can be assumed to be Leak-Before-Burst if the inner layer fails in fast fracture, the outer body can hold 120% of the design pressure without resulting in collapse.

The collapse pressure of a cylindrical shell can be determined by multiplying the design pressure from KD-221.1 with the design margin of 1.732. The design pressure of the outer shell is:

$$P_D = min\left(2.986 * K_{ut} * S_y * (Y^{0.268} - 1), 1.0773 * \left(S_y + S_u\right) * (Y^{0.268} - 1)\right) \qquad KD - 221.1$$

$$P_D = min\big(2.986 * 0.871 * 120{,}000psi * (2.088^{0.268} - 1), 1.0773 * (120{,}000psi + 135{,}000psi) * (2.088^{0.268} - 1)\big) = min(68{,}030psi, 59{,}905psi) = 59{,}905psi$$

Where:

$$K_{ut} = 1.244 - 0.42 * \left(\frac{120{,}000psi}{135{,}000psi}\right) = 0.871$$

$$Y = \frac{D_o}{D_i} = \frac{50in}{23.95in} = 2.088$$

And the collapse pressure of the outer shell is therefore:

$$1.732 * P_D = 1.732 * 59{,}905psi = 103{,}755\ psi$$

And 120% of the design pressure of the dual walled vessel is:

$$120\% * P_{D.Dual} = 1.20 * 97{,}029psi = 116{,}435psi$$

Therefore, this cylinder is not considered to be Leak-Before-Burst.

4.3 Example Problem E-KD-3.1.3 – Fatigue Assessment of Welds – Elastic Analysis and Structural Stress

Evaluate an open-ended vessel with the same dimensions as that given in Example E-KD-2.1.1 in accordance with the fatigue methodology provided in KD-340. For this problem, the material of the vessel is SA-182 Grade F22. Design pressure and resulting stresses were calculated using the same methodology as in the previous problems with no autofrettage. Design requirements include only the pressure loading at an operating pressure of 24,500 psi for 10,000 cycles. Note that the vessel is in non-corrosive service with respect to environmental effects upon the fatigue behavior. Perform fatigue assessment for a theoretical radial-axial crack along the heat affected zone of a longitudinal seam weld in the vessel.

STEP 1 – Determine a load history for the vessel.

Per the User's Design Specification as described above, a full internal pressure cycle is the only loading event to be considered. The internal pressure is expected to cycle 10,000 times between 0 psi and the operating pressure of 24,500 psi.

STEP 2 – Determine the individual stress-strain cycles.

Since the full pressure cycle is the only event under consideration, the cyclic stress range is between the stress in the vessel at 0 and at 24,500 psi.

STEP 3 – Determine the elastically calculated membrane and bending stress normal to the assumed hypothetical crack plane at the start and end points (${}^{m}t$ and ${}^{n}t$, respectively) for the cycle in Step 2. Using this data, calculate the membrane and bending stress ranges and the maximum, minimum and mean stress. Note that for this problem, there is only one load range, so "k" equals one.

The membrane stresses are:

$$^{m}\sigma^{e}_{m,k} = 24.543ksi$$

$$^{n}\sigma^{e}_{m,k} = 0ksi$$

The bending stresses are:

$$^{m}\sigma^{e}_{b,k} = 11.177ksi$$

$$^{n}\sigma^{e}_{b,k} = 0ksi$$

Assume the end point of the cycle (${}^{n}t$) is in the shutdown condition where internal pressure is at 0. Stress distributions were calculated for the radial and tangential (hoop) components due to the internal pressure on the vessel. Note, since this is an open-ended vessel, there is no axial stress component due to internal pressure. The longitudinal seam weld crack to be considered will be assumed to be radial and axial in orientation, meaning the stress component normal to the hypothetical crack plane is the hoop stress.

The equations for membrane, bending, maximum, minimum and mean stress are evaluated as follows using the through thickness hoop stress distribution from similar to that found in Example Problem E-KD-2.1.1, except at 24,500 psi (refer to Figure E-KD-3.1.3-1):

$$\Delta\sigma^{e}_{m,k} = {}^{m}\sigma^{e}_{m,k} - {}^{n}\sigma^{e}_{m,k} = 24.543ksi - 0ksi = 24.543ksi \qquad KD-341.1$$

$$\Delta\sigma^{e}_{b,k} = {}^{m}\sigma^{e}_{b,k} - {}^{n}\sigma^{e}_{b,k} = 11.177ksi - 0ksi = 11.177ksi \qquad KD-341.2$$

$$\sigma_{max,k} = max\left(\left({}^{m}\sigma^{e}_{m,k} + {}^{m}\sigma^{e}_{b,k}\right), \left({}^{n}\sigma^{e}_{m,k} + {}^{n}\sigma^{e}_{b,k}\right)\right) \qquad KD-341.3$$

$$\sigma_{max,k} = max\big((24.543\ ksi + 11.177\ ksi), (0ksi + 0ksi)\big) = 35.720ksi$$

$$\sigma_{min,k} = min\left(\left({}^{m}\sigma^{e}_{m,k} + {}^{m}\sigma^{e}_{b,k}\right), \left({}^{n}\sigma^{e}_{m,k} + {}^{n}\sigma^{e}_{b,k}\right)\right) \qquad KD-341.4$$

$$\sigma_{min} = min\big((24.543\ ksi + 11.177\ ksi), (0ksi + 0ksi)\big) = 0ksi$$

$$\sigma_{mean,k} = \frac{\sigma_{max,k} + \sigma_{min,k}}{2} = \frac{35.720ksi + 0ksi}{2} = 17.860ksi \qquad KD-341.5$$

STEP 4 – Determine the elastically calculated structural stress range.

$$\Delta\sigma^{e}_{k} = \Delta\sigma^{e}_{m,k} + \Delta\sigma^{e}_{b,k} = 24.543ksi + 11.177ksi = 35.720ksi \qquad KD-341.6$$

STEP 5 – Determine the elastically calculated structural strain and the elastically calculated structural stress obtained in Step 4.

$$\Delta\varepsilon^{e}_{k} = \frac{\Delta\sigma^{e}_{k}}{E_{ya,k}} = \frac{35.720ksi}{30{,}600ksi} = 1.167 * 10^{-3} \qquad KD-341.7$$

Where E_{ya} = 30.6 x 10^{6} psi (modulus of elasticity for 2¼Cr - 1 Mo material at 70°F) from BPVC Section II, Part D

The corresponding local nonlinear structural stress and strain ranges are determined by simultaneously solving Neuber's Rule (equation KD-341.8) and the material hysteresis loop stress-strain curve model (equation KD-341.9).

$$\Delta\sigma_{k} * \Delta\varepsilon_{k} = \Delta\sigma^{e}_{k} * \Delta\varepsilon^{e}_{k} \qquad KD-341.8$$

$$\Delta\sigma_{k} = \frac{35.720ksi * 1.167 * 10^{-3}}{1.167 * 10^{-3}} = 35.720ksi$$

$$\Delta\varepsilon_{k} = \frac{\Delta\sigma_{k}}{E_{ya,k}} + 2 * \left(\frac{\Delta\sigma_{k}}{2 * K_{css}}\right)^{\frac{1}{n_{css}}} \qquad KD-341.9$$

$$\Delta\varepsilon_{k} = \frac{35.720ksi}{30{,}600ksi} + 2 * \left(\frac{35.720ksi}{2 * 115.5ksi}\right)^{\frac{1}{0.100}} = 1.167 * 10^{-3}$$

The two unknowns, $\Delta\sigma$ and $\Delta\varepsilon$, are found iteratively using the above equations.

The values for the coefficients K_{css} = 115.5 ksi and n_{css} = 0.100 are obtained from Table KM-630 at 70°F for 2¼ Cr material.

Next, the nonlinear structural stress range is to be modified for low-cycle fatigue, as the transition between low and high cycle fatigue is not known. Equation KD-341.10 performs this modification.

$$\Delta\sigma_{k} = \left(\frac{E_{ya,k}}{1-\nu^{2}}\right) * \Delta\varepsilon_{k} = \left(\frac{30{,}600ksi}{1-0.3^{2}}\right) * 1.167 * 10^{-3} = 39.253ksi \qquad KD-341.10$$

STEP 6 – Compute the equivalent structural stress range parameter.

$$\Delta S_{ess,k} = \frac{\Delta\sigma_{k}}{t_{ess}^{\frac{2-m_{ss}}{2m_{ss}}} * I^{\frac{1}{m_{ss}}} * f_{M,k}} \qquad KD-341.11$$

$$\Delta S_{ess} = \frac{39.253ksi}{(3in)^{\frac{2-3.6}{2*3.6}} * 2.107^{\frac{1}{3.6}} * 1.0} = 40.735 \frac{ksi}{in^{\frac{2-m_{ss}}{2m_{ss}}}}$$

Where:

$$m_{ss} = 3.6 \qquad KD-341.12$$

$$t_{ess} = t\ for\ 0.625in < t < 6in = 3in \qquad KD-341.14$$

$$I^{\frac{1}{m_{ss}}} = \frac{1.23 - 0.364 * R_{b,k} - 0.17 * R_{b,k}^2}{1.007 - 0.306 * R_{b,k} - 0.178 * R_{b,k}^2} \qquad KD-341.16$$

Solving for I yields:

$$I = \left(\frac{1.23 - 0.364 * R_{b,k} - 0.17 * R_{b,k}^2}{1.007 - 0.306 * R_{b,k} - 0.178 * R_{b,k}^2}\right)^{m_{ss}}$$

$$I = \left(\frac{1.23 - 0.364 * 0.313 - 0.17 * 0.313^2}{1.007 - 0.306 * 0.313 - 0.178 * 0.313^2}\right)^{3.6} = 2.107$$

Where:

$$R_{b,k} = \frac{|\Delta\sigma_{b,k}^e|}{|\Delta\sigma_{m,k}^e| + |\Delta\sigma_{b,k}^e|} = \frac{11.177ksi}{24.543ksi + 11.177ksi} = 0.313 \qquad KD-341.17$$

$$f_{M,k} = 1.0\ for\ \sigma_{meann,k} \geq 0.5\ S_{y,k}, and\ R_k > 0, and\ |\Delta\sigma_{m,k}^e + \Delta\sigma_{b,k}^e| \leq 2S_{y,k} \qquad KD-341.19$$

$$f_{M,k} = 1.0\ for\ 17.860\ ksi \geq 0.5\ (115\ ksi), and\ R_k > 0, and\ |24.543\ ksi + 11.177\ ksi| \leq 2(115\ ksi)$$

$$R_k = \frac{\sigma_{min,k}}{\sigma_{max,k}} = \frac{0ksi}{40.833ksi} = 0 \qquad KD-341.20$$

STEP 7 – Determine the permissible number of cycles per the fatigue curves in KD-370 based on the equivalent structural stress range parameter as computed in Step 6.

The number of allowable design cycles, *N*, can be computed using equation KD-3.50 using constants provided in Table KD-370.1 for the lower 99% Prediction Interval (-3σ). These constants are to be used unless otherwise agreed upon by the Owner-User and the Manufacturer.

$$N = \frac{f_I}{f_E} * \left(\frac{f_{MT} * C}{\Delta S_{ess}}\right)^{\frac{1}{h}} = \frac{1}{1} * \left(\frac{1.041 * 818.3}{40.735}\right)^{\frac{1}{0.31950}} = 13{,}566 \qquad KD-372.1$$

Where:

$f_I = 1$	No fatigue improvement performed
$f_E = 1$	Non-corrosive service
E_T = 30.6 x 103 ksi	Elastic Modulus for Grade 22 at 70°F
E_{ACS} = 29.4 x 103 ksi	Elastic Modulus for Carbon Steel at 70°F
$f_{MT} = \frac{E_T}{E_{ACS}} = 1.041$	Temp./Material adjustment for fatigue curves (Eq. KD-372.6)
C = 818.3	Welded Joint Fatigue Curve coefficients for Lower 99%

$h = 0.31950$ Prediction Interval per Table KD-370.1

STEP 8 – Determine the fatigue damage for the cycle history.

Per the User Design Specification above, 10,000 full pressure cycles are required (n = 10,000). Using equation KD-341.21 and the results of Step 7, the fatigue damage fraction can be calculated.

$$D_f = \frac{n}{N} = \frac{10,000}{13,566} = 0.737 \qquad KD-341.21$$

STEPS 9-11 – Assessment of Steps 9-11 are not required for this vessel as the only stress range considered in this design is the 0 to 24,500 psi internal pressure operational cycle (i.e. k = 1).

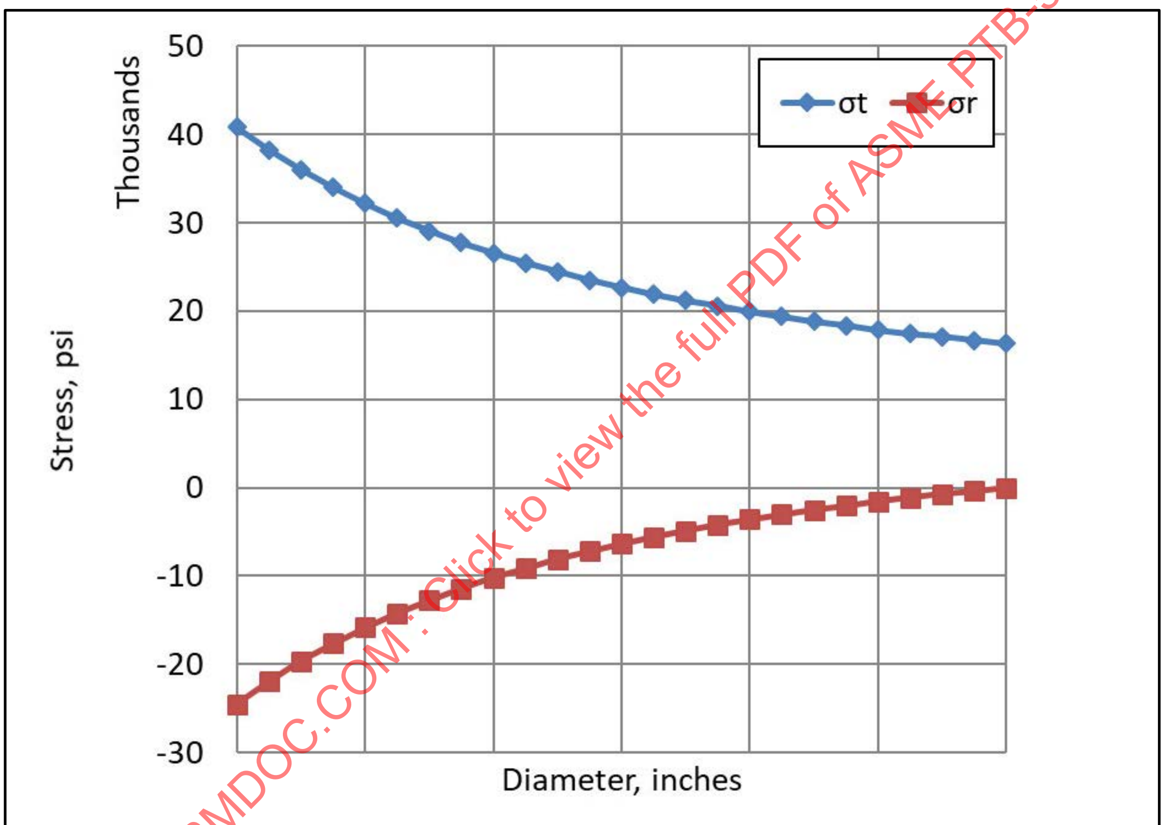

Figure 35 – E-KD-3.1.3-1 – Stress Distribution in Monoblock Open End Shell (E-KD-2.1.1) Evaluated at 24,500 psi

4.4 Example Problem E-KD-3.1.4 – Non-Welded Vessel using Design Fatigue Curves

Evaluate an open ended monobloc vessel described in Example E-KD-2.1.1 in accordance with the fatigue methodology provided in KD-340. Design pressure and residual stresses were calculated using the methodology found in problems E-KD-2.1.1 and E-KD-3.1.3. Design requirements include only the cyclic pressure loading from zero to an operating pressure of 30,000 psi for 3,500 cycles, as listed in its User's Design Specification. Note that the vessel is in non-corrosive service with respect to environmental effects upon the fatigue behavior. The objective of this problem is to perform fatigue of the vessel bore and determine the cumulative effect of the number of design cycles on the vessel results in a number of design cycles in excess of this number.

It should be noted that based on past operational experience, this vessel is considered Leak-Before-Break in accordance with KD-141(d). The User's Design report includes documentation for vessels of similar size that all resulted in Leak-Before-Break Mode of Failure.

The surface roughness of the bore of the vessel is noted as 125 R_a on the drawing. The modulus of elasticity used in the analysis of the vessel is 28.5 x 10^6 psi from Table TM-1 from BPVC Section II, Part D.

STEP 1 – Determine a load history for the vessel and the associated stresses at each of the load cycles specified.

Per the User's Design Specification as described above, a full internal pressure cycle is the only loading event to be considered. The internal pressure is expected to cycle 3,500 times between 0 and the operating pressure of 24,500 psi. E-KD-3.1.4-1 is a compilation of these principal stresses.

Table 7 – E-KD-3.1.4-1 – Principal Stresses in Cylinder

	Operating Pressure	**Zero Pressure**
First Principal Stress (s_1)	50,000 psi	0 psi
Second Principal Stress (s_2)	0 psi	0 psi
Third Principal Stress (s_3)	-30,000 psi	0 psi

STEP 2 – Determine the Stress Intensities (S_{ij}) – The second step is to determine the operational stress intensities for the complete operating cycle. In this point, there are only two points to be evaluated, at operating pressure and at zero pressure. Table E-KD-3.1.4-2 has the results of these calculations at the top of the table listed as S_{ij} for each of the differences (1-2, 2-3, and 3-1).

STEP 3 – Determine the Alternating Stress Intensities ($S_{alt\ i,j}$) – The next step is to evaluate the alternating stress intensities by the absolute value of the difference of maximum and minimum stress intensities throughout the complete operational cycle. These are listed in Table E-KD-3.1.4-2 as $S_{alt\ i,j}$.

STEP 4 – Determine the Associated Mean Stress ($\sigma_{n\ i,j}$)– The next step is to determine the associated mean stress normal to the plane of the maximum shear stress, associated with the three $S_{alt\ i,j}$. These are listed in Table E-KD-3.1.4-2 as $\sigma_{n\ i,j}$.

STEP 5 – Determine the Stress Normal to the Plane of Maximum Shear ($\sigma_{n\ i,j}$) – The next step is to determine the associated mean normal stress. It is noted here that the cylinder is a non-welded monobloc construction and not made of austenitic stainless steel. The values calculated are shown in Table E-KD-3.1.4-2 as $\sigma_{n\ i,j}$ for both the operating and zero pressure case

STEP 6 – Determine the Associated Mean Stress ($\sigma_{nm\ i,j}$) – The next step is to determine the mean normal stress, $\sigma_{nm\ i,j}$. This is determined by taking the average of the stresses normal to the plane of maximum shear calculated in Step 5 for the operating and zero pressure cases.

STEP 7 – Determine the Appropriate Fatigue Curve for Use and Surface Roughness Factor (K_r) – Figure KD-320.3 is the curve to be used for pressure equipment made of this material per KD-322(c). The influence of the surface roughness of this cylinder is taken into account by inclusion of the surface roughness factor, K_r, which is found using Figure KD-320.5(b). The equation for this is:

$$K_r = max\left(1, \frac{1}{-0.16998\,log(R_a) + 1.2166}\right) = max\left(1, \frac{1}{-0.16998 * log(125) + 1.2166}\right) = 1.163$$

Further, it should also be noted that the modulus of elasticity for Figure KD-320.4 is 29 x 10^6 psi and that the allowable amplitude of the alternating stress component ($S`_a$) when σ_{nm} equals 0 and $N = 10^6$ cycles is 42,800 psi.

STEP 8 – Determine the Equivalent Alternating Stress Intensity ($S_{eq\,i,j}$) – This cylinder being evaluated is for non-welded construction. Paragraph KD-312.4 states that for 17-4 or 15-5 stainless steel, the value of β shall be either 0.2 or 0.5 depending on the Associated Mean Stress ($\sigma_{nm\,i,j}$) determined in Step 6. These are listed in Table E-KD-3.1.4-2 along with a calculation for the denominator of equation KD-312.13 from paragraph KD-312.4. The limit of this factor is 0.9, but the table shows that this is below 0.9 for all cases considered. The value of Equivalent Alternating Stress for each of the directions considered are shown in the table. The fatigue life will be evaluated based on the maximum value calculated of 42,481 psi.

STEP 9 – Determine the Alternating Stress for Use in Evaluation of Design Life (S_a) – The alternating stress that will be used in conjunction with the curve is then found using equation KD-322.4:

$$S_a = K_f K_r K_e S_{eq} \frac{E(curve)}{E(analysis)} = 1.0 * 1.163 * 1.0 * 42{,}481 psi * \frac{29 * 10^6 psi}{28.5 * 10^6 psi} = 50{,}254\, psi$$

Where:

$$K_f = 1.0 \; \textit{(There are no local effects as this failure mode is for a straight inner bore)}$$

$$K_e = 1.0\; for\; \Delta S_n \le 2S_y \qquad KD-322.1$$

$$K_e = 1.0\; for\; 32{,}666\, psi \le 2(115\, ksi)$$

Table 8 – E-KD-3.1.4-2 – Calculated Stress Intensities and other Values for Fatigue

	1 - 2	2 – 3	3 – 1
Stress Intensities (psi)	$S_{ij} = \sigma_i - \sigma_j$		
Operating Pressure	50,000	30,000	-80,000
Zero Pressure	0	0	0
Alternating Stress Intensities (psi)	$S_{alt\,i,j} = \lvert 0.5(S_{ij\,max} - S_{ij\,min})\rvert$		
	25,000	15,000	40,000
Stress Normal to the Plane of Maximum Shear (psi)	$\sigma_{n\,i,j} = 0.5\,(\sigma_i + \sigma_j)$		
Operating Pressure	25,000	-15,000	10,000
Zero Pressure	0	0	0
Associated Mean Normal Stresses (psi)	$\sigma_{nm\,i,j} = 0.5\,(\sigma_{n\,i,j\,max} + \sigma_{n\,i,j\,min})$		
	12,500	-7,500	5,000
β	KD 312.4		
	0.5	0.2	0.5
Factor	$1 - \beta\sigma_{nm\,i,j}/S`_a$		
	0.854	1.035	0.942

Equivalent Alternating Stress Intensity (psi)	$S_{eq\,i,j} = S_{alt\,i,j}\dfrac{1}{1-\beta\sigma_{nm\,i,j}/S`_a}$		
	29,275	14,492	42,481

STEP 10 – Determine the Design Life of the Vessel (N_f) – The design life of the vessel is then determined by using Figure KD-320.4. There are three acceptable methods for determining the number of cycles including interpolation from tabular values listed in Table KD-320.1 for Figure KD-320.4, use of the equations below Table KD-320.1 for Figure KD-320.4 or graphically from Figure KD-320.4. The method of interpolation was used here. Table E-KD-3.1.4-3 contains the values from Table KD-320.1 used for interpolation.

The equation for interpolation of the fatigue curve from the notes to Table KD-320.1 is re-written here as:

$$N_f = N_{fi}\left(\frac{N_{fj}}{N_{fi}}\right)^{\frac{log\left(\frac{S_{ai}}{S_a}\right)}{log\left(\frac{S_{ai}}{S_{aj}}\right)}} \qquad Table\ KD-320.1$$

$$N_f = 1E5 * \left(\frac{2E5}{1E5}\right)^{\frac{log\left(\frac{51.6}{42.481}\right)}{log\left(\frac{51.6}{48.7}\right)}} = 137{,}271\,cycles$$

On that basis, the cylinder meets the requirements specified for a design life of 3,500 cycles.

Table 9 – E-KD-3.1.4-3 – Values for Interpolation from Table KD-320.1 for Figure KD-320.4

	i	**j**
S_a	51,600 psi	48,700 psi
N_f	100,000	200,000

4.5 Example Problem E-KD-3.1.5 – Autofrettaged, Non-Welded Vessel using Design Fatigue Curves

Evaluate an autofrettaged, open-ended, mono-wall vessel described in Example E-KD-2.1.1 in accordance with the fatigue methodology provided in KD-340. Design pressure and residual stresses were calculated using the methodology found in problems E-KD-2.1.1 and E-KD-5.1.1. Design requirements include only the cyclic pressure loading from zero to an operating pressure of 30,000 psi for 3,500 cycles, as listed in its User's Design Specification. Note that the vessel is in non-corrosive service with respect to environmental effects upon the fatigue behavior. The objective of this problem is to perform a fatigue evaluation of the vessel bore and determine if the number of design cycles results is greater than the number required by the UDS.

It should be noted that this is an example of the principles shown in KD-313 – Calculation of Fatigue Stresses when Principal Stress Axes Change. The axes in this problem are in the same general orientation (i.e. aligned in the hoop, radial and longitudinal directions) for each stress state. However, the direction of each principal stress changes when changing between the two stress states.

It should be noted that based on past operational experience, this vessel is considered Leak-Before-Break in accordance with KD-141(d). The User's Design report includes documentation for vessels of similar size that all resulted in Leak-Before-Break Mode of Failure.

The surface roughness of the bore of the vessel is noted as 125 R_a on the drawing. The modulus of elasticity used in the analysis of the vessel is 28.5 x 10^6 psi from Table TM-1 from Section II Part D.

STEP 1 – Determine a load history for the vessel and the associated stresses at each of the load cycles specified.

Per the User's Design Specification as described above, a full internal pressure cycle is the only loading event to be considered. The internal pressure is expected to cycle 3,500 times between 0 and the operating pressure of 24,500 psi. E-KD-3.1.4-1 is a compilation of these principal stresses.

Table 10 – E-KD-3.1.5-1 – Principal Stresses in Cylinder

	Operating Pressure	**Zero Pressure**
First Principal Stress (σ_1)	12,147 psi (hoop)	0 (radial)
Second Principal Stress (σ_2)	0 (longitudinal)	0 (longitudinal)
Third Principal Stress (σ_3)	-30,000 psi (radial)	-7853 psi (hoop)

STEP 2 – Determine the Stress Intensities (S_{ij}) – The second step is to determine the operational stress intensities for the complete operating cycle. In this point, there are only two load conditions to be evaluated, at operating pressure and at zero pressure. Table E-KD-3.1.4-2 has the results of these calculations at the top of the table listed as S_{ij} for each of the differences (1-2, 2-3, and 3-1).

It should be noted that in this case, the "zero pressure principle stresses" need to be re-ordered to establish them for determination of the planes of maximum shear stress.

Table 11 – E-KD-3.1.5-2 –Stresses in Cylinder (Ordered for Evaluation of Shear)

	Operating Pressure	**Zero Pressure**
First Principal Stress (σ_1)	12,147 psi (hoop)	-37,853 psi (hoop)
Second Principal Stress (σ_2)	0 (longitudinal)	0 (longitudinal)
Third Principal Stress (σ_3)	-30,000 psi (radial)	0 (radial)

STEP 3 – Determine the Alternating Stress Intensities ($S_{alt\ i,j}$) – The next step is to evaluate the alternating stress intensities by the absolute value of the difference of maximum and minimum stress intensities throughout the complete operational cycle. These are listed in Table E-KD-3.1.4-3 as $S_{alt\ i,j}$.

STEP 4 – Determine the Associated Mean Stress ($\sigma_{n\ i,j}$)– The next step is to determine the associated mean stress normal to the plane of the maximum shear stress, associated with the three $S_{alt\ i,j}$. These are listed in Table E-KD-3.1.4-3 as $\sigma_{n\ i,j}$.

STEP 5 – Determine the Stress Normal to the Plane of Maximum Shear ($\sigma_{n\ i,j}$) – The next step is to determine the associated mean normal stress. It is noted here that the cylinder is a non-welded monobloc construction and not made of austenitic stainless steel. The values calculated are shown in Table E-KD-3.1.4-3 as $\sigma_{n\ i,j}$ for both the operating and zero pressure case

STEP 6 – Determine the Associated Mean Stress ($\sigma_{nm\ i,j}$) – The next step is to determine the mean normal stress, $\sigma_{nm\ i,j}$. This is determined by taking the average of the stress normal to the plane of maximum shear calculated in Step 5.

STEP 7 – Determine the appropriate Fatigue Curve for use and Surface Roughness Factor (K_r) – Figure KD-320.3 is the curve to be used for pressure equipment made of this material per KD-322(c). The influence of the surface roughness of this cylinder is taken into account by inclusion of the surface roughness factor, K_r. which is found using Figure KD-320.5(b). The equation for this is:

$$K_r = max\left(1, \frac{1}{-0.16998\,log(R_a) + 1.2166}\right) = max\left(1, \frac{1}{-0.16998 * log(125) + 1.2166}\right) = 1.163$$

Further, it should also be noted that the modulus of elasticity for Figure KD-320.4 is 29 x 10^6 psi and that the allowable amplitude of the alternating stress component ($S`_a$) when σ_{nm} = 0 and $N = 10^6$ cycles is 42,800 psi.

STEP 8 – Determine the Equivalent Alternating Stress Intensity ($S_{eq\ i,j}$) – This cylinder being evaluated is for non-welded construction. Paragraph KD-312.4 states that for 17-4 or 15-5 stainless steel, the value of β shall be either 0.2 or 0.5 depending on the Associated Mean Stress ($\sigma_{nm\ i,j}$) determined in Step 6. These are listed in Table E-KD-3.1.4-2 along with a calculation for the denominator of equation KD-3.11 from paragraph KD-312.4. The limit of this factor is 0.9, but the table shows that this is below 0.9 for all cases considered. The value of Equivalent Alternating Stress for each of the directions considered are shown in the table. The fatigue life will be evaluated based on the maximum value calculated of 37,440 psi.

STEP 9 – Determine the Alternating Stress for Use in Evaluation of Design Life (S_a) – The alternating stress that will be used in conjunction with the curve is then found using equation KD-322.4:

$$S_a = K_f K_r K_e S_{eq} \frac{E(curve)}{E(analysis)} = 1.0 * 1.163 * 1.0 * 37{,}556\ psi * \frac{28.3 * 10^6 psi}{28.5 * 10^6 psi} = 43{,}355 psi$$

Where:

$$K_f = 1.0\ \textit{(local effects are accounted for in the model)}$$

$$K_e = 1.0\ for\ \Delta S_n \le 2S_y \qquad KD-322.1$$

$$K_e = 1.0\ for\ 32{,}667\ psi \le 2(115 ksi)$$

Table 12 – E-KD-3.1.5-2 – Calculated Stress Intensities and other Values for Fatigue

	1 - 2	2 – 3	3 – 1
Stress Intensities (psi)	$S_{ij} = s_i - s_j$		
Operating Pressure	12,147	30,000	-42,147
Zero Pressure	-37,853	0	37,853
Alternating Stress Intensities (psi)	$S_{alt\ i,j} = \lvert 0.5(S_{ij\ max} - S_{ij\ min})\rvert$		
	25,000	15,000	40,000
Stress Normal to the Plane of Maximum Shear (psi)	$\sigma_{n\ i,j} = 0.5\ (\sigma_i + \sigma_j)$		
Operating Pressure	6,074	-15,000	-8,926
Zero Pressure	-18,927	0	-18,927
Associated Mean Normal Stresses (psi)	$\sigma_{nm\ i,j} = 0.5\ (\sigma_{n\ i,j\ max} + \sigma_{n\ i,j\ min})$		
	-6,426	-7,500	-13,926
β	KD 312.4		
	0.2	0.2	0.2

Factor	$1 - \beta\sigma_{nm\,i,j}/S`_a$		
	1.03	1.035	1.065
Equivalent Alternating Stress Intensity (psi)	$S_{eq\,i,j} = S_{alt\,i,j}\dfrac{1}{1 - \beta\sigma_{nm\,i,j}/S`_a}$		
	24,271	14,492	37,556

STEP 10 – Determine the Design Life of the Vessel (N_f) – The design life of the vessel is then determined by using Figure KD-320.4. There are three acceptable methods for determining the number of cycles including interpolation from tabular values listed in Table KD-320.1 for Figure KD-320.4, use of the equations below Table KD-320.1 for Figure KD-320.4 or graphically from Figure KD-320.4. The stress determined here is smaller than the smallest stress in Table KD-320.1 for Figure 320.4.

The equation for interpolation of the fatigue curve from the notes to Table KD-320.1 is re-written here as:

$$N_f = N_{fi}\left(\frac{N_{fj}}{N_{fi}}\right)^{\frac{log\left(\frac{S_{ai}}{S_a}\right)}{log\left(\frac{S_{ai}}{S_{aj}}\right)}} \qquad Table\ KD-320.1$$

$$N_f = 2E5 * \left(\frac{5E5}{2E5}\right)^{\frac{log\left(\frac{48.7}{43.355}\right)}{log\left(\frac{48.7}{45.2}\right)}} = 849{,}049 cycles$$

On that basis, the cylinder meets the requirements specified for a design life of 3,500 cycles.

Table 13 – E-KD-3.1.4-3 – Values for Interpolation from Table KD-320.1 for Figure KD-320.4

	i	**j**
S_a	45,200 psi	42,800 psi
N_f	500,000	1,000,000

PART 5

Example Problems: Life Assessment Using Fracture Mechanics

5 EXAMPLE PROBLEMS LIFE ASSESSMENT USING FRACTURE MECHANICS

5.1 Example Problem E-KD-4.1.1 – Determine the Design Life of a Vessel from E-KD-2.1.1

Determine the design life of the vessel wall found in E-KD-2.1.1 using the fracture mechanics design approach of KD-4. The vessel wall under consideration is an open-end vessel where the end load is not supported by the vessel. The failure mode to be analyzed is a semi-elliptical surface breaking flaw in the ID of the wall in the axial-radial plane.

The cylinder being analyzed is from E-KD-2.1.1.

- Outside Diameter (D_O) = 12.0 inches
- Design Pressure (P_D) = 45,000 psi

  Max Design Pressure per E-KD-2.1.1 = 56,079 psi
- Operating Pressure = 40,000 psi

  Assumed to be approximately 90% of P_D
- Operational Temperature = 70°F
- Assumed Initial Crack Size = 0.0625 inch (a) x 0.188 inch ($2c$)
- Assumed Crack Aspect Ratio ($2c/a$) = 3:1 per KD-410(b)
- Material fracture toughness (K_{Ic}) = 100 ksi-in$^{0.5}$
- Vessel is to be operated cycling between the operational pressure and the zero-pressure state
- No autofrettage is to be considered
- The number of design cycles is to be 10,000 cycles per the User's Design Specification

The initial crack size is on the basis of the size flaw specified in the User's Design Specification for the vessel. The User's Design Specification is on the basis of this size on the maximum acceptable defect by the non-destructive examination (NDE) method (refer to KD-411(a)).

Residual stresses due to autofrettage are not considered as part of this analysis.

<u>**STEP 1** – Evaluate the stresses at the extremes of the operational conditions</u>

The stresses due to operating pressure were evaluated based on the equations of Appendix 9-300. Figure E-KD-4.1.1-1 shows a plot of these stress distributions for the vessel wall. There are no residual stresses considered in the evaluation of design life of the vessel. Pressure was added to the stress distribution during the evaluation of the crack growth but is not shown in the figure.

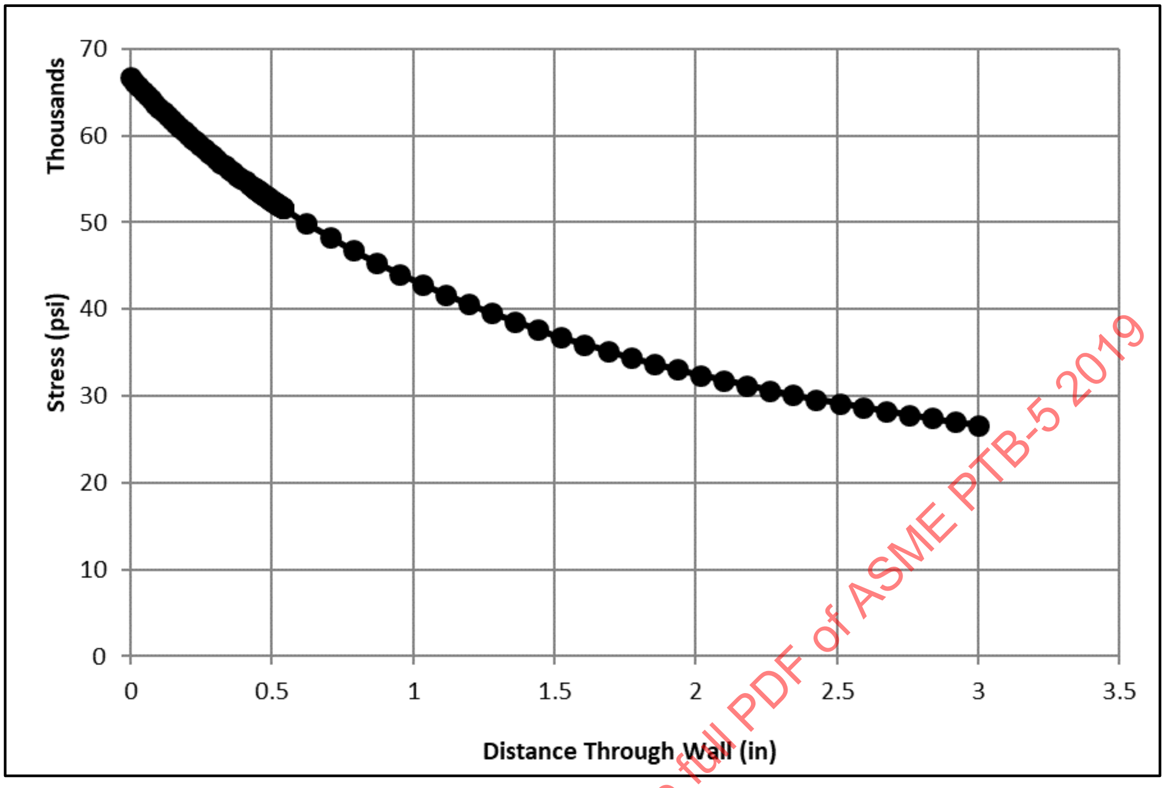

Figure 36 – E-KD-4.1.1-1 – Stress through the Vessel Wall due to Operating Pressure (40 ksi)

STEP 2 – Calculate the critical crack growth for the crack following the method of KD-430

The crack growth should be calculated by means of the equations in KD-430. Specifically, the equation for crack growth at the deepest point in a two-dimensional crack is shown in equation 1 as:

$$\frac{da}{dN} = C\,(f(R_k))\,(\Delta K)^m \qquad KD-430.1$$

And the growth along the free surface is:

$$\frac{dl}{dN} = 2C\,(f(R_k))\,(\Delta K)^m \qquad KD-430.2$$

Where $l = 2c$, and

$$\Delta K = K^*_{I\,max} - K^*_{I\,min}$$

$$R_K = \frac{K^*_{I.min} + K_{I.res}}{K^*_{I.max} + K_{I.res}}$$

Function $f(R_K)$ is based on the function from Table D-500 of Appendix D of VIII-3 for martensitic precipitation hardened steels, where $C_3 = 3.48$ and $C_2 = 1.5$ and

$$R_K \geq 0.67 \rightarrow f(R_K) = 30.53\,R_K - 17.0$$

$$0 < R_K \leq 0.67 \rightarrow f(R_K) = 1.0 + C_3\,R_K$$

$$R_K < 0 \rightarrow f(R_K) = \left(\frac{C_2}{C_2 - R_K}\right)^m$$

The stress intensity at the crack tips due to the various loading conditions is calculated separately. The stress intensity factor for the pressurized state will follow the methodology found in problem E-KD-3.1.1 for a semi-elliptical ID surface breaking crack in the longitudinal direction. This methodology will not be repeated here.

In this instance the $K^*_{I\,min}$ and $K^*_{I\,max}$ are the stress intensity at the zero pressure and operating pressure states, respectively. It should be noted that the crack is a surface breaking flaw which results in pressure acting on the crack faces. This is included in the calculation of stress intensity by adding the pressure to the stresses shown in Figure E-KD-4.1.1. The $K_{I\,res}$ is the stress intensity due to any residual stresses that may be present including due to shrink fitting of the component, autofrettage, or yielding during normal hydrostatic testing operations. In the failure mode that is being evaluated, there are no residual stresses in the component and the zero-pressure state will respectively result in $K_{I\,res}$ and $K^*_{I\,min}$ equaling zero.

Figure E-KD-4.1.1-2 shows a plot of the stress intensity factors calculated for the case in question here. Note that the stress is assumed to be constant along the inside surface of the vessel and the crack is assumed to be remote from any discontinuities that may affect it throughout the life of the crack.

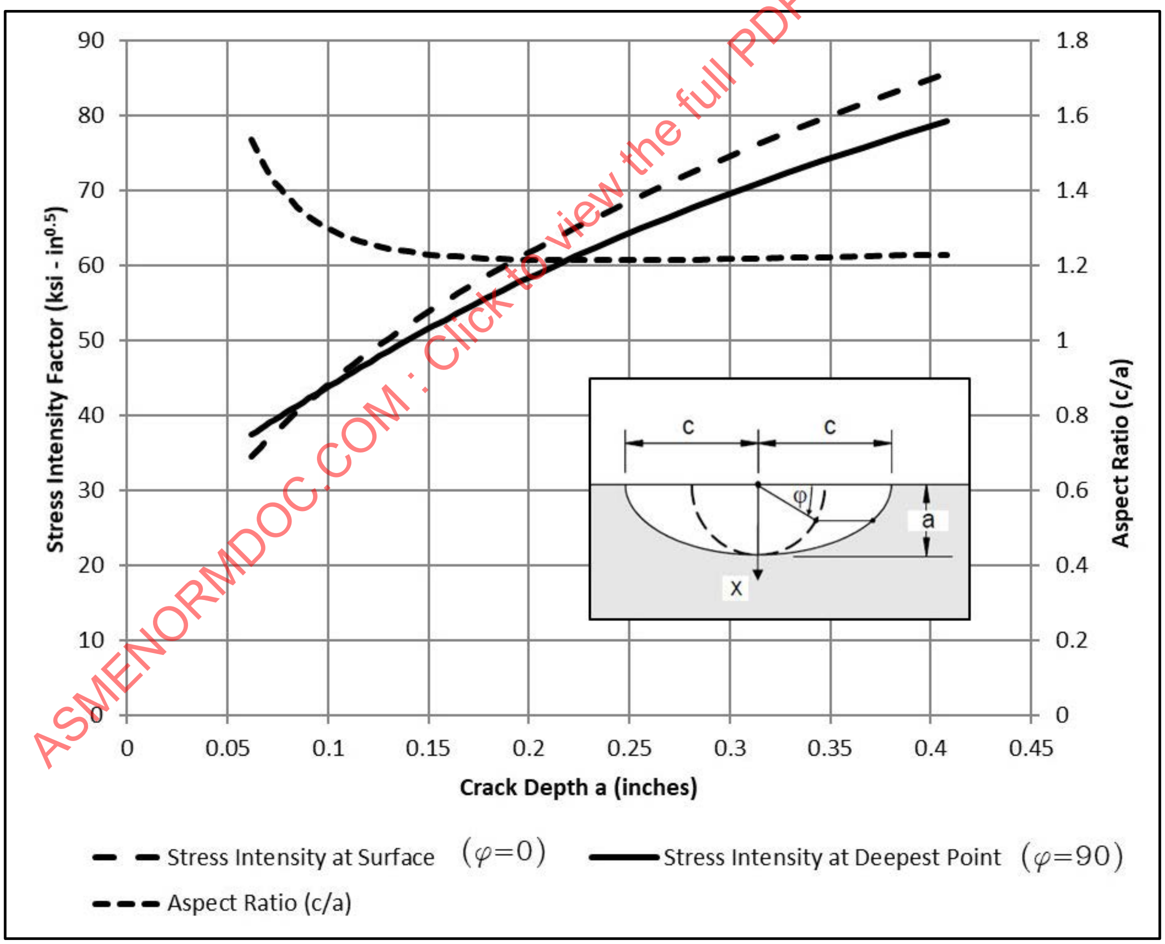

Figure 37 – E-KD-4.1.1-2 – Stress Intensity Factor for the Crack and Aspect Ratio vs. Crack Depth

The crack growth is required to be a two-dimensional crack growth per KD-430. Therefore, the aspect ratio of the crack changes as the crack grows. The stress intensities shown in the figure are based on the numerical integration of the two crack growth equations from KD-430. The stress intensities shown correspond to the stress intensity of the crack at the surface corresponds to the stress of the crack at the deepest point based on the aspect ratio of the crack for that dimension.

The number of cycles at the final crack size evaluated is 4600 cycles. Figure E-KD-4.1.1-3 shows the crack growth in both directions.

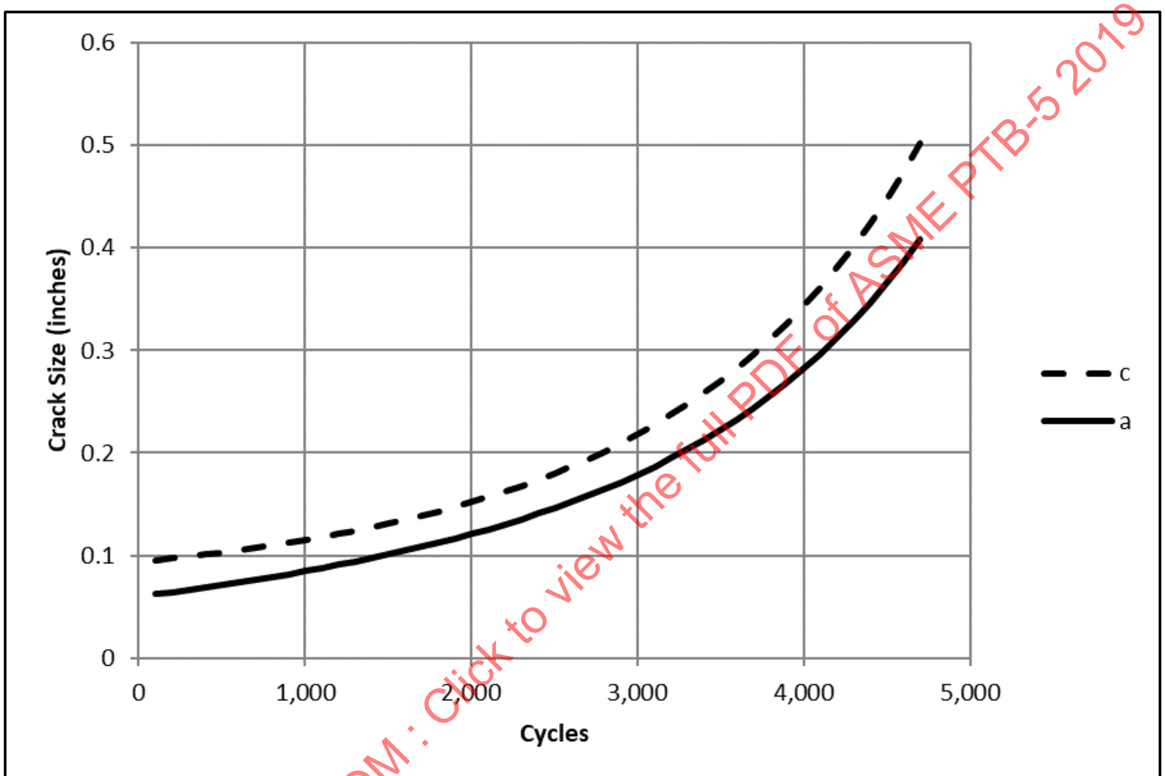

Figure 38 – E-KD-4.1.1-3 – Crack Size vs. Number of Cycles

<u>**STEP 3** – Determine the Critical Crack size in accordance with the Failure Assessment Diagram of API 579-1 / ASME FFS-1</u>

The critical crack depth for the failure mode being evaluated is determined using the failure assessment diagram from API 579-1 / ASME FFS-1. This is performed in accordance with paragraph 9.4.3.

In this methodology, the cracking is plotted K_r vs. L^P_r on the FAD as shown in Figure E-KD-4.1.1-4, where:

$$L_r^P = \frac{\sigma_{ref}^P}{\sigma_{ys}}$$

Where σ_{ref}^P is the reference stress for the crack in question from Annex D - D.5.10 and σ_{ys} is the yield strength of the material.

And:

$$K_r = \frac{K_I^P + \Phi K_I^{SR}}{K_{mat}}$$

Where K_I^P is the applied stress intensity due to the primary stress distribution, K_I^{SR} is the applied stress intensity due to the secondary and residual stress distributions, K_{mat} is the material toughness and Φ is the plasticity correction factor. The complete details of this are found in Part 9 of API 579-1 / ASME FFS-1.

Figure E-KD-4.1.1-4 specifically shows the bounding curve for the FAD including the "acceptable region" and the unacceptable regions.

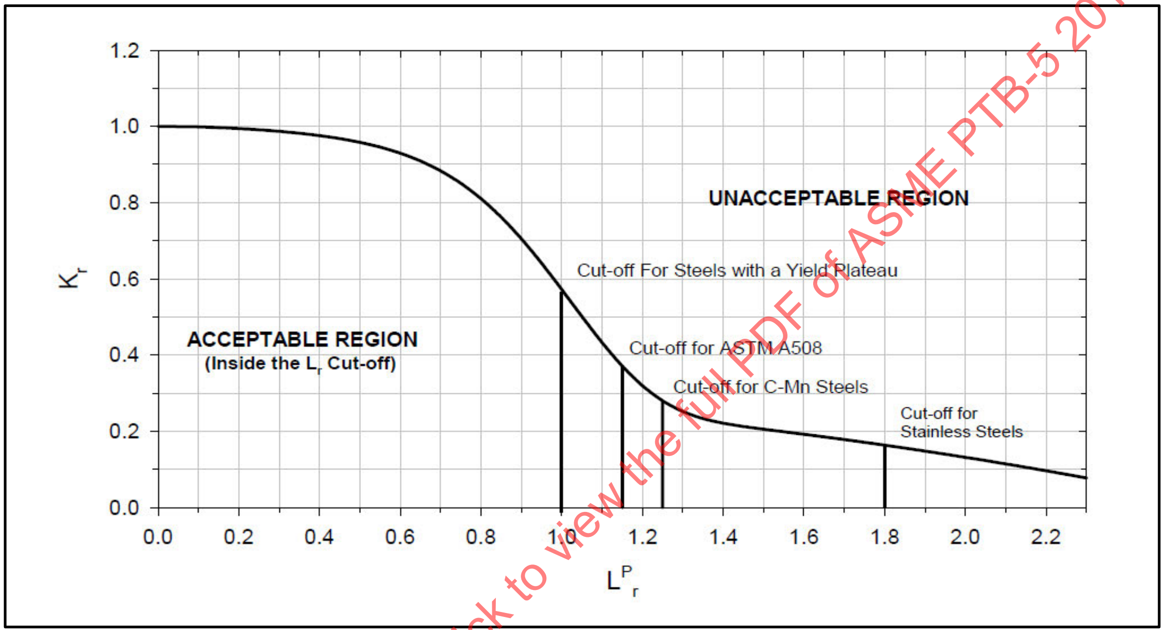

Figure 39 – E-KD-4.1.1-4 – Example of a Failure Assessment Diagram (from API 579-1 / ASME FFS-1 Fig 9.20)

The bounding curve is defined as:

$$K_r = (1 - 0.14(L_R^P)^2) * \left(0.3 + 0.7 * e^{-0.65*(L_R^P)^6}\right)$$

The bounding curve is stopped at a value of L_r^P equal to 1.109 for this problem based on the SA-705 XM-12 H1100 steel used.

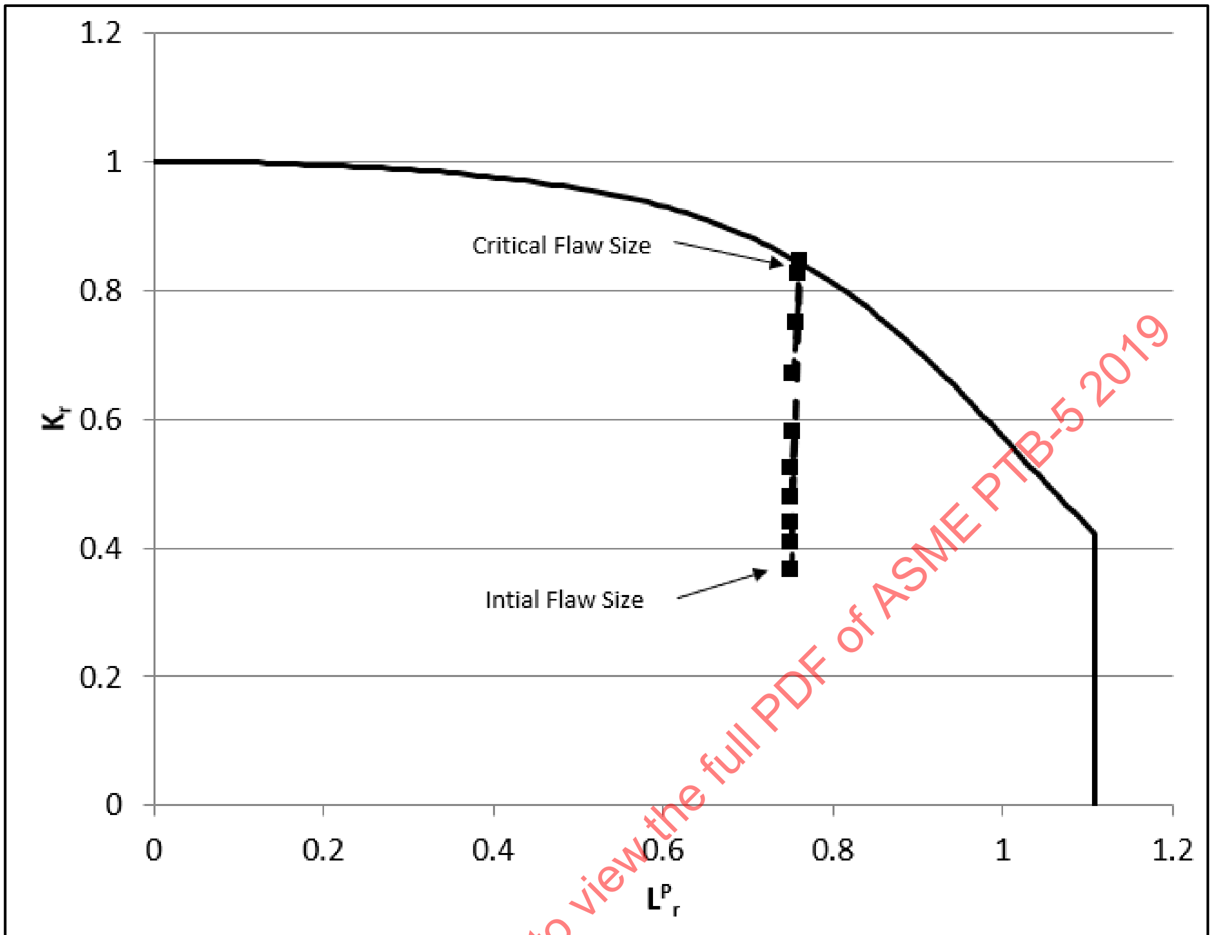

Figure 40 – E-KD-4.1.1-5 – Failure Assessment Diagram for E-KD-4.1.1

Figure E-KD-4.1.1-5 shows a plot of the failure assessment diagram. The assessment curve is the collection of points with coordinates $L^P{}_r, K_r$ generated as the crack is numerically grown in the analysis. The initial crack size of 0.188 inch long x 0.0625 inch deep resulted in the point at the lower end of the assessment curve. The crack was then grown as described in Step 2, analyzed and plotted at those sizes. The result is a crack who intersects the FAD bounding curve at a size of 1.0032 inch long (2c) x 0.408 inch deep (*a*). This size crack has a value of K_r equal to 0.845 and L_r^P equal to 0.761.

<u>**STEP 4** – Determine the Allowable Final Crack Depth and the Number of Design Cycles (N_P) for the Cylinder</u>

The allowable final crack depth is determined in accordance with KD-412. Note, the total number of cycles at the critical flaw size as predicted by the FAD is 4,600 cycles.

The allowable final crack depth per KD-412.1 is the lesser of:

- 25% of the section thickness considered = 0.75 inch
- the assumed initial flaw depth plus 25% of the dimensional difference between the critical crack depth and the assumed initial flaw = 0.062 + 25% x (0.408 – 0.062) inch deep = 0.149 inch

KD-412 states for this case that the number of “design” cycles (N_P) is the lesser of:

- half of the cycles to reach the critical crack depth = (4600 cycles) / 2 = 2,300 cycles
- the number of cycles to reach the “allowable final crack depth” (0.149 inches) = 2,500 cycles

Therefore, the number of “design” cycles (N_P) in accordance with KD-4 is 2,300 cycles.

Table 14 – E-KD-3.1.5-2 – Calculated Stress Intensities and other Values for Fatigue

	Stress Intensity at Deepest Point (f = 90)						Stress Intensity at Surface (f = 0)					
Cycles	Kmax	Kmin	Delta K	DaDn	a	a/t	Kmax	Kmin	Delta K	DaDn	c	(c/a)
100	37.479	0	37.479	2.16E-05	0.062	0.021	34.527	0	34.527	1.67E-05	0.095	1.54
200	37.859	0	37.859	2.23E-05	0.064	0.021	35.168	0	35.168	1.77E-05	0.097	1.51
300	38.249	0	38.249	2.30E-05	0.067	0.022	35.816	0	35.816	1.87E-05	0.099	1.49
400	38.650	0	38.650	2.38E-05	0.069	0.023	36.471	0	36.471	1.98E-05	0.101	1.46
500	39.062	0	39.062	2.46E-05	0.071	0.024	37.134	0	37.134	2.10E-05	0.103	1.44
600	39.486	0	39.486	2.54E-05	0.074	0.025	37.805	0	37.805	2.22E-05	0.105	1.42
700	39.924	0	39.924	2.63E-05	0.076	0.026	38.485	0	38.485	2.35E-05	0.107	1.40
800	40.376	0	40.376	2.73E-05	0.079	0.026	39.175	0	39.175	2.48E-05	0.110	1.39
900	40.843	0	40.843	2.83E-05	0.082	0.027	39.874	0	39.874	2.62E-05	0.112	1.37
1000	41.325	0	41.325	2.94E-05	0.085	0.028	40.585	0	40.585	2.77E-05	0.115	1.36
1100	41.825	0	41.825	3.05E-05	0.088	0.029	41.307	0	41.307	2.93E-05	0.118	1.34
1200	42.342	0	42.342	3.17E-05	0.091	0.030	42.041	0	42.041	3.10E-05	0.121	1.33
1300	42.877	0	42.877	3.30E-05	0.094	0.031	42.789	0	42.789	3.28E-05	0.124	1.32
1400	43.431	0	43.431	3.43E-05	0.098	0.033	43.552	0	43.552	3.46E-05	0.127	1.31
1500	44.006	0	44.006	3.58E-05	0.101	0.034	44.329	0	44.329	3.66E-05	0.131	1.30
1600	44.601	0	44.601	3.73E-05	0.105	0.035	45.122	0	45.122	3.87E-05	0.135	1.29
1700	45.219	0	45.219	3.90E-05	0.108	0.036	45.933	0	45.933	4.10E-05	0.139	1.28
1800	45.859	0	45.859	4.07E-05	0.112	0.038	46.762	0	46.762	4.33E-05	0.143	1.27
1900	46.522	0	46.522	4.26E-05	0.117	0.039	47.611	0	47.611	4.59E-05	0.147	1.26
2000	47.210	0	47.210	4.47E-05	0.121	0.040	48.480	0	48.480	4.85E-05	0.152	1.26
2100	47.924	0	47.924	4.68E-05	0.126	0.042	49.371	0	49.371	5.14E-05	0.157	1.25
2200	48.664	0	48.664	4.91E-05	0.130	0.043	50.285	0	50.285	5.45E-05	0.162	1.25
2300	49.431	0	49.431	5.16E-05	0.135	0.045	51.224	0	51.224	5.77E-05	0.168	1.24
2400	50.227	0	50.227	5.43E-05	0.141	0.047	52.188	0	52.188	6.12E-05	0.174	1.24
2500	51.052	0	51.052	5.71E-05	0.146	0.049	53.180	0	53.180	6.50E-05	0.180	1.23
2600	51.908	0	51.908	6.02E-05	0.152	0.051	54.201	0	54.201	6.90E-05	0.187	1.23
2700	52.796	0	52.796	6.35E-05	0.158	0.053	55.253	0	55.253	7.33E-05	0.194	1.23
2800	53.717	0	53.717	6.71E-05	0.165	0.055	56.337	0	56.337	7.79E-05	0.202	1.22
2900	54.672	0	54.672	7.09E-05	0.172	0.057	57.455	0	57.455	8.29E-05	0.210	1.22
3000	55.662	0	55.662	7.50E-05	0.179	0.060	58.608	0	58.608	8.82E-05	0.218	1.22
3100	56.689	0	56.689	7.95E-05	0.187	0.062	59.799	0	59.799	9.40E-05	0.227	1.22
3200	57.754	0	57.754	8.43E-05	0.195	0.065	61.030	0	61.030	1.00E-04	0.237	1.22
3300	58.859	0	58.859	8.94E-05	0.204	0.068	62.302	0	62.302	1.07E-04	0.247	1.22
3400	60.004	0	60.004	9.50E-05	0.213	0.071	63.618	0	63.618	1.14E-04	0.259	1.21
3500	61.192	0	61.192	1.01E-04	0.223	0.074	64.979	0	64.979	1.22E-04	0.270	1.21
3600	62.423	0	62.423	1.08E-04	0.233	0.078	66.388	0	66.388	1.31E-04	0.283	1.21
3700	63.699	0	63.699	1.15E-04	0.244	0.081	67.847	0	67.847	1.40E-04	0.296	1.21
3800	65.022	0	65.022	1.22E-04	0.256	0.085	69.358	0	69.358	1.50E-04	0.311	1.21
3900	66.392	0	66.392	1.31E-04	0.269	0.090	70.924	0	70.924	1.61E-04	0.327	1.22
4000	67.812	0	67.812	1.40E-04	0.282	0.094	72.546	0	72.546	1.73E-04	0.343	1.22
4100	69.282	0	69.282	1.50E-04	0.297	0.099	74.228	0	74.228	1.86E-04	0.361	1.22
4200	70.804	0	70.804	1.60E-04	0.312	0.104	75.970	0	75.970	2.00E-04	0.380	1.22
4300	72.378	0	72.378	1.72E-04	0.329	0.110	77.775	0	77.775	2.15E-04	0.401	1.22
4400	74.006	0	74.006	1.84E-04	0.347	0.116	79.646	0	79.646	2.32E-04	0.423	1.22
4500	75.688	0	75.688	1.98E-04	0.366	0.122	81.583	0	81.583	2.50E-04	0.448	1.22
4600	77.424	0	77.424	2.12E-04	0.386	0.129	83.588	0	83.588	2.70E-04	0.474	1.23

PART 6

Example Problems: Residual Stresses using Autofrettage

6 EXAMPLE PROBLEMS ON RESIDUAL STRESSES USING AUTOFRETTAGE

6.1 Example Problem E-KD-5.1.1 – Determine Residual Stresses in Autofrettaged Cylinder Wall with Known Autofrettage Pressure

Determine the residual stress in the vessel wall found in E-KD-2.1.1 for an autofrettaged cylinder with an autofrettage pressure of 65,000 psi using the methodology in KD-521. Assume this is an open-end vessel where the end load is not supported by the vessel.

The cylinder being analyzed is from E-KD-2.1.1.

- Outside Diameter (D_O) = 12.0 inches
- Inside Diameter (D_I) = 6.0 inches
- Design Pressure (P_D) = 56,079 psi (Calculated in E-KD-2.1.1)
- Autofrettage Pressure (P_A) = 65,000 psi

It should be noted that for an actual vessel using this calculation, the actual measured yield strength for the material (KD-502) of the vessel should be used, in lieu of the specification minimum. The specification minimum is used in this example.

STEP 1 – Calculate the elastic-plastic interface for the given autofrettage pressure using KD-521.3

The elastic-plastic interface (D_p) is calculated iteratively using the following equation:

$$P_A = 1.15 * S_y * \left(ln\left(\frac{D_p}{D_I}\right) + \frac{D_O^2 - D_P^2}{2 * D_O^2} \right) \rightarrow 65{,}000psi = 1.15 * 115{,}000psi * \left(ln\left(\frac{D_p}{6in}\right) + \frac{(12in)^2 - D_P^2}{2 * (12in)^2} \right)$$

Therefore, the elastic-plastic interface diameter is (iteratively):

$$D_P = 7.080in$$

Note that the maximum overstrain ratio from KD-521(c) is 0.4. The overstrain ratio here is:

$$\frac{D_P - D_I}{D_O - D_I} = \frac{7.080in - 6in}{12in - 6in} = 0.180 < 0.4$$

STEP 2 – Determine the Linear Elastic Stress at the ID at Autofrettage Pressure

The theoretical linear elastic stress at the ID of the vessel can be found using the equations of KD-250:

$$\sigma_t = P_A * \frac{D_O^2 + D_I^2}{D_O^2 - D_I^2} = 65{,}000psi * \frac{(12in)^2 + (6in)^2}{(12in)^2 - (6in)^2} = 108{,}333psi$$

$$\sigma_r = -P_A = -65{,}000psi$$

$$\sigma_l = 0psi$$

Note the longitudinal stress (σ_l) is zero for a vessel that does not support the pressure end load as in this case.

STEP 3 – Verify of the Average permanent tangential strain at ID

Paragraph KD-510 limits the permanent tangential strain at the bore surface resulting from the autofrettage operation to not exceed 2%. The equations of KD-521.2 can be used for this purpose by solving the left side of the equation for ε_p:

$$FAC = (1-2\nu)*\left(ln\left(\left(\frac{D_I}{D_P}\right)^2\right)-1\right)+(2-\nu)*\left(\frac{D_P}{D_I}\right)^2+(1-\nu)*\left(\frac{D_P}{D_O}\right)^2$$

$$FAC = (1-2*0.3)*\left(ln\left(\left(\frac{6in}{7.080in}\right)^2\right)-1\right)+(2-0.3)*\left(\frac{7.080in}{6in}\right)^2+(1-0.3)*\left(\frac{7.080in}{12in}\right)^2 = 2.088$$

$$\epsilon_P = \frac{1.15*S_y}{2*E}*\left(FAC-\frac{\left(ln\left(\left(\frac{D_P}{D_I}\right)^2\right)+\frac{D_O^2-D_P^2}{D_O^2}\right)*\left(1-\nu+(1+\nu)*\left(\frac{D_O}{D_I}\right)^2\right)}{\left(\frac{D_O}{D_I}\right)^2-1}\right)$$

$$\epsilon_P = \frac{1.15*115{,}000psi}{2*28{,}300{,}000psi}*\left(2.088-\frac{\left(ln\left(\left(\frac{7.080in}{6in}\right)^2\right)+\frac{(12in)^2-(7.080in)^2}{(12in)^2}\right)*\left(1-0.3+(1+0.3)*\left(\frac{12in}{6in}\right)^2\right)}{\left(\frac{12}{6in}\right)^2-1}\right)$$

$$\epsilon_P = 0.0338\% < 2\%$$

Therefore, this is acceptable per KD-510.

<u>**STEP 4** – Calculate the Theoretical Residual Stresses Between the Bore and the Elastic-Plastic Interface</u>

The theoretical residual stress distribution for the cylinder without the effect of reverse yielding should be determined using the equations of KD-522.1.

$$\sigma_{tRA}(D) = S_y*\left(\frac{D_P^2+D_O^2}{2*D_O^2}+ln\left(\frac{D}{D_P}\right)-\left(\frac{D_I^2}{D_O^2-D_I^2}\right)*\left(\frac{D_O^2-D_P^2}{2*D_O^2}+ln\left(\frac{D_P}{D_I}\right)\right)*\left(1+\frac{D_O^2}{D^2}\right)\right)$$

$$\sigma_{tRA}(6in) = 115{,}000psi*\left(\frac{7.080^2+12^2}{2*12^2}+ln\left(\frac{6}{7.080}\right)-\left(\frac{6^2}{12^2-6^2}\right)*\left(\frac{12^2-7.080^2}{2*12^2}+ln\left(\frac{7.080}{6}\right)\right)*\left(1+\frac{12^2}{6^2}\right)\right)$$

$$\sigma_{tRA}(6in) = -35.724ksi$$

$$\sigma_{rRA}(D) = S_y*\left(\frac{D_P^2-D_O^2}{2*D_O^2}+ln\left(\frac{D}{D_P}\right)-\left(\frac{D_I^2}{D_O^2-D_I^2}\right)*\left(\frac{D_O^2-D_P^2}{2*D_O^2}+ln\left(\frac{D_P}{D_I}\right)\right)*\left(1-\frac{D_O^2}{D^2}\right)\right)$$

$$\sigma_{rRA}(6in) = 115{,}000psi*\left(\frac{7.080^2-12^2}{2*12^2}+ln\left(\frac{6}{7.080}\right)-\left(\frac{6^2}{12^2-6^2}\right)*\left(\frac{12^2-7.080^2}{2*12^2}+ln\left(\frac{7.080}{6}\right)\right)*\left(1-\frac{12^2}{6^2}\right)\right)$$

$$\sigma_{rRA}(6in) = 0ksi$$

<u>**STEP 5** – Correction for Bauschinger Factor for Reverse Yielding</u>

The residual stresses are then corrected for the effect of a reduced compressive yield strength after tensile yielding, termed as the "Bauschinger Effect Factor". This is done in accordance with the methods of KD-522.2.

a) The first step is to determine the diameter where the residual hoop stress minus the radial stress is equal to zero. This is done using the equations in KD-522.1 as shown in Step 4 iteratively. This is defined as D_Z, which for this problem equals 6.869 inches.

$$\sigma_{rRA}(D_Z) = 115{,}000psi * \left(\frac{7.080^2 - 12^2}{2 * 12^2} + ln\left(\frac{6.869}{7.080}\right) - \left(\frac{6^2}{12^2 - 6^2}\right) * \left(\frac{12^2 - 7.080^2}{2 * 12^2} + ln\left(\frac{7.080}{6}\right)\right) * \left(1 - \frac{12^2}{6.869^2}\right)\right)$$

$$\sigma_{rRA}(D_Z) = \sigma_{rRA}(6.869in) = -2.307ksi$$

$$\sigma_{tRA}(D_Z) = 115{,}000psi * \left(\frac{7.080^2 + 12^2}{2 * 12^2} + ln\left(\frac{6.869}{7.080}\right) - \left(\frac{6^2}{12^2 - 6^2}\right) * \left(\frac{12^2 - 7.080^2}{2 * 12^2} + ln\left(\frac{7.080}{6}\right)\right) * \left(1 + \frac{12^2}{6.869^2}\right)\right)$$

$$\sigma_{tRA}(D_Z) = \sigma_{tRA}(6.869in) = -2.307ksi$$

b) Define the hoop stress at the ID without reverse yielding as σ_{AD}:

$$\sigma_{AD} = -35.724ksi$$

c) Determine the overstrain ratio as:

$$M = \frac{D_P - D_I}{D_O - D_I} = \frac{7.080in - 6in}{12in - 6in} = 0.18$$

d) Determine the corrected value of tangential stress at the ID based on the equations in KD-522.2(b):

$$\sigma_{CD1} = \sigma_{AD} * (1.6695 - 0.1651Y - 1.8871M + 1.9837M^2 - 0.7296M^3)$$

$$\sigma_{CD1} = -35.724ksi * (1.6695 - 0.1651 * 2.0 - 1.8871 * 0.18 + 1.9837 * 0.18^2 - 0.7296 * 0.18^3)$$

$$\sigma_{CD1} = -37.853ksi$$

$$\sigma_{CD2} = \sigma_{AD} * (-0.5484 + 1.8141Y - 0.6502Y^2 + 0.0791Y^3)$$

$$\sigma_{CD2} = -35.724ksi * (-0.5484 + 1.8141 * 2.0 - 0.6502 * 2.0^2 + 0.0791 * 2.0^3) = -39.718ksi$$

$$\sigma_{CD} = max(\sigma_{CD1}, \sigma_{CD2}) = max(-37.853ksi, -39.718ksi) = -37.853ksi$$

Note:

$$\frac{\sigma_{CD}}{S_y} = \frac{-37.853ksi}{115ksi} = -0.329 > -0.7$$

So no further correction is needed (refer to KD-522.2(c)).

e) The residual stresses are then calculated in a piecewise continuous fashion.

1) From $D_I < D < D_Z$

$$\sigma_{tR}(D) = \sigma_{CD} * \left(\frac{D_Z * \left(ln\left(\frac{D}{D_I}\right) + 1\right) + D_I - 2D}{D_Z - D_I}\right)$$

$$\sigma_{tR}(D_I) = \sigma_{tR}(6in) = -37.853ksi * \left(\frac{6.869in * \left(ln\left(\frac{6in}{6in}\right) + 1\right) + 6in - 2 * 6in}{6.869in - 6in}\right) = -37.853ksi$$

$$\sigma_{tR}(D_z) = \sigma_{tR}(6.869in) = -37.853ksi * \left(\frac{6.869in * \left(ln\left(\frac{6.869in}{6in}\right) + 1\right) + 6in - 2 * 6.869in}{6.869in - 6in}\right) = -2.618ksi$$

$$\sigma_{rR}(D) = \sigma_{CD} * \left(\frac{D_Z * ln\left(\frac{D}{D_I}\right) + D_I - D}{D_Z - D_I} \right)$$

$$\sigma_{rR}(D_I) = \sigma_{rR}(6in) = -37.853ksi * \left(\frac{6.869in * ln\left(\frac{6in}{6in}\right) + 6in - 6in}{6.869in - 6in} \right) = 0ksi$$

$$\sigma_{rR}(D_Z) = \sigma_{rR}(6.869in) = -37.853ksi * \left(\frac{6.869in * ln\left(\frac{6.869in}{6in}\right) + 6in - 6.869in}{6.869in - 6in} \right) = -2.618ksi$$

2) A correction factor is then applied for the stresses for $D > D_Z$:

$$F_b = \frac{\sigma_{rR}(D_Z)}{\sigma_{rRA}(D_Z)} = \frac{-2.618ksi}{-2.307ksi} = 1.135$$

3) The stresses for $D_Z < D < D_P$ are then calculated by multiplying stresses from KD-522.1 by the correction factor F_b.

4) The stresses for $D_P < D < D_O$ are then calculated using the equations from KD-523:

$$\sigma_{tRB}(D) = S_y * F_b * \left(1 + \frac{D_O^2}{D^2}\right) * \left(\frac{D_P^2}{2 * D_O^2} + \frac{D_I^2}{D_O^2 - D_I^2} * \left(\frac{D_P^2 - D_O^2}{2 * D_O^2} - ln\left(\frac{D_P}{D_I}\right) \right) \right)$$

$$\sigma_{rRB}(D) = S_y * F_b * \left(1 - \frac{D_O^2}{D^2}\right) * \left(\frac{D_P^2}{2 * D_O^2} + \frac{D_I^2}{D_O^2 - D_I^2} * \left(\frac{D_P^2 - D_O^2}{2 * D_O^2} - ln\left(\frac{D_P}{D_I}\right) \right) \right)$$

At the elastic-plastic interface:

$$\sigma_{tRB}(D_P) = 115{,}00psi * 1.135 * \left(1 + \frac{12^2}{7.080^2}\right) * \left(\frac{7.080^2}{2 * 12^2} + \frac{6^2}{12^2 - 6^2} * \left(\frac{7.080^2 - 12^2}{2 * 12^2} - ln\left(\frac{7.080}{6}\right) \right) \right)$$

$$\sigma_{tRB}(D_P) = \sigma_{tRB}(7.080in) = 5.171ksi$$

$$\sigma_{rRB}(D_P) = 115{,}00psi * 1.135 * \left(1 - \frac{12^2}{7.080^2}\right) * \left(\frac{7.080^2}{2 * 12^2} + \frac{6^2}{12^2 - 6^2} * \left(\frac{7.080^2 - 12^2}{2 * 12^2} - ln\left(\frac{7.080}{6}\right) \right) \right)$$

$$\sigma_{rRB}(D_P) = \sigma_{rRB}(7.080in) = -2.500ksi$$

And at the outside diameter:

$$\sigma_{tRB}(D_O) = 115{,}00psi * 1.135 * \left(1 + \frac{12^2}{12^2}\right) * \left(\frac{7.080^2}{2 * 12^2} + \frac{6^2}{12^2 - 6^2} * \left(\frac{7.080^2 - 12^2}{2 * 12^2} - ln\left(\frac{7.080}{6}\right) \right) \right)$$

$$\sigma_{tRB}(D_O) = \sigma_{tRB}(12in) = 2.671ksi$$

$$\sigma_{rRB}(D_O) = 115{,}00psi * 1.135 * \left(1 - \frac{12^2}{12^2}\right) * \left(\frac{7.080^2}{2 * 12^2} + \frac{6^2}{12^2 - 6^2} * \left(\frac{7.080^2 - 12^2}{2 * 12^2} - ln\left(\frac{7.080}{6}\right) \right) \right)$$

$$\sigma_{rRB}(D_O) = \sigma_{rRB}(12in) = 0ksi$$

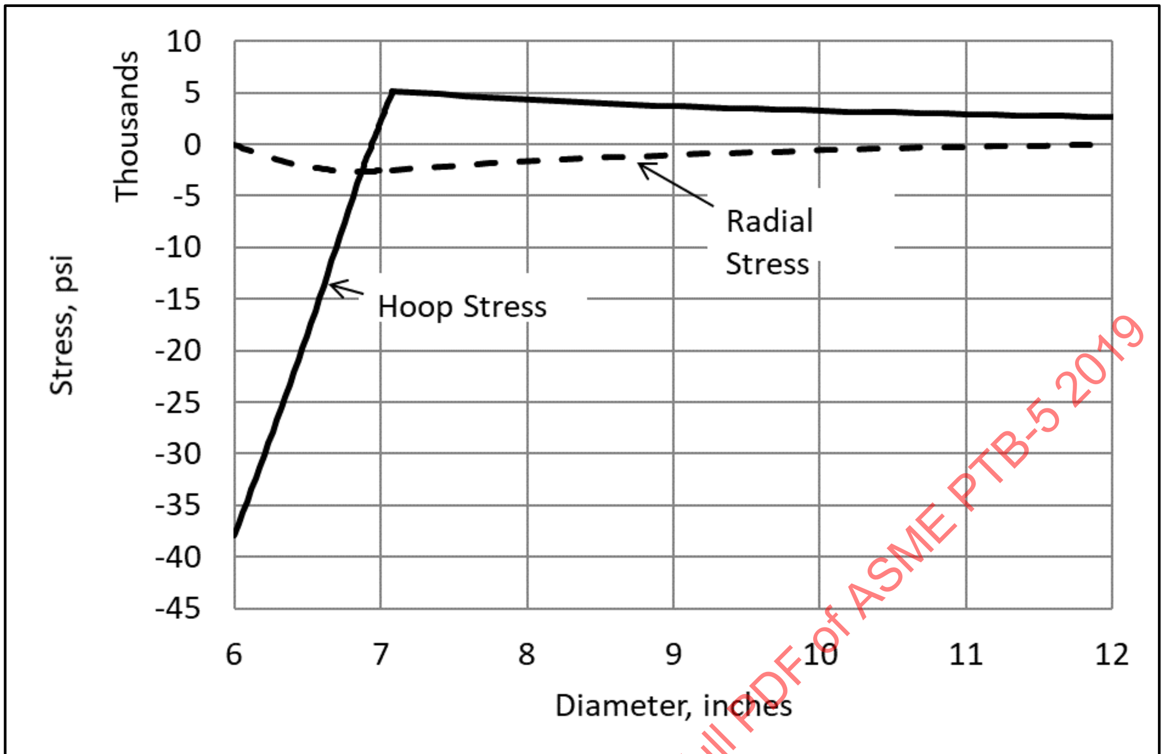

Figure 41 – E-KD-5.1.2-1 – Residual Stress Distribution in Vessel Wall

6.2 Example Problem E-KD-5.1.2 – Determine the Autofrettage Pressure in a Cylinder Wall with known Residual ID Tangential Strain

Determine the autofrettage pressure (P_A) in the vessel wall found in E-KD-2.1.1 when the residual tangential ID strain is known using the methodology in KD-521. Assume this is an open-end vessel where the end load is not supported by the vessel.

The cylinder being analyzed is from E-KD-2.1.1.

- Outside Diameter (D_O) = 12.0 inches
- Design Pressure (P_D) = 56,079 psi (Calculated in E-KD-2.1.1)
- Residual ID tangential strain (ε_P) = 0.0338%

It should be noted that for an actual vessel using this calculation, the actual measured yield strength for the material (KD-502) of the vessel should be used. The minimum yield strength of 115 ksi from E-KD-2.1.1 is used here.

<u>**STEP 1** – Calculate the elastic-plastic interface (D_p) for the given autofrettage pressure using KD-521.2</u>

The elastic-plastic interface (D_p) is calculated iteratively using the following equation:

$$\epsilon_P = \frac{1.15S_y}{2E} * \left((1-2\nu) * \left(ln\left(\left(\frac{D_I}{D_P}\right)^2\right) - 1 \right) + (2-\nu) * \left(\frac{D_P}{D_I}\right)^2 + (1-\nu) * \left(\frac{D_P}{D_O}\right)^2 - \frac{\left(ln\left(\left(\frac{D_P}{D_I}\right)^2\right) + \frac{D_O^2 - D_P^2}{D_O^2} \right) * \left(1 - \nu + (1+\nu) * \left(\frac{D_O}{D_I}\right)^2 \right)}{\left(\frac{D_o}{D_I}\right)^2 - 1} \right)$$

$$0.0338\% = \frac{1.15 * 115ksi}{2 * 28{,}300ksi} * \left((1 - 2 * 0.3) * \left(ln\left(\left(\frac{6in}{D_P}\right)^2\right) - 1 \right) + (2 - 0.3) * \left(\frac{D_P}{6in}\right)^2 + (1 - 0.3) * \left(\frac{D_P}{12in}\right)^2 - \frac{\left(ln\left(\left(\frac{D_P}{6in}\right)^2\right) + \frac{(12in)^2 - D_P^2}{(12in)^2} \right) * \left(1 - 0.3 + (1 + 0.3) * \left(\frac{12in}{6in}\right)^2 \right)}{\left(\frac{12in}{6in}\right)^2 - 1} \right)$$

The elastic plastic interface can then be found to be 7.080 inches.

<u>**STEP 2** – Calculate the autofrettage pressure (P_A) using KD-521.3</u>

The autofrettage pressure can then be determined using the equation in KD-521.3.

$$P_A = 1.15 * S_y * \left(ln\left(\frac{D_p}{D_I}\right) + \frac{D_O^2 - D_P^2}{2 * D_o^2} \right) = 1.15 * 115{,}000psi * \left(ln\left(\frac{7.080in}{6in}\right) + \frac{(12in)^2 - (7.080in)^2}{2 * (12in)^2} \right) = 65{,}000psi$$

PART 7

Example Problems: Closures and Connections

7 EXAMPLE PROBLEMS IN CLOSURES AND CONNECTIONS

7.1 Example Problem E-KD-6.1.1 – Evaluation of a Connection in a 60-ksi Pressure Vessel at 100°F

Determine the suitability of an industry standard 9/16 tubing connection [6][7] for use in an BPVC Section VIII, Division 3 pressure vessel made of SA-723 Grade 2 Class 2 material rated at 60 ksi at 100°F as shown in Figure E-KD-6.1.1-1. The vessel will be designed with sufficient wall thickness to accommodate an opening of this size using the elastic-plastic finite element methods of KD-230.

Note, that it is typical to consider the boundary of a pressure vessel to end at the first connection. However, it is noted that in KD-6, the rules regarding the geometry of the connection machined into the vessel are mandatory for all vessels.

Also, it is assumed that this is a connection on the exterior surface of a pressure vessel such as a center connection on a head. These assumptions mean that the dimension of the material surrounding the connection (in the direction perpendicular to the centerline of the connection as shown in Figure E-KD-6.1.1-1) is large compared with the dimensions of the connection. Therefore, it is assumed that the radial displacement is negligible.

The UDS states that there are no externally imposed loads on this connection.

There are two methods for evaluation of this type of connection in KD-6, elastic-plastic basis in KD-621 and the linear-elastic basis in KD-623. This example will approach the problem from the linear elastic basis.

Note that if the connection is to be under cyclic operation, it may need to be evaluated as a potential failure mode in accordance with KD-3 / KD-4 but will not be as part of this example.

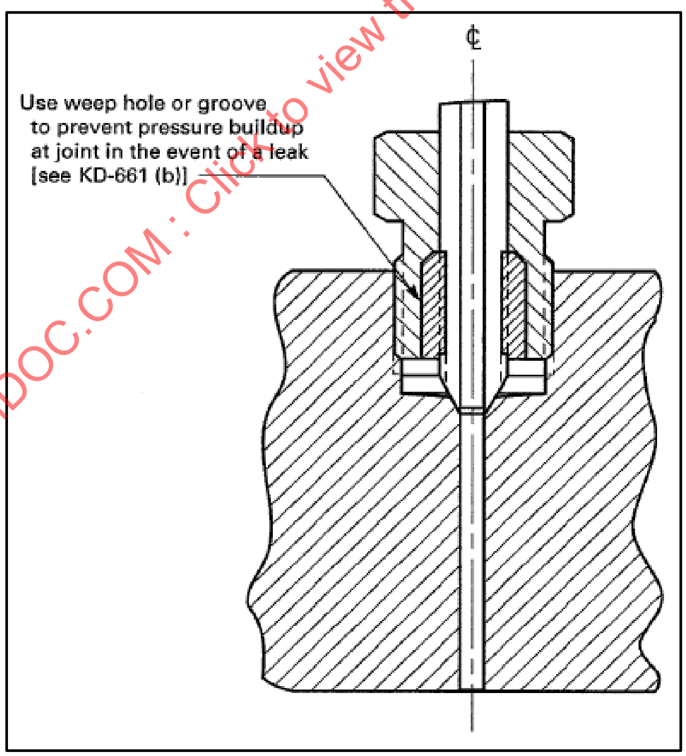

Figure 42 – E-KD-6.1.1-1 – Typical High-Pressure Connection (VIII-3, Appendix H)

Material

Vessel Material

SA-723 Grade 2 Class 2 material rated at 100°F

S_y = 120,000 psi at 100°F — Yield Strength of the material

S_u = 135,000 psi at 100°F — Tensile Strength of the material

Vessel and Opening Dimension and Loading Data

D_s = 0.296 inch	Opening Seal Diameter
b = 0.005 inch	Assumed Seal Width
m = 6.5	Seal Factor
P = 60,000 psi	Pressure Load
$W = 0.785D_s^2P + (2b \times 3.14D_smP)$	
W = 7,755 lbf	End load on connection using BPVC Section VIII, Division 1[5] Appendix 2 Section 2-5 for connections with metal seat.
Thread	1/8-14UNS Class 2B thread machined into the body
p = 1/14 inch	Thread pitch
D_{major} = 1.113 inches	Min material condition of Major Diameter of Male Thread
D_{minor} = 1.064 inches	Min material condition of Minor Diameter of Female Thread
D_{pitch} = 1.071 inches	Pitch Diameter of Connection Thread
D_{root} = 1.0384 inches	Nominal Root Diameter of Male Thread
L = 0.438 inch	Minimum Engaged Gland Thread Length per KD-626 (this does not include incomplete or partial threads)

STEP 1 – Determine the Average Thread Shear Stress (KD-623(f))

The average thread shear stress is limited to 30,000 psi ($0.25S_y$). This is calculated by:

$$\tau = \frac{W}{\pi D_{pitch}\frac{L}{2}} = \frac{7{,}755lbf}{\pi * 1.071in * \frac{0.438in}{2}} = 10{,}525\ psi$$

Where:

$$W = 0.785D_s^2P + (2b * 3.14D_smP)$$

$$W = 0.785 * (0.296in)^2 * 60{,}000psi + (2 * 0.005in * 3.14 * 0.296in * 6.5 * 60{,}000psi) = 7{,}755lbf$$

STEP 2 –Determination of the Average Thread Bearing Stress (KD-623(g))

The average thread bearing stress due to the maximum design load is limited to 90,000 psi ($0.75S_y$). This is calculated by:

$$\sigma = \frac{W}{A_{bearing}} = \frac{7{,}755lbf}{1.011in^2} = 7{,}671\ psi$$

Where:

$$A_{bearing} = \pi D_{pitch}\left(D_{major} - D_{minor}\right)\frac{L}{p} = \pi * 1.071in * (1.113in - 1.064in) * \frac{0.438in}{\frac{1}{14}} = 1.011\ in^2$$

STEP 3 – Determine the Length of Engagement Required (KD-623(k))

The minimum thread engagement length is the minimum based on the drawing tolerances and without credit for the first and last partial thread in the engaged length. KD-623(j) states that connections with imposed loads must comply with the length of engagement for bolts in KD-623(k). The UDS states that there are no externally imposed loads on these connections, so therefore, this requirement does not apply.

Note: The connections listed here are "industry standard" but typically machined to manufacturer's published standards such as listed in the references for this manual.

It is also noted that KD-623(k) states that the engaged thread length shall not be less than the larger of d_s or $0.75d_s$ (S_y of stud material at design temperature / S_y of tapped material at design temperature). The engaged thread length would not meet this requirement.

7.2 Example Problem E-KD-6.1.2 – Alternative Evaluation of Stresses in Threaded End Closures

In lieu of performing a numerical simulation, such as a finite element analysis, of a closure to determine the stresses for a fatigue or fracture mechanics analysis, KD-630 provides guidance on the evaluation of these stresses. This problem is to evaluate the stresses at the first thread in the pressure vessel evaluated in example problem E-AE-2.2.1 and E-KD-2.3.1.

It is noted that a vent hole will be incorporated into the closure for use in the event of seal failure, as required by KD-661. This will either be as a small weep hole through the side of the vessel or by venting the nut by grooving the face and possibly drilling an intersecting hole axially through the nut.

Vessel Dimension and Loading Data (refer to E-AE-2.2.1 and E-KD-2.3.1 for complete details)

Design Pressure *(P_D)*	=	11,000 psi
Outside Diameter of the Vessel *(D_O)*	=	12 inches
Inside Diameter of the Vessel *(D_I)*	=	10 inches
Pitch Diameter of the Threads *(D_p)*	=	10.443 inches
Root Diameter of the Threads (D_{root})	=	10.769 inches
Thread Pitch *(P_T)*	=	0.5 inch
Total number of threads *(n)*	=	10
F_s (from E-AE-2.2.1)	=	863,938 lbf

STEP 1 – Evaluate the Longitudinal Bending Stress at the First Thread (KD-631.1)

The primary longitudinal bending stress in the vessel at the first thread is found using:

$$\sigma_{LB} = 3.0\frac{F_s}{A_{Longitudinal}} = 3.0 * \frac{863{,}938lbf}{22.014} = 117{,}737\ psi$$

Where:

$$A_{Longintudinal} = \frac{\pi}{4}\left(D_O^2 - D_{root}^2\right) = \frac{\pi}{4} * \left((12in)^2 - (10.769in)^2\right) = 22.014\ in^2$$