

**ASME B31J-2017**

**[Revision of ASME B31J-2008 (R2013)]**

# **Stress Intensification Factors (*i*-Factors), Flexibility Factors (*k*-Factors), and Their Determination for Metallic Piping Components**

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**ASME Code for Pressure Piping, B31**

**AN AMERICAN NATIONAL STANDARD**



**The American Society of  
Mechanical Engineers**

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**The American Society of  
Mechanical Engineers**

Two Park Avenue • New York, NY • 10016 USA

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# FOREWORD

In 1990 The American Society of Mechanical Engineers (ASME) B31 Code for Pressure Piping Technical Committee on Mechanical Design (MDC) realized that there was a need for a standard method to develop stress intensification factors (SIFs or *i*-factors) for ASME piping components and joints. At the time, the B31 Code books provided SIFs for various standard fittings and joints but did not provide guidance on how to conduct further research on existing SIFs or how to establish SIFs for nonstandard and other standard fittings or joints.

In 2001 the Committee realized that SIFs and *k*-factors in the various B31 Code books were not consistent or up to date, and so ASME initiated a research project completed by the MDC to incorporate recent research and current manufacturing practices in the SIF and *k*-factor test procedures, to provide a consistent and up-to-date table of SIFs and *k*-factors for metallic piping components.

This document provides a standard approach for the development of SIFs, *k*-factors, and sustained stress multipliers for piping components and joints of all types, including standard, nonstandard, and proprietary fittings.

Sustained stress multipliers are used to multiply the nominal bending stress due to sustained loading and reflect the collapse capacity of the metallic piping component or joint. Multipliers of the nominal bending stress due to sustained loads currently exist explicitly in some, but not all, B31 books. Where more accurate sustained stresses are needed but an equation for the sustained stress is not given in the B31 Code book, nominal stresses due to sustained moments computed using the section modulus of the matching pipe should be multiplied by the appropriate sustained stress multiplier. Where the sustained stress is needed and an equation for the sustained stress is given in the Code book as a function of the SIF and provided in lieu of more applicable data, the sustained stress multipliers developed using the method in this Standard may be substituted as more applicable data and used with the nominal stress computed using the section modulus of the matching pipe.

The most applicable currently available stress intensification and flexibility factors compiled from test and analysis data for standard commercially available metallic components are included in [Table 1-1](#) and should be used with the section modulus of the matching pipe (not an “effective” section modulus). [Nonmandatory Appendix A](#) provides the standard method to develop stress intensification factors. [Nonmandatory Appendix B](#) provides the standard method to develop branch connection flexibility factors. [Nonmandatory Appendix C](#) demonstrates how the new branch connection *k*-factors should be used in the elastic analysis of piping systems, and [Nonmandatory Appendix D](#) provides a standard method to develop sustained stress factors. A procedure to develop *k*-factors for bends, elbows, and straight pipe is available in WRC Bulletin 463, “Report 1: Standardized Method for Developing Flexibility Factors for Piping Components,” E. C. Rodabaugh and E. A. Wais (July 2001).

In its development, this Standard has been reviewed by individuals and appropriate subcommittees of the Boiler and Pressure Vessel Code, B31, and B16 Committees. Comments resulting from the review have been considered and responded to, with revisions made to the Standard, as appropriate. The 2017 edition has been revised in its entirety. It was approved as an American National Standard by the American National Standards Institute on January 11, 2017.

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The Committee welcomes proposals for revisions to this Standard. Such proposals should be as specific as possible, citing the paragraph number(s), the proposed wording, and a detailed description of the reasons for the proposal, including any pertinent documentation.

**Interpretations.** Upon request, the B31 Standards Committee will render an interpretation of any requirement of the Standard. Interpretations can only be rendered in response to a written request sent to the Secretary of the B31 Standards Committee.

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If the Inquirer is unable to use the online form, he/she may mail the request to the Secretary of the B31 Standards Committee at the above address. The request for an interpretation should be clear and unambiguous. It is further recommended that the Inquirer submit his/her request in the following format:

Subject:	Cite the applicable paragraph number(s) and the topic of the inquiry in one or two words.
Edition:	Cite the applicable edition of the Standard for which the interpretation is being requested.
Question:	Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. Please provide a condensed and precise question, composed in such a way that a "yes" or "no" reply is acceptable.
Proposed Reply(ies):	Provide a proposed reply(ies) in the form of "Yes" or "No," with explanation as needed. If entering replies to more than one question, please number the questions and replies.
Background Information:	Provide the Committee with any background information that will assist the Committee in understanding the inquiry. The Inquirer may also include any plans or drawings that are necessary to explain the question; however, they should not contain proprietary names or information.

Requests that are not in the format described above may be rewritten in the appropriate format by the Committee prior to being answered, which may inadvertently change the intent of the original request.

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# INTRODUCTION

The ASME B31 Code for Pressure Piping consists of a number of individually published Sections and Standards, each an American National Standard, under the direction of the ASME B31 Code for Pressure Piping Committee.

Rules for each Section reflect the kinds of piping installations considered during its development, as follows:

(a) B31.1, Power Piping: piping typically found in electric generating stations, in industrial and institutional plants, in geothermal and solar power applications, and in central and district heating and cooling systems

(b) B31.3, Process Piping: piping typically found in petroleum refineries and in chemical, pharmaceutical, textile, paper, semiconductor, cryogenic, and related processing plants and terminals

(c) B31.4, Pipeline Transportation Systems for Liquid Hydrocarbons and Other Liquids: piping that transports products that are predominately liquid between plants and terminals, and within terminals and pumping, regulating, and metering stations

(d) B31.5, Refrigeration Piping: piping for refrigerants and secondary coolants

(e) B31.8, Gas Transportation and Distribution Piping Systems: piping that transports products that are predominately gas between sources and terminals, including compressor, regulating, and metering stations and gas gathering pipelines

(f) B31.9, Building Services Piping: piping typically found in industrial, institutional, commercial, and public buildings and multiunit residences that do not require the range of sizes, pressures, and temperatures covered by B31.1

(g) B31.12, Hydrogen Piping and Pipelines: piping in gaseous and liquid hydrogen service and pipelines for gaseous hydrogen service

Rules for each Standard provide guidance for a specific task found in one or more B31 Section publications, as follows:

(a) B31E, Seismic Design and Retrofit of Above-Ground Piping Systems, establishes a method for the seismic design of above-ground metallic piping systems in the scope of the ASME B31 Code for Pressure Piping.

(b) B31G, Remaining Strength of Corroded Pipelines, provides a simplified procedure to determine the effect of wall loss due to corrosion or corrosion-like defects on the pressure integrity in pipeline systems.

(c) B31H, Standard Method to Establish Maximum Allowable Design Pressure for Piping Components, provides a standardized method to perform a proof (burst) test for piping components and joints (under development).

(d) B31J, Stress Intensification Factors (*i*-Factors), Flexibility Factors (*k*-Factors), and Their Determination for Metallic Piping Components, provides a standardized method to develop the stress intensification factors (*i*-factors), flexibility factors (*k*-factors), and sustained stress factors used in ASME B31 piping analysis.

(e) B31T, Standard Toughness Requirements for Piping, provides requirements for evaluating the suitability of materials used in piping systems for piping that may be subject to brittle failure due to low-temperature service conditions.

This B31J Standard provides stress intensification factors (*i*-factors) and flexibility factors (*k*-factors), with procedures for their determination for metallic piping components and joints. Stress intensification and flexibility factor equations for common piping components are provided in [Table 1-1](#). The sustained load test procedure can be used to determine more applicable nominal stress multipliers for use in sustained and occasional ASME B31 analyses. Hereafter, in this Introduction and throughout the text of this B31 Standard, where the word *Standard* is used without further identification, it means this B31J Standard.

This Standard sets forth stress intensification factors, flexibility factors, and engineering procedures deemed appropriate for the safe determination of the fatigue and sustained load capacity of metallic piping components or joints in typical services. The procedure cannot foresee all geometries and services possible, and the use of competent engineering judgment may be necessary to extend the procedure to cover unusual geometries and service conditions or to ensure a safe testing environment.

The ASME B31 Committee is organized and operates under procedures of The American Society of Mechanical Engineers, which have been accredited by the American National Standards Institute. The Committee is continuing and keeps all Code Sections and Standards current with new developments in methods, materials, construction, and industrial practice. New editions are published or reaffirmed at intervals of 3 years to 5 years.

It is intended that this edition of the B31J Standard not be retroactive. Unless agreement is specifically made between contracting parties to use another edition, or a regulatory body having jurisdiction imposes the use of another edition, the latest edition issued at least 6 months prior to the original contract date for the piping installation activity in which a component or joint qualified by this Standard is to be used shall be the governing document for the determination of SIFs

and  $k$ -factors. Users of this Standard are cautioned against making use of Standard revisions without assurance that they are acceptable to the proper authorities in the jurisdiction where the piping component is to be installed.

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# Stress Intensification Factors (*i*-Factors), Flexibility Factors (*k*-Factors), and Their Determination for Metallic Piping Components

## 1 General

The ASME B31 Code for Pressure Piping and the ASME Boiler and Pressure Vessel Code, Section III, Nuclear Components, Subsections NC and ND piping rules require the use of stress intensification factors (SIFs or *i*-factors) and flexibility factors (*k*-factors) when checking the adequacy of components and joints (welded and nonwelded) in piping subject to various loads, including cyclic loads, that may produce fatigue failures. As used herein, where the word *Code* is used without specific identification, it means the Code that incorporates or references this Standard. Experimental methods to determine SIFs, flexibility factors, and sustained load factors are provided in the Nonmandatory Appendices. Compiled stress intensification and flexibility factor equations for common piping components are included in [Table 1-1](#); see also [Tables 1-2](#) and [1-3](#) and [Figures 1-1](#) through [1-7](#).

## 2 Definitions

*flexibility factor*: for branch connections and reducers, a ratio that defines the rotation of one end of a zero- or negligible-length element with respect to the opposite end of the same element when equal and opposite moments are applied at each end; for bends, a factor based on an effective length of matching pipe that increases the element flexibility to simulate the effect of bend ovalization that applies over the entire arc length of the bend.

*i*-factor: same as the stress intensification factor.

*k*-factor: same as the flexibility factor.

*pipe stress analyst*: individual responsible for the accuracy of *i*-factors, *k*-factors, and sustained load factors used in the analysis of the piping system.

*piping components*: mechanical elements suitable for joining or assembly into pressure-tight, fluid-containing piping systems. Components include pipe, tubing, fittings, flanges, gaskets, bolting, valves, and devices such as expansion joints, flexible joints, pressure hoses, traps, strainers, in-line portions of instruments, and separators.

*stress intensification factor (SIF)*: a piping component fatigue strength factor that is the ratio of the elastically calculated nominal stress in matching pipe that causes a through-wall crack to appear in a given number of cycles in a straight pipe butt weld to the elastically calculated nominal stress in the matching pipe used with the component that produces a through-wall crack in the same number of cycles in the component or attached pipe.

*verified numerical analysis*: typically, a finite element analysis of a particular piping system component whose results have been verified against existing test data.

## 3 Document Contents

(a) Several different types of tests are often conducted on metallic piping components by the manufacturer or user of a piping component to demonstrate Code adequacy. These tests include burst tests, load-deflection tests (*k*-factor tests), SIF tests (*i*-factor tests), and sustained load tests. Multiple tests may be performed on the same specimen. For example, SIF tests can follow multiple *k*-factor tests, and sustained load tests can often follow SIF tests when the specimen has been suitably repaired.

(b) Typical tests conducted as part of a piping component evaluation include, but are not limited to, the following:

- (1) burst test
- (2) SIF test (in accordance with [Nonmandatory Appendix A](#))
- (3) *k*-factor test (in accordance with [Nonmandatory Appendix B](#))
- (4) sustained load test (in accordance with [Nonmandatory Appendix D](#))

Procedures for the tests in subparas. (2) through (4) above are described in the Nonmandatory Appendices in this Standard.

(c) Stress intensification and flexibility factors for metallic piping components are included in [Table 1-1](#) and were developed using the test procedures in this Standard and numerical methods.

Table 1-1 Flexibility and Stress Intensification Factors

Term	Equation	Sketch
<b>1.1 Pipe Bend or Welding Elbow Meeting ASME B16.9 [Notes (1), (2), (3), (4)]</b>		
Flexibility characteristic, $h$	$TR_1 / R^2$	
Flexibility factor in plane, $k_i$	$1.65/h$	
Flexibility factor out of plane, $k_o$	$1.65/h$	
SIF in plane, $i_i$	$0.9h^{2/3}$	
SIF out of plane, $i_o$	$0.75h^{2/3}$	
SIF torsional, $i_t$	1	
<b>1.2 Closely Spaced Miter Bend, <math>s &lt; R</math> (<math>1 + \tan \theta</math>) [Notes (1), (2), (4)]</b>		
Flexibility characteristic, $h$	$sT \cot \theta / (2R^2)$	
Flexibility factor in plane, $k_i$	$1.52/h^{5/6}$	
Flexibility factor out of plane, $k_o$	$1.52/h^{5/6}$	
SIF in plane, $i_i$	$0.9/h^{2/3}$	
SIF out of plane, $i_o$	$0.9/h^{2/3}$	
SIF torsional, $i_t$	1	
<b>1.3 Widely Spaced Miter Bend, <math>s \geq R</math> (<math>1 + \tan \theta</math>) [Notes (1), (4), (5)]</b>		
Flexibility characteristic, $h$	$T (1 + \cot \theta) / (2R)$	
Flexibility factor in plane, $k_i$	$1.52/h^{5/6}$	
Flexibility factor out of plane, $k_o$	$1.52/h^{5/6}$	
SIF in plane, $i_i$	$0.9/h^{2/3}$	
SIF out of plane, $i_o$	$0.9/h^{2/3}$	
SIF torsional, $i_t$	1	

Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

Term	Equation	Sketch
<b>2.1 Welding Tee Meeting ASME B16.9 [Notes (1), (6), (7)]</b>		
Run in-plane flexibility factor, $k_{ir}$	$0.18(R/T)^{0.8} (d/D)^5$	
Run out-of-plane flexibility factor, $k_{or}$	1	
Run torsional flexibility factor, $k_{tr}$	$0.08(R/T)^{0.91} (d/D)^{5.7}$	
Branch in-plane flexibility factor, $k_{ib}$	$[1.9(d/D) - 4.32(d/D)^2 + 2.7(d/D)^3][R/T]^{0.77} (d/D)^{0.47} (t/T)$	
Branch out-of-plane flexibility factor, $k_{ob}$	$[0.34(d/D) - 0.49(d/D)^2 + 0.18(d/D)^3][R/T]^{1.46} (t/T)$	
Branch torsional flexibility factor, $k_{tb}$	$[1.08(d/D) - 2.44(d/D)^2 + 1.52(d/D)^3][R/T]^{0.77} (d/D)^{1.61} (t/T)$	
Run SIF in plane, $i_{ir}$	$0.98(R/T)^{0.35} (d/D)^{0.72} (t/T)^{-0.52}$	
Run SIF out of plane, $i_{or}$	$0.61(R/T)^{0.29} (d/D)^{1.95} (t/T)^{-0.53}$	
Run SIF torsional, $i_{tr}$	$0.34(R/T)^{2/3} (d/D)(t/T)^{-0.5}$	
Branch SIF in plane, $i_{ib}$	$0.33(R/T)^{2/3} (d/D)^{0.18} (t/T)^{0.7} (d/D)^{0.2}$	
Branch SIF out of plane, $i_{ob}$	$0.42(R/T)^{2/3} (d/D)^{0.37} (t/T)^{0.37}$	
Branch SIF torsional, $i_{tb}$	$0.42(R/T)^{2/3} (d/D)^{1.1} (t/T)^{1.1}$	
<b>2.2 Reinforced Fabricated Tee (When <math>t_p &gt; 1.5T</math>, Use <math>t_p = 1.5T</math>) [Notes (1), (7)]</b>		
Run in-plane flexibility factor, $k_{ir}$	$0.21[R/(T + 0.5t_p)]^{0.97} (t/T)^{-0.65} (d/D)^{6.2}$	
Run out-of-plane flexibility factor, $k_{or}$	1	
Run torsional flexibility factor, $k_{tr}$	$0.12[R/(T + 0.5t_p)]^{1.39} (t/T)^{-0.74} (d/D)^{8.5}$	
Branch in-plane flexibility factor, $k_{ib}$	$[1.29(d/D) - 2.73(d/D)^2 + 1.62(d/D)^3][R/(T + 0.5t_p)]^{1.2} (t/T)^{0.56} (d/D)^{0.33}$	
Branch out-of-plane flexibility factor, $k_{ob}$	$[0.84(d/D) - 1.27(d/D)^2 + 0.5(d/D)^3][R/(T + 0.5t_p)]^{1.69} (t/T)^{0.68} (d/D)^{0.21}$	
Branch torsional flexibility factor, $k_{tb}$	$1.1[R/(T + 0.5t_p)]^{0.5} (d/D)^{5.42}$	
Run SIF in plane, $i_{ir}$	$[R/(T + 0.5t_p)]^{0.45} (d/D)^{0.54} (t/T)^{0.34} \geq 1.5$	
Run SIF out of plane, $i_{or}$	$[1.29(d/D) - 2.87(d/D)^2 + 2.39(d/D)^3][t/T]^{-0.25} [R/(T + 0.5t_p)]^{0.35}$	
Run SIF torsional, $i_{tr}$	$0.36[R/(T + 0.5t_p)]^{2/3} (t/T)^{-0.6} (d/D)^{1.4}$	
Branch SIF in plane, $i_{ib}$	$[3.33(d/D) - 5.49(d/D)^2 + 2.94(d/D)^3][TR^2/3]^{5/3} (t/T)^{0.3}$	
Branch SIF out of plane, $i_{ob}$	$[2.86(d/D) + 2.4(d/D)^2 - 4.34(d/D)^3][TR^2/3]^{5/3} (t/T)^{0.3}$ (when $t/T < 0.85$ , use $t/T = 0.85$ )	
Branch SIF torsional, $i_{tb}$	$0.642(d/D)^2 [TR^2/3]^{5/3} (t/T)^{0.3}$	

Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

Term	Equation	Sketch
<b>2.3 Fabricated Tee [Notes (1), (7), (8)]</b>		
Run in-plane flexibility factor, $k_{ir}$	$1.23(R/T)^{0.47} (t/T)^{-0.47} (d/D)^{5.3}$	
Run out-of-plane flexibility factor, $k_{or}$	1	
Run torsional flexibility factor, $k_{tr}$	$(R/T)^{0.78} (t/T)^{-0.8} (d/D)^{7.8}$	
Branch in-plane flexibility factor, $k_{ib}$	$13.15(d/D) - 6.4(d/D)^2 + 4(d/D)^3 [(R/T)^{0.83} (t/T)^{0.49} (d/D)^{-0.2}]$	
Branch out-of-plane flexibility factor, $k_{ob}$	$12.05(d/D) - 2.94(d/D)^2 + 1.1(d/D)^3 [(R/T)^{1.4} (t/T)^{0.6} (d/D)^{0.12}]$	
Branch torsional flexibility factor, $k_{tb}$	$0.95(R/T)^{0.83} (d/D)^{5.42}$	
Run SIF in plane, $i_{ir}$	$1.2(d/D)^{0.5} (R/T)^{0.4} (t/T)^{-0.35} \geq 1.5$	
Run SIF out of plane, $i_{or}$	$[(d/D) - 2.7(d/D)^2 + 2.62(d/D)^3] (R/T)^{0.43} (t/T)^{-0.7}$ (when $d/D < 0.5$ , use $d/D = 0.5$ ; when $t/T < 0.5$ , use $t/T = 0.5$ )	
Run SIF torsional, $i_{tr}$	$1.2 (R/T)^{0.46} (t/T)^{-0.45} (d/D)^{1.37}$ (when $t/T < 0.15$ , use $t/T = 0.15$ )	
Branch SIF in plane, $i_{ib}$	$[0.038 + 1.45(d/D) - 2.39(d/D)^2 + 1.34(d/D)^3] (R/T)^{0.76} (t/T)^{0.74}$ (when $t/T < 1$ , use $t/T = 1$ )	
Branch SIF out of plane, $i_{ob}$	$[0.038 + 2(d/D) + 2(d/D)^2 - 3.1(d/D)^3] (R/T)^{2/3} (t/T)$ (when $t/T < 0.85$ , use $t/T = 0.85$ )	
Branch SIF torsional, $i_{tb}$	$0.45(R/T)^{0.8} (t/T)^{0.29} (d/D)^2$	

Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

Term	Equation	Sketch
<b>2.4 Extruded Outlet With <math>r_x \geq 0.05d_o</math> and <math>T &lt; T_c &lt; 1.5T</math> [Notes (1), (7), (9)]</b>		
Run in-plane flexibility factor, $k_{ir}$	$0.18(R/T)^{0.8} (d/D)^5$	
Run out-of-plane flexibility factor, $k_{or}$	1	
Run torsional flexibility factor, $k_{tr}$	$0.08(R/T)^{0.91} (d/D)^{5.7}$	
Branch in-plane flexibility factor, $k_{ib}$	$[1.9(d/D) - 4.32(d/D)^2 + 2.7(d/D)^3](R/T)^{0.77} (d/D)^{0.47} (t/T)$	
Branch out-of-plane flexibility factor, $k_{ob}$	$[0.34(d/D) - 0.49(d/D)^2 + 0.18(d/D)^3](R/T)^{1.46} (t/T)$	
Branch torsional flexibility factor, $k_{tb}$	$[1.08(d/D) - 2.44(d/D)^2 + 1.52(d/D)^3](R/T)^{0.77} (d/D)^{1.79} (t/T)$	
Run SIF in plane, $i_{ir}$	$1.45(1 + r_x/R)^{2/3} (R/T)^{0.35} (d/D)^{0.72} (t/T)^{0.52}$	
Run SIF out of plane, $i_{or}$	$0.58(1 + r_x/R)^{-2/3} (R/T)^{2/3} (d/D)^{2.69}$	
Run SIF torsional, $i_{tr}$	$0.55(1 + r_x/R)^{-2/3} (R/T)^{2/3} (d/D)(t/T)^{-0.5}$	
Branch SIF in plane, $i_{ib}$	$0.56(1 + r_x/R)^{-2/3} (R/T)^{2/3} (d/D)^{0.68}$	
Branch SIF out of plane, $i_{ob}$	$0.85(1 + r_x/R)^{-2/3} (R/T)^{2/3} (d/D)^{0.5}$	
Branch SIF torsional, $i_{tb}$	$0.71(1 + r_x/R)^{-2/3} (R/T)^{2/3} (d/D)^2$	
<b>2.5 Welded-in Contour Insert (When <math>r_x</math> Is Not Provided, Use <math>r_x = 0</math>) [Notes (1), (6), (7)]</b>		
Run in-plane flexibility factor, $k_{ir}$	$0.18(R/T)^{0.84} (d/D)^5$	
Run out-of-plane flexibility factor, $k_{or}$	1	
Run torsional flexibility factor, $k_{tr}$	$0.1(R/T)^{0.91} (d/D)^{5.7}$	
Branch in-plane flexibility factor, $k_{ib}$	$[2.36(d/D) - 5.33(d/D)^2 + 3.33(d/D)^3](R/T)^{0.77} (d/D)^{0.47} (t/T)$	
Branch out-of-plane flexibility factor, $k_{ob}$	$(1 + r_x/R)[0.67(d/D) - 0.97(d/D)^2 + 0.36(d/D)^3](R/T)^{1.46} (t/T)$	
Branch torsional flexibility factor, $k_{tb}$	$[1.05(d/D) - 2.36(d/D)^2 + 1.49(d/D)^3](R/T)^{0.77} (d/D)^{1.61} (t/T)$	
Run SIF in plane, $i_{ir}$	$(R/T)^{0.35} (d/D)^{0.72} (t/T)^{-0.52}$	
Run SIF out of plane, $i_{or}$	$0.72(R/T)^{0.29} (d/D)^{1.95} (t/T)^{-0.53}$	
Run SIF torsional, $i_{tr}$	$0.36(R/T)^{2/3} (d/D)(t/T)^{-0.5}$	
Branch SIF in plane, $i_{ib}$	$0.35(R/T)^{2/3} (d/D)^{0.18} (t/T)^{0.7}$	
Branch SIF out of plane, $i_{ob}$	$0.48(R/T)^{2/3} (d/D)^{0.37} (t/T)^{0.37}$	
Branch SIF torsional, $i_{tb}$	$0.44(R/T)^{2/3} (d/D)^{1.1} (t/T)^{1.1}$	



Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

Term	Equation	Sketch
2.6 Integrally Reinforced Branch Welded-on Fittings [Notes (1), (7), (10)]		
Run in-plane flexibility factor, $k_{ir}$	$0.5(R/T)^{0.5} (d/D)^5$	
Run out-of-plane flexibility factor, $k_{or}$	1	
Run torsional flexibility factor, $k_{tr}$	$0.1(R/T)(d/D)^{5.7}$	
Branch in-plane flexibility factor, $k_{ib}$	$[0.55(d/D) - 1.13(d/D)^2 + 0.69(d/D)^3](R/T)(t/T)$	
Branch out-of-plane flexibility factor, $k_{ob}$	$[1.03(d/D) - 1.55(d/D)^2 + 0.59(d/D)^3](R/T)^{1.4} (t/T)(d/D)^{0.33}$	
Branch torsional flexibility factor, $k_{tb}$	$[0.37(d/D) - 0.75(d/D)^2 + 0.46(d/D)^3](R/T)(t/T)(d/D)^{1.2}$	
Run SIF in plane, $i_{ir}$	$(R/T)^{0.43} (d/D)^{0.2} \geq 1.5$	
Run SIF out of plane, $i_{or}$	$[0.02 + 0.88(d/D) - 2.56(d/D)^2 + 2.58(d/D)^3](R/T)^{0.43}$	
Run SIF torsional, $i_{tr}$	$1.3(R/T)^{0.45} (d/D)^{1.37}$	
Branch SIF in plane, $i_{ib}$	$[0.08 + 1.28(d/D) - 2.35(d/D)^2 + 1.45(d/D)^3](R/T)^{0.81} (t/T)(r/r_p)$	
Branch SIF out of plane, $i_{ob}$	$[1.83(d/D) - 1.07(d/D)^3](R/T)^{0.82} (t/T)(r/r_p)^{1.18}$	
Branch SIF torsional, $i_{tb}$	$0.77(R/T)^{2/3} (t/T) (d/D)^2 (r/r_p)$	
3.1 Concentric or Eccentric Reducer Meeting ASME B16.9 [Note (11)]		
SIF in plane, $i_i$	$0.6 + 0.003(\alpha T_2 / T_1)^{0.8} (D_2 / T_2)^{0.25} (D_2 / r_2)$	
SIF out of plane, $i_o$	$0.6 + 0.003(\alpha T_2 / T_1)^{0.8} (D_2 / T_2)^{0.25} (D_2 / r_2)$	
SIF torsional, $i_t$	$0.3 + 0.0015(\alpha T_2 / T_1)^{0.8} (D_2 / T_2)^{0.25} (D_2 / r_2)$	

Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

Term	Equation	Sketch
4.1 Butt Weld, $T \geq 6$ mm (0.237 in.), $\delta_{\max} \leq 1.5$ mm (1/16 in.), and $\delta_{\text{avg}}/T \leq 0.13$ [Note (12)]		
SIF in plane, $i_i$	1.0	
SIF out of plane, $i_o$	1.0	
SIF torsional, $i_t$	1.0	
4.2 Butt Weld, $T \geq 6$ mm (0.237 in.), $\delta_{\max} \leq 3$ mm (1/8 in.), and $\delta_{\text{avg}}/T = \text{Any Value OR } T < 6$ mm (0.237 in.), $\delta_{\max} \leq 1.5$ mm (1/16 in.), and $\delta_{\text{avg}}/T \leq 0.33$ [Note (12)]		
SIF in plane, $i_i$	1.9 max. or $0.9 + 2.7(\delta_{\text{avg}}/T)$ but not less than 1.0	
SIF out of plane, $i_o$	1.9 max. or $0.9 + 2.7(\delta_{\text{avg}}/T)$ but not less than 1.0	
SIF torsional, $i_t$	$0.45 + 1.35(\delta_{\text{avg}}/T)$ but not less than 1.0	
4.3 Fillet-Welded Joint or Socket-Welded Flange Meeting ASME B16.5 or Socket-Welded Fitting Meeting ASME B16.11 [Note (13)]		
SIF in plane, $i_i$	1.3	
SIF out of plane, $i_o$	1.3	
SIF torsional, $i_t$	1.3	
4.4 Tapered Transition in Accordance With Applicable Code Sections and ASME B16.25		
SIF in plane, $i_i$	$1.9 \text{ max. or } 1.3 + 0.0036(D_o/T) + 3.6(\delta/T)$	
SIF out of plane, $i_o$	$1.9 \text{ max. or } 1.3 + 0.0036(D_o/T) + 3.6(\delta/T)$	
SIF torsional, $i_t$	1.3	
4.5 Weld Neck Flange		
SIF in plane, $i_i$	1.0	...
SIF out of plane, $i_o$	1.0	
SIF torsional, $i_t$	1.0	

Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

Term	Equation	Sketch
<b>4.6 Single Welded Slip-on Flange</b>		
SIF in plane, $i_i$	1.3	...
SIF out of plane, $i_o$	1.3	
SIF torsional, $i_t$	1.3	
<b>4.7 Double Welded Slip-on Flange</b>		
SIF in plane, $i_i$	1.2	...
SIF out of plane, $i_o$	1.2	
SIF torsional, $i_t$	1.2	
<b>4.8 Lap Joint Flange (With ASME B16.9 Lap Joint Stub)</b>		
SIF in plane, $i_i$	1.6	...
SIF out of plane, $i_o$	1.6	
SIF torsional, $i_t$	1.6	
<b>5.1 Threaded Pipe Joint or Threaded Flange in Accordance With Acceptable Code Detail</b>		
SIF in plane, $i_i$	2.3	...
SIF out of plane, $i_o$	2.3	
SIF torsional, $i_t$	2.3	

## GENERAL NOTES:

(a) The following symbols are used in this table:

- $A_p$  = metal area of pipe cross section, in.<sup>2</sup> (mm<sup>2</sup>)  
 $B$  = length of miter segment at crotch, in. (mm)  
 $b$  = branch subscript corresponding to Leg 3 in Figure 1-1  
 $c$  = factor for rigid ends adjacent to bends, miters, and branch connections in Sketches 1.1, 1.2, and 2.1 through 2.6  
 $C_x$  = minimum socket weld leg length, in. (mm)  
 $D$  = mean diameter of matching pipe found from  $(D_o - T)$ , in. (mm); for Sketches 2.1 through 2.6, the mean diameter of the matching run pipe  
 $d$  = mean diameter of matching branch pipe found from  $(d_o - t)$ , in. (mm)  
 $d'$  = effective branch diameter used with Figure 1-3, illustrations (a), (b), and (c), in. (mm)  
 $D_1, D_2$  = outside diameter of matching pipe at large and small ends of reducer, respectively, in. (mm)  
 $D_i$  = inside diameter of matching run pipe found from  $(D_o - 2T)$ , in. (mm)  
 $d_i$  = inside diameter of matching branch pipe found from  $(d_o - 2t)$ , in. (mm)  
 $D_o$  = outside diameter of matching pipe, in. (mm)  
 $d_o$  = outside diameter of the matching branch pipe, in. (mm); for Sketches 2.1 through 2.6, the outside diameter of the matching run pipe  
 $E$  = modulus of elasticity, psi (kPa)  
 $I$  = stress intensification factor (SIF)  
 $I_w$  = matching branch and run pipe moment of inertia used in Table 1-2, in.<sup>4</sup> (mm<sup>4</sup>)  
 $k$  = flexibility factor with respect to the plane and component indicated (see Nonmandatory Appendix B, Ref. [1] for a more detailed definition of flexibility factor as it applies to straight and curved pipe and branch connections)  
 $L_1$  = length of taper or thicker branch section in Figure 1-2, in. (mm)  
 $L_2$  = length of the cylindrical portion at the small end of the reducer in Sketch 3.1, in. (mm)  
 $M$  = moment on branch or run legs shown in Figure 1-1, in.-lb (N-mm)

Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

## GENERAL NOTES (Cont'd):

- $N_c$  = number of flanges or other rigid components adjacent to the run pipe end of a branch connection (1 or 2)  
 $P$  = gage pressure, psi (MPa)  
 $R$  = mean radius of matching pipe found from  $(D_o - T)/2$ , in. (mm)  
 $r$  = mean radius of matching branch pipe found from  $(d_o - t)/2$  for Sketches 2.1 through 2.6, in. (mm)  
 $R_1$  = bend radius of welding elbow or pipe bend, in. (mm)  
 $r_2$  = radii used with Figure 1-3 and in Sketch 3.1, in. (mm)  
 $r_i$  = inside radius used with Figure 1-3, in. (mm)  
 $r_p$  = radius to outside edge of fitting for Sketches 2.3 and 2.6 measured in longitudinal plane, in. (mm)  
 $r_x$  = external crotch radius of welding tee in accordance with ASME B16.9, extruded outlet and welded-in contour insert (Sketches 2.1, 2.4, and 2.5), measured in the plane containing the centerline axes of the run and branch, in. (mm)  
 $s$  = miter spacing at centerline, in. (mm)  
 SIF = stress intensification factor  
 $T$  = nominal wall thickness of matching pipe or the average wall thickness of the fitting, if available, for welding elbows in Sketch 1.1; nominal wall thickness of pipe or the average wall thickness of the fitting, if available, for pipe bends in Sketch 1.1; nominal wall thickness of pipe for miter bends in Sketches 1.2 and 1.3; nominal wall thickness of matching run pipe for tees in Sketches 2.1 and 2.4; nominal wall thickness of run pipe for tees in Sketches 2.2, 2.3, 2.5, and 2.6; nominal wall thickness of pipe for weld joints in Sketches 4.1 through 4.4  
 $t$  = nominal wall thickness of matching branch pipe, in. (mm)  
 $t'$  = effective branch thickness used with Figure 1-3, illustrations (a), (b), and (c), in. (mm)  
 $T_1$ ,  $T_2$  = nominal wall thickness of matching pipe at the large and small ends of the reducer, respectively, in. (mm)  
 $T_c$  = crotch thickness in Sketches 2.1, 2.4, and 2.5 measured at the center of the crotch and in the plane shown, in. (mm)  
 $t_n$  = local branch pipe thickness used with Figure 1-3, illustrations (a) and (b), in. (mm)  
 $t_p$  = reinforcement pad or saddle thickness, in. (mm)  
 $y$  = large end of taper used with Figure 1-3, illustration (c), and found from  $(d_o \tan \theta_n)$ , in. (mm)  
 $Z$  = section modulus of pipe, in.<sup>3</sup> (mm<sup>3</sup>) [see Note (7)]  
 $Z_b$  = section modulus of matching branch pipe, in.<sup>3</sup> (mm<sup>3</sup>) [see Note (7)]  
 $\alpha$  = reducer cone angle, deg  
 $\delta$  = mismatch, in. (mm)  
 $\theta$  = one-half angle between adjacent miter axes, deg  
 $\theta_n$  = angle used with Figure 1-3, illustration (c), deg  
 $\theta_{lb}$ ,  $\theta_{ob}$ ,  $\theta_{lb}$ ,  $\theta_{lb}$ ,  $\theta_{lb}$ ,  $\theta_{lb}$ ,  $\theta_{lb}$ ,  $\theta_{lb}$  = rotations at branch or run legs shown in Figure 1-1, rad

(b) Stress intensification and flexibility factor data in this table shall be used in the absence of more directly applicable data. Their validity has been demonstrated for  $D/T \leq 100$ . Other limits are provided as needed below.

(1) Flexibility and stress intensification factors shall not be less than 1.0.

(2) Stress intensification factors may be used without flexibility factors.

(3) Stress intensification and flexibility factors in this table have been developed from fatigue tests of representative commercially available matching product forms with assemblies manufactured from ductile ferrous materials and from numerical analysis using finite elements. Caution should be exercised when applying these rules for certain nonferrous materials (e.g., copper and aluminum alloys) for other than low-cycle applications.

(4) Corrugated straight pipe or corrugated or creased bends should be designed using the principles found in ASME B31.3 Nonmandatory Appendix X, standards from the Expansion Joint Manufacturers Association, or similar standards.

(c) The highest in-plane or out-of-plane stress intensification factor shall be used when only a single stress intensification factor is needed. Flexibility factors should always be used with the orientation specified. For Sketches 3.1 through 5.1, the in-plane and out-of-plane orientations must be orthogonal to each other and to the pipe axis.

Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)

## GENERAL NOTES (Cont'd):

- (d) Where sustained stress or moment factors are required by the applicable Code (e.g., ASME B31.1, ASME B31.3), and in lieu of more applicable data, for components of Sketches 1.1, 1.3, 2.1, and 2.2, the directional sustained stress or moment multiplier can be taken as the component stress intensification factor. For all components of Sketch 1.2, the directional sustained stress or moment multiplier can be conservatively taken as the smaller of (1) and either (2) or (3) below
- (1) 0.75 times the applicable stress intensification factor
  - (2)  $(t/T)$  times the square root of the applicable stress intensification factor when  $t/T > 1$
  - (3) the square root of the applicable stress intensification factor when  $t/T \leq 1$
- The sustained stress or moment factors should always be used with the section modulus of the matching pipe and should not be less than 1.0.

## NOTES:

- (1) Stress intensification and flexibility factors apply over the effective arc length (shown by heavy centerlines in the sketches) for curved and miter bends and may be read from Figure 1-4. Stress intensification factors for Sketches 2.1 through 2.6 apply to the intersection point for Legs 1 and 2 as shown in Figures 1-1 and 1-6. Stress intensification factors apply to the intersection point for branch Leg 3 in Figures 1-1 and 1-6 when  $d_o/D_o > 0.5$ , and to the branch centerline at the surface of the run pipe when  $d_o/D_o \leq 0.5$ . Flexibility factors for Sketches 2.1 through 2.6 shall be applied as shown in Figures 1.1 and 1.6 for all  $d_o/D_o$ .
- (2) Where flanges or other rigid components are attached to one or both ends, the in-plane and out-of-plane values of  $k$  and  $i$  shall be multiplied by the factor  $c$  from Figure 1-5, entering with the computed  $h$ .
- (3) When the bend angle is 90 deg and the thickness of the bend is equal to the thickness of the matching pipe, the flexibility factors  $k_i$  and  $k_o$  may be found from 1.3/ $h$  and adjusted by the factor  $c$  from Figure 1-5 where applicable.
- (4) In large-diameter thin-wall elbows and bends, pressure can affect the magnitudes of  $k$  and  $i$ . To correct values from this table, divide  $k$  by

$$\left[ 1 + 6 \left( \frac{P}{E} \right) \left( \frac{R}{T} \right)^{7/3} \left( \frac{R_i}{R} \right)^{1/3} \right]$$

and divide  $i$  by

$$\left[ 1 + 3.25 \left( \frac{P}{E} \right) \left( \frac{R}{T} \right)^{5/2} \left( \frac{R_i}{R} \right)^{2/3} \right]$$

For consistency, use kPa and mm for SI and psi and in. for U.S. customary notation. Stress intensification factors shall be used with the section modulus of the matching pipe or the section modulus of the bend, whichever is smaller.

- (5) Sketch 1.3 includes single miter joints.
- (6) If  $r_x \geq (1/8)(d_o)$  and  $T_c \geq 1.5T$ , the factors  $k$  and  $i$  may be divided by 1.26.

- (7) The flexibility and stress intensification factors apply only if the following conditions are satisfied:

- (a) The branch pipe axis is normal to within 5 deg of the surface of the run pipe unless otherwise noted.
- (b)  $R/T \leq 50$
- (c)  $d/D \leq 1$
- (d)  $r/t \leq 50$
- (e) The matching run pipe thickness,  $T$ , and diameter,  $D$ , are maintained for at least two run pipe diameters on each side of the branch centerline.
- (f) For Sketches 2.1, 2.4, and 2.5,  $t/T \leq 1.2$  and  $T_c/T > 1.1$ .

When a Table 1-2 flexibility factor is less than or equal to 1.0, the stiffness associated with that flexibility factor shall be rigid. Flexibility factors  $k_{ib}$ ,  $k_{ob}$ , and  $k_{ib}$  in Sketches 2.1 through 2.6 shall be multiplied by the factor  $c$  from Table 1-3 when flanges or other rigid components are adjacent to one or more of the run pipe ends. A flange or other rigid component is adjacent to the run pipe end when the length of any straight run pipe between the branch and the flange or rigid component is less than  $0.1D^{1.4}/r^{0.4}$ . Stress intensification factors  $k_{ib}$ ,  $k_{ob}$ ,  $k_{ib}$ ,  $k_{or}$ , and  $k_{or}$  in Sketches 2.1, 2.2, 2.4, 2.5, and 2.6 shall not be greater than the corresponding stress intensification and flexibility factors for Sketch 2.3 and Figure 1-3, illustration (d), calculated using matching branch and run pipe dimensions and  $r_2 = 0$ . Stress intensification factors  $k_{ib}$ ,  $k_{ob}$ ,  $k_{ib}$ ,  $k_{or}$ , and  $k_{or}$  in Sketches 2.2, 2.3, 2.4, 2.5, and 2.6 shall not be less than the corresponding stress intensification and flexibility factors for Sketch 2.1 calculated using  $T_c = 1.1T$ .

**Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)**

NOTES (Cont'd):

If  $i_{ob} < i_{ib}$  for any of Sketches 2.1 through 2.6, then use  $i_{ob} = i_{ib}$ . If  $i_{ir} < i_{or}$ , then use  $i_{ir} = i_{or}$ . Stress intensification factors  $i_{ib}$ ,  $i_{ob}$ ,  $i_{ir}$ ,  $i_{or}$ , and  $i_{tr}$  from this table can be used for Sketches 2.2, 2.3, 2.5, and 2.6 when the branch pipe axis is in the same plane as the run pipe axis and is normal to within 45 deg of the surface of the run pipe, provided  $D/T < 50$  and  $d/D \leq 0.6$ ; in the absence of more applicable data,  $i_{ib}$  can be taken equal to  $i_{ob}$ . The stress intensification factor  $i_{ob}$  in Sketch 2.3 shall be multiplied by the larger of  $[0.75(t/T)^2 - 0.89(t/T)^2 + 0.18](D/T)^{0.34}$ , or 1.0 when  $t/T \leq 0.85$ ,  $d/D < 1$ , and  $D/T \geq 25$ .

The stress intensification factor  $i_{ob}$  in Sketch 2.2 shall be multiplied by the larger of  $[1.07(t/T)^2 - 1.08(t/T)^2 + 0.026](D/T)^{0.34}$ , or 1.0 when  $t/T \leq 0.85$ ,  $d/D < 1$ , and  $D/T \geq 25$ .

The designer must be satisfied that the branch connection pressure rating is greater than or equal to that of the matching run pipe. Branch connection stress intensification factors shall be used with the section modulus of the matching pipe. The section modulus shall be calculated using the following equation for the run:

$$Z = \left( \frac{\pi}{32} \right) \left( \frac{D_o^4 - D_i^4}{D_o} \right)$$

and by the following equation for the branch:

$$Z_b = \left( \frac{\pi}{32} \right) \left( \frac{d_o^4 - d_i^4}{d_o} \right)$$

(8) The in-plane, out-of-plane, and torsional stress intensification factors for both the branch and the run may be multiplied by the factor 0.7 for the geometries shown in Figure 1-3 when the outer radius  $r_2$  is provided and is not less than the smallest of  $T/2$ ,  $t/2$ ,  $(r_p - r_i)/2$ , or  $(t+y)/2$ . For Figure 1-3, illustrations (a), (b), and (c), the following hold:

(a) Flexibility and stress intensification factors shall be calculated by replacing the parameters  $t/T$  with  $t'/T$  and  $d/D$  with  $d'/D$ .

(b) Stress intensification factors  $i_{ib}$ ,  $i_{ob}$ , and  $i_{tr}$  and flexibility factors  $k_{ib}$ ,  $k_{ob}$ , and  $k_{tr}$  shall also be multiplied by  $(t/t')(d/d')^2$ .

(c) Calculate  $t'$  as follows:

(-1) For Figure 1-3, illustrations (a) and (b)

$$\begin{aligned} t' &= t_n \text{ if } L_1 \geq 0.5(2r_p)^{1/2} \\ &= t \text{ if } L_1 < 0.5(2r_p)^{1/2} \end{aligned}$$

(-2) For Figure 1-3, illustration (c)

$$\begin{aligned} t' &= t + (2/3)y \text{ if } \theta_n \leq 30 \text{ deg} \\ &= t + 0.38SL_1 \text{ if } \theta_n > 30 \text{ deg} \end{aligned}$$

(d) Calculate  $d'$  as follows:

$$d' = d - t + t'$$

(e) Stress intensification factors  $i_{ib}$ ,  $i_{ob}$ ,  $i_{ir}$ , and  $i_{tr}$  shall not be less than 1.5.

(9) When  $r_x$  is not provided, use  $r_x = 0.05d_o$ . If  $r_x/r > 1$  use  $r_x/r = 1$ .

(10) When  $r/r_p$  is not available, a value of 0.85 may be used. If  $r/r_p < 0.6$ , then use  $r/r_p = 0.6$ . For size-on-size branch connections when  $D/T < 40$ ,  $i_{ob}$  may be multiplied by 0.75. When the weld sizes and actual dimensions of the fitting are available,  $r_p$  can be taken as the distance along the surface of the run pipe from the branch centerline to the toe of the attachment fillet weld in the longitudinal plane. When  $d/D > 0.8$ , the geometry of commercially available fittings varies considerably from manufacturer to manufacturer. More applicable data from the manufacturer should be used when available. Results from tests where  $D/T < 40$  should not be extrapolated to branch connections where  $D/T > 40$ .

(11) The flexibility and stress intensification factors apply only if the following conditions are satisfied:

(a)  $5 \text{ deg} < \alpha < 60 \text{ deg}$

(b)  $5 < D_2/T_2 < 80$

(c) The wall thickness is not less than  $T_1$  throughout the body of the reducer, except in and immediately adjacent to the cylindrical portion on the small end, where the thickness shall not be less than  $T_2$ .

(d)  $0.08 < r_2/D_2 < 0.7$

(e)  $1 < T_1/T_2 < 2.12$

(f) If  $L_2 < (D_2T_2)^{0.5}$ , the stress intensification factors should be multiplied by  $[2 - L_2/(D_2T_2)^{0.5}]$ .

**Table 1-1 Flexibility and Stress Intensification Factors (Cont'd)****NOTES (Cont'd):**

The maximum stress intensification factor need not be greater than 2.0 but shall in no case be less than 1.0. Reducers with  $D_2/T_2 \leq 55$  can be modeled as a step change in diameter and thickness from  $D_2, T_2$  to  $D_1, T_1$  at the middle of the reducer, or with any more applicable geometry. When  $D_2/T_2 > 55$ , consideration should be given to adding flexibility to the beam model to more accurately represent the stiffness of the reducer. For eccentric reducers, the dimensions shown in Sketch 3.1 are to be taken at the location on the circumference where  $\alpha$  is the maximum. When  $r_2$  is not given, use  $r_2 = 0.1D_1$ . When  $L_2$  is not given, use  $L_2 = 0.1D_2$ . When  $\alpha$  is not given, use  $\alpha$  equal to the smaller of  $60(D_1/D_2 - 1)$  or 60.

(12) The stress intensification factors apply to girth butt welds between two items for which the wall thicknesses are between  $0.875T$  and  $1.10T$  for an axial distance of  $(D_o T)^{1/2}$ .  $D_o$  and  $T$  are the nominal outside diameter and nominal wall thickness, respectively.  $\delta_{avg}$  is the average mismatch or offset.

(13) For welds to socket-welded flanges and fittings, the stress intensification factor is based on the following:

(a) the assumption that the pipe and fitting are matched in accordance with ASME B16.11 or that socket-welded pipe, flanges, and other fittings greater than NPS 2 meet the fabrication requirements of the applicable Code

(b) the weld is made as shown in Sketch 4.3

(c) the pipe wall thickness is greater than the lesser of schedule 40 or standard weight

(d) the weld size  $C_x$  is in accordance with the applicable Code. For pipe whose wall thickness is thinner than the lesser of schedule 40 or standard weight, the stress intensification factor for all directions shall be equal to 2.1 unless otherwise justified. Blending the toe of the fillet weld with no undercut smoothly into the pipe wall, as shown in Figure 1-7, illustrations (b) and (d), has been shown to improve the fatigue performance of the weld. Large-diameter socket-welded and slip-on flanges with welds smaller than those required by the applicable Code may induce stresses not considered by the stress intensification factors.



**Table 1-2 Moment-Rotation Relationships for Sketches 2.1 Through 2.6 of Table 1-1**

Moment (Figure 1-1)	Flexibility Factor, $k$	Stiffness, in.-lb/rad (N-mm/rad)	Stiffness, in.-lb/rad (N-mm/rad)
$M_{i3}$ (Leg 3)	$k_{ib}$	$M_{ib} / \theta_{ib}$	$(E)(I_b) / (k_{ib} d)$
$M_{o3}$ (Leg 3)	$k_{ob}$	$M_{ob} / \theta_{ob}$	$(E)(I_b) / (k_{ob} d)$
$M_{t3}$ (Leg 3)	$k_{tb}$	$M_{tb} / \theta_{tb}$	$(E)(I_b) / (k_{tb} d)$
$M_{i1,2}$ (Legs 1, 2)	$k_{ir}$	$M_{ir} / \theta_{ir}$	$(E)(I_r) / (k_{ir} D)$
$M_{o1,2}$ (Legs 1, 2)	$k_{or}$	$M_{or} / \theta_{or}$	$(E)(I_r) / (k_{or} D)$
$M_{t1,2}$ (Legs 1, 2)	$k_{tr}$	$M_{tr} / \theta_{tr}$	$(E)(I_r) / (k_{tr} D)$

GENERAL NOTE: The moment-rotation relationships in this table are developed by independently applying moments to the respective run or branch leg. Simultaneous run and branch moment-rotation interaction must be accommodated by the model.

**Table 1-3 Flanged End Correction Coefficients for Sketches 2.1 through 2.6 of Table 1-1**

Flexibility Factor	Flexibility Factor Multiplier, $c$
$k_{ib}$	$1 - 0.032 N_c^{1.345} (D/T)^{0.431} (d/D)^{0.903}$
$k_{ob}$	$1 - 0.07 N_c^{0.61} (D/T)^{0.44} (d/D)^{0.339}$
$k_{tb}$	$1 - 0.003 N_c^{3.962} (D/T)^{0.548} (d/D)^{0.693}$

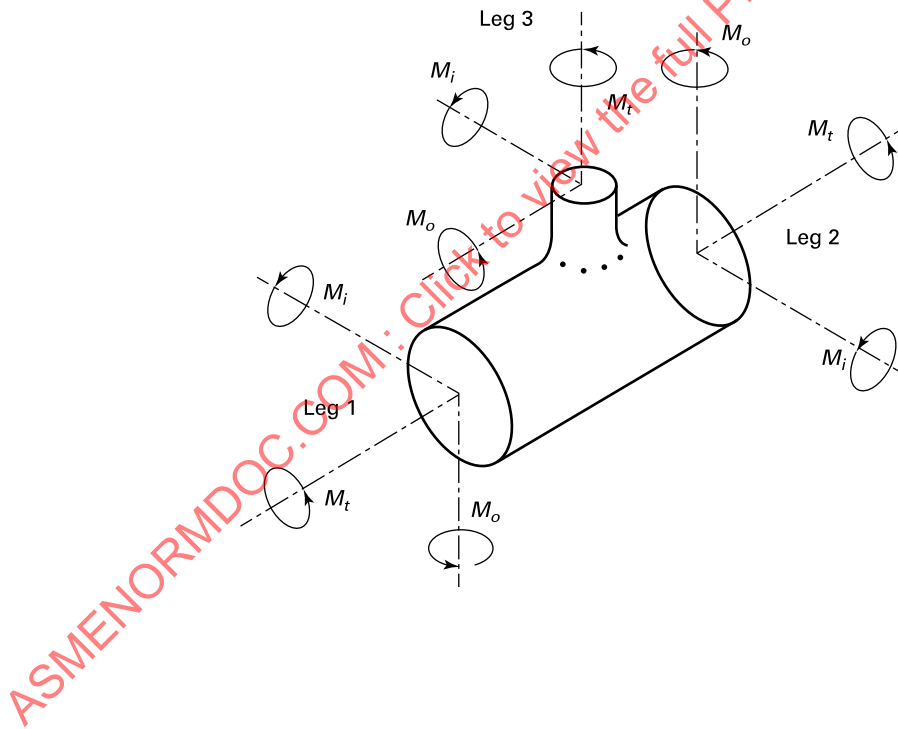
**Figure 1-1 Orientations for Sketches 2.1 Through 2.6 of Table 1-1**



Figure 1-2 Orientations for Bends

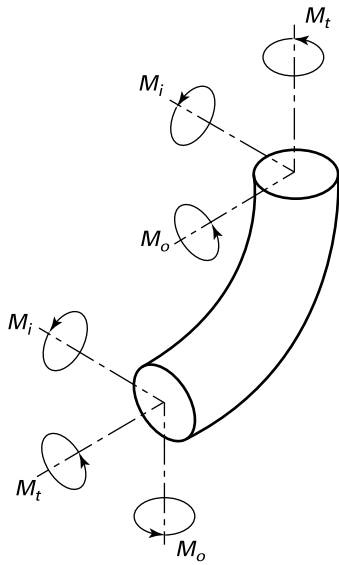
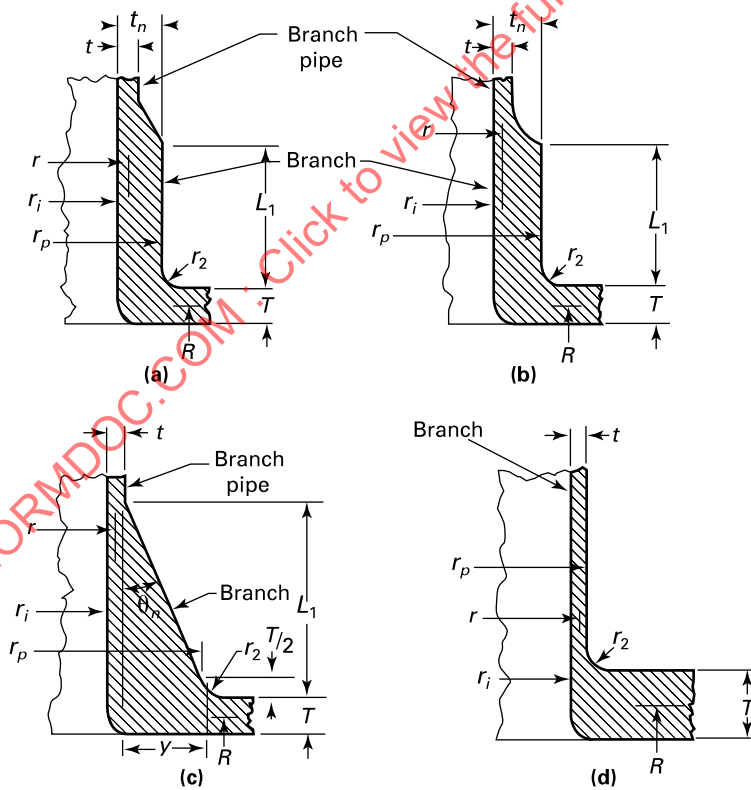
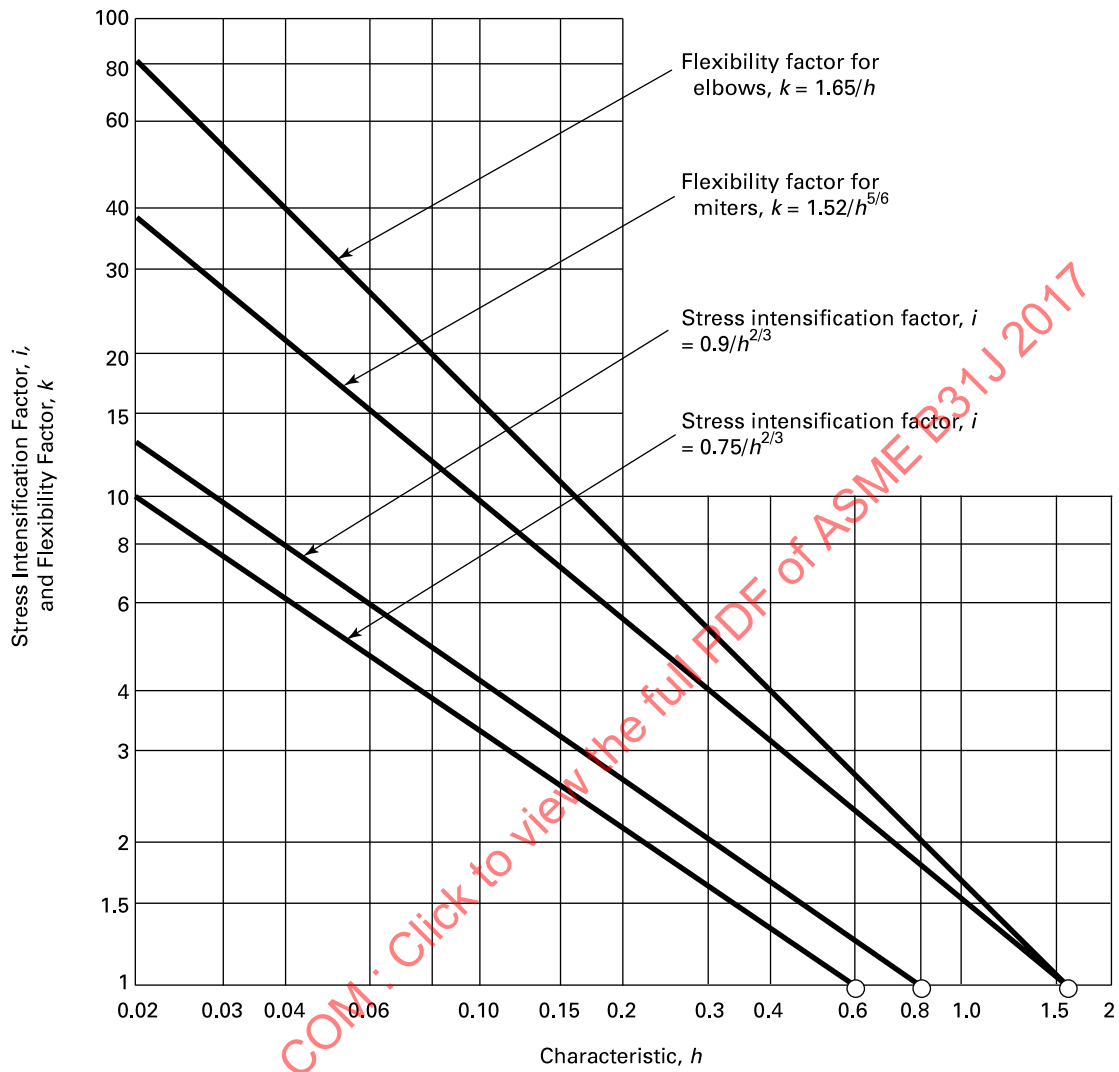
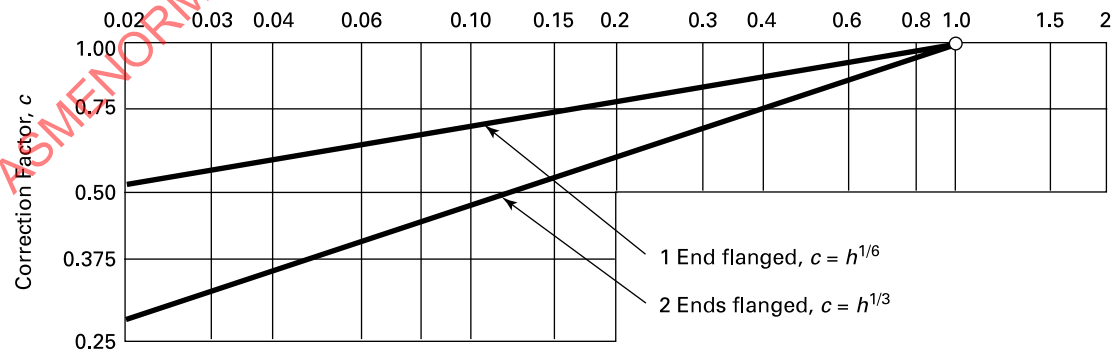
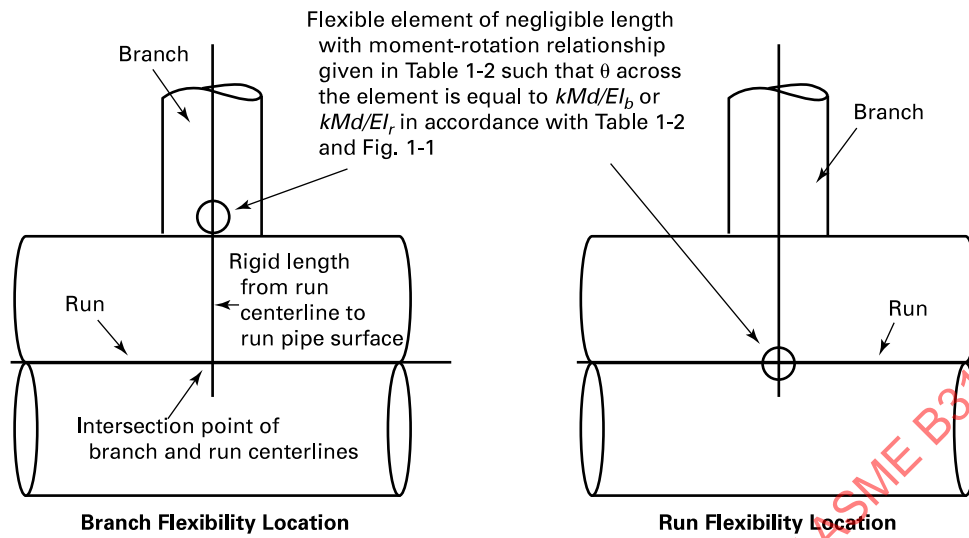


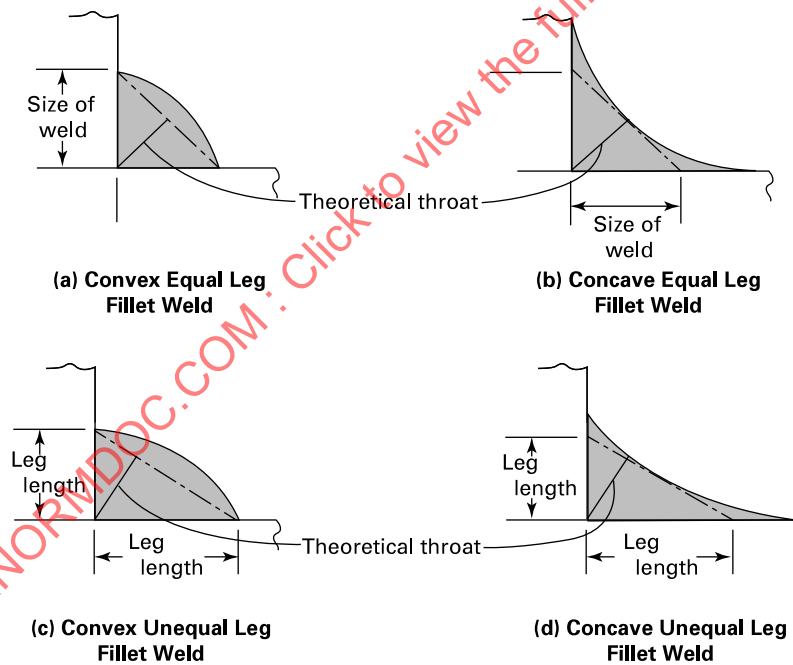
Figure 1-3 Branch Dimensions



**Figure 1-4 Flexibility and Stress Intensification Factors for Bends and Miters****Figure 1-5 Flanged End Corrections for Bends and Miters**

**Figure 1-6 Flexibility Element Locations**

GENERAL NOTE: See Figure 1-1 for flexibility orientations.

**Figure 1-7 Fillet Weld Contours**

## NONMANDATORY APPENDIX A

### Stress Intensification Factor (SIF) Test Procedure

#### A-1 General

##### A-1.1 Test Equipment

A schematic of a test arrangement is given in [Figure A-1.1-1](#).

(a) The fatigue test machine framework must be sufficiently rigid to prevent sliding, ratcheted movement, or any unwanted displacements or rotations at the fixed end of the assembly. An unwanted rotation is defined by the movements with respect to the mounting flange as shown in [para. B-4.3](#) in [Nonmandatory Appendix B](#).

(b) The component to be tested shall be mounted no closer than two pipe diameters to the fixed end of the test assembly unless the component is bolted or flanged, in which case the component can be attached directly to the framework if a condition typically employed in service is replicated. The mounting configuration is an important aspect of producing an accurate result. Failures must occur at the part and not at the attachment

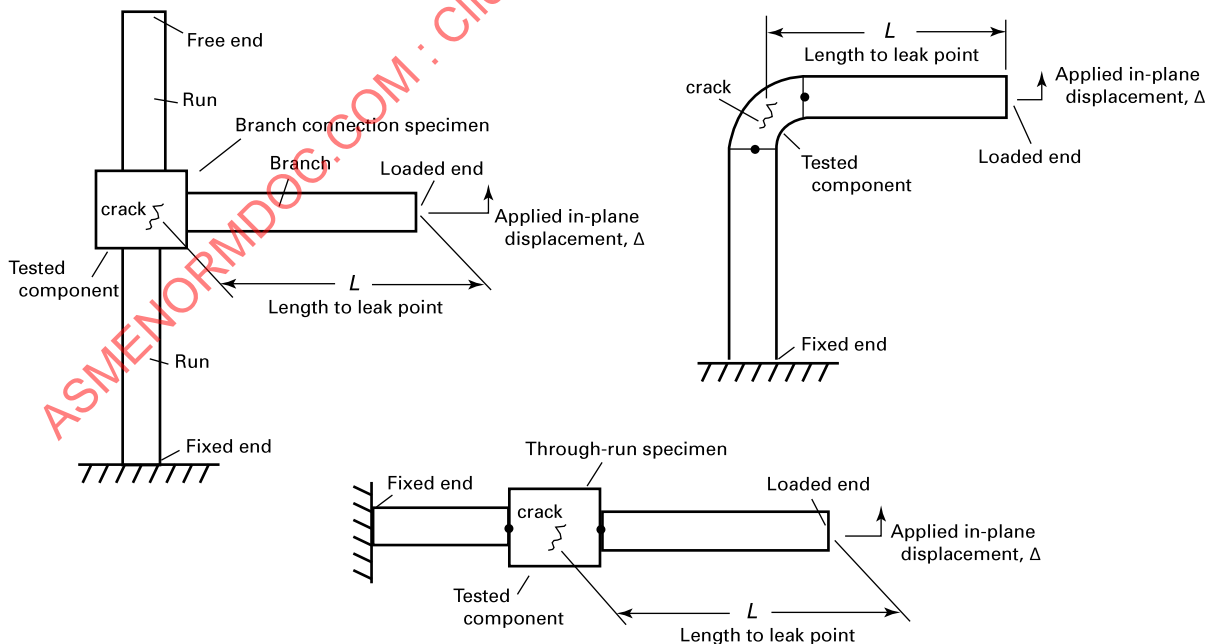
to the framework; additionally, the resistance to ovalization at the connection to the framework should not influence the SIF results. When  $D/T > 40$ , the length of pipe between the component and the framework should be numerically evaluated to ensure that unwanted interaction does not occur.

(c) The test rig shall be capable of applying a fully reversed displacement at the point of applied load without binding in a direction transverse to the direction of loading.

(d) The test equipment shall be calibrated to read displacements with an accuracy of 1% of the imposed displacement amplitude.

(e) The piping attached to the tested component should be equal to the diameter and wall thickness intended to be used with the component.

**Figure A-1.1-1 Representative Cantilever Test Arrangements**



GENERAL NOTE: Other supported test configurations, such as the four-point bend arrangements, should be used with caution in the low-cycle range. See WRC Bulletin 433, "Report No. 2: Effect of Testing Methods on Stress Intensification Factors," para 5.7, p. 74, E. C. Rodabaugh and R. J. Scavuzzo (July 1988).

### A-1.2 Test Specimen

The test specimen may be fabricated from a lower-strength carbon steel, such as ASTM A106 Grade B pipe or ASTM A234 Grade WPB fittings, or equivalent plates and forgings, corresponding to the “UTS < 80 ksi” curve in ASME Boiler and Pressure Vessel Code, Section VIII, Division 2, Mandatory Appendix 3-F(a). For other materials, the material constant,  $C$ , shall be modified or derived as described in para. A-3.1 if needed.

The fabrication, welding, and examination of the tested component shall be the same as that expected to be used in service. Weld contours and procedures should also be representative of those intended to be used in practice. Weld locations where fatigue cracks are likely to originate should be inspected for undercut, welding starts and stops, or other anomalies that may affect fatigue life. Where welding starts and stops, undercuts, or other irregularities are visible at potential crack sites, these imperfections should be documented. All weld profiles, sizes, repairs, and/or photographs should be recorded and included in the Test Report.

Where leakage is anticipated in or adjacent to a weld, the dimensions of the weld in that area should be recorded carefully and variations in the weld contours noted in the Test Report. After the test is completed, the inside weld profile at the failure location should be described in the Test Report with photographs if possible and any anomalies noted.

### A-1.3 Applied Displacement Calibration

(a) The test specimen shall be mounted in the test assembly to develop a load-deflection diagram using the same procedure that will be used during the cyclic loading portion of the fatigue test. To develop the load-deflection diagram, displacements shall be applied in positive steps in the linear range to obtain a load-displacement diagram similar to that shown in Figure A-1.3-1. At least five points shall be recorded in the linear region of the diagram. A point can be considered in the linear range if after the displacement is applied the load does not change for a period of at least 3 minutes.

(b) The initial loading sequence described in para. (a) shall be stopped when the load-displacement plot is no longer linear or when a sufficient linear portion of the diagram has been produced. To accommodate this requirement, the loading sequence may require one or two steps into the nonlinear range. If the cycle range is known to be well within the linear portion of the load-deflection plot range produced in para. (a), deflections in the nonlinear portion of the load-deflection diagram are not required. Several cycles within the linear range may be applied to remove fabrication and installation residual effects before the load-displacement plot is produced.

(c) If there is an expectation that deflection in the opposite direction will produce a different load-displacement diagram, the specimen should be unloaded following the same recording sequence used during the loading steps in paras. (a) and (b), and displacement applied in the opposite direction to approximately the same displacement magnitude used in the positive displacement loading sequence in para. (b). If the slope of the reverse-direction load-displacement diagram is sufficiently different from the slope of the positive-direction load-displacement diagram, the smaller value of the slope shall be used when determining  $F_e$  in eq. (A-1).

(d) The linear region of the load-displacement curve and its straight-line extension will be used in determining the force,  $F_e$ , used in para. A-2.1.

### A-1.4 Cycles to Leakage

(a) Once the load-deflection plot is established, the test specimen shall be fixed in the test machine with the cyclic loading mechanism attached to the test specimen. The test specimen shall be pressurized with water sufficient to detect leakage. A head pressure of 12 in. (300 mm) of water at the expected failure location (leak point) is usually sufficient. Equivalent methods of through-wall crack detection are permissible. If a closed system is used to provide water pressure, provisions should be made for a possible separation of the test piece at the anticipated failure location, and the pressure should be monitored throughout the test.

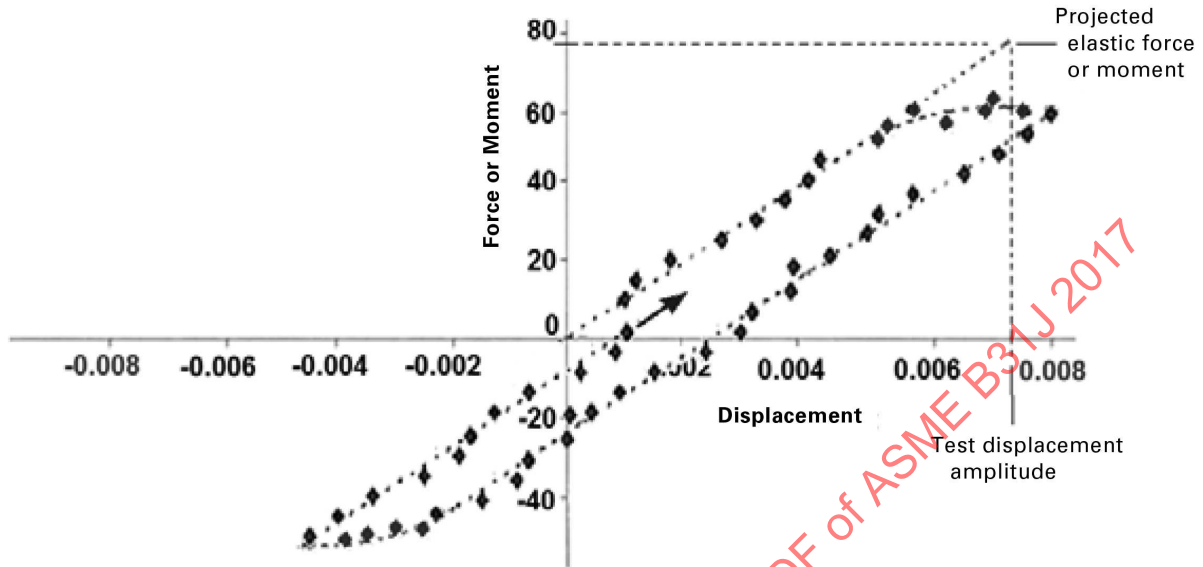
(b) The specimen shall be subjected to a prespecified fully reversed cyclic displacement until a through-wall crack is detected in the component or its weld to the attached pipe. The displacement shall be selected so that the test failure occurs within an expected cycle range in accordance with para. (d) below.

(c) The fully reversed displacement shall be applied at a frequency not to exceed 120 cycles per minute. Higher frequencies are permitted, provided there are no deleterious effects due to temperature or dynamic effects that alter the static character of the test, i.e., natural frequencies of the test piece should not coincide with the loading frequency.

(d) The number of cycles,  $N$ , at which the through-wall crack occurs shall be recorded immediately when leakage is detected at the through-wall crack. The cyclic displacements shall be selected such that failure occurs in a minimum of 5,000 cycles of reversed displacements and at no more than  $2 \times 10^6$  cycles of reversed displacement. If the through-wall crack occurs at more than  $2 \times 10^5$  cycles, the values of  $C$  and  $b$  used in eq. (A-3) must be validated.

(e) The number of cycles to leakage shall be determined as described in para. A-2.6 if the applied test displacement magnitude is changed during the test because of accidental deviations in prescribed displacements, to accommodate failure, or for any other reason.

**Figure A-1.3-1 Displacement and Force or Moment Recorded During Loading and Unloading of a Test Specimen in Both Positive and Negative Directions, With Linear Displacement**



GENERAL NOTE: The slope of the best-fit straight line is used in subsequent tests to determine the stress intensification factor.

## A-2 Stress Intensification Factor

### A-2.1 Calculated Stress

The applied moment at the leak point,  $M_e$ , is calculated from eq. (A-1).

$$M_e = F_e L \quad (A-1)$$

where

$F_e$  = force corresponding to the applied displacement amplitude, taken from the straight-line portion of the load-deflection diagram developed in para. A-1.3, lb (N)

$L$  = distance between the point of applied displacement and the leak point, along a line perpendicular to the imposed displacement, in. (mm)

$M_e$  = applied elastic moment amplitude, in.-lb (N-mm)

The elastically calculated stress amplitude corresponding to the elastic moment at the leak point is

$$S = M_e / Z \quad (A-2)$$

where

$S$  = stress amplitude at the leak point, psi (MPa)

$Z$  = section modulus as defined in para. A-2.2, in.<sup>3</sup> (mm<sup>3</sup>)

### A-2.2 Section Modulus

The value of the section modulus,  $Z$ , used in calculating the stress amplitude at the leak point in para. A-4(a) shall be that of the matching nominal wall pipe intended to be

used with the component. If the stress at the leak point is computed using  $Z$  other than that of the matching pipe, the manner in which  $Z$  is computed must be specified in the definition of the stress intensification factor so that an appropriate value for  $Z$  can be used in design.

Unless otherwise defined by the Code, the value of  $Z$  shall be calculated using the following equation for matching pipe with an outside diameter  $D_o$  and an inside diameter  $D_i$ , where the difference between  $D_o$  and  $D_i$  is twice the nominal wall thickness of the matching pipe:

$$Z = \left( \frac{\pi}{32} \right) \left( \frac{D_o^4 - D_i^4}{D_o} \right) \quad (A-2a)$$

Additional care may be needed when defining  $Z$  for  $i$ -factors developed for reduced outlet branch connections since many piping Codes use a modified value of  $Z$  in such cases. Stress intensification factors given in Table 1-1 are for use with the section modulus of the attached matching pipe for all connections unless otherwise noted.

### A-2.3 Stress Intensification Factor

The stress intensification factor is established as

$$i = C / (S N^b) \quad (A-3)$$

where

$b$  = material exponent, 0.2 for metals. If the number of cycles to failure is less than 5,000 or greater than 200,000 [see para. A-1.4(d)], the user must validate the values of  $b$  and  $C$  used. If more appropriate

**Table A-2.4-1 Stress Intensification Increase Factor**

Number of Test Specimens	Testing Factor, $R_i$
1	1.2
2	1.1
3	1.05
≥4	1.0

values for  $b$  and  $C$  are available, those values may be used and should be included in the Test Report; alternate values of  $b$  typically used are between 0.2 and 0.35.

$C$  = material constant, 245,000 psi (1 690 MPa) for a carbon-steel test specimen

$i$  = stress intensification factor

$N$  = number of equivalent cycles to failure, where  $N$  is the first cycle in which through-wall leakage occurs. See para. A-2.6 when variable-amplitude displacements are used in the test.

$S$  = nominal stress amplitude at the leak point, psi (MPa)

#### A-2.4 Number of Test Specimens

(a) The value of the stress intensification factor,  $i$ , shall be the average value of a minimum of four cyclic displacement tests when no analytically determined stress intensification factor is available.

(b) When fewer than four tests are conducted and there is no analytically determined stress intensification factor available, the average stress intensification factor,  $i$ , shall be increased by the factor  $R_i$  given in Table A-2.4-1.

(c) When an analytically determined stress intensification factor and one or more test-determined stress intensification factors are available, the value of the stress intensification factor,  $i$ , shall be determined by a detailed rational analysis included in the Test Report.

#### A-2.5 Directional Stress Intensification Factors

(a) For components that do not have a single axis of symmetry (e.g., branch connections), a directional stress intensification factor shall be established for each independent direction of bending. When a verified numerical analysis is available, the verified numerical analysis may be used to predict the stress intensification factor for directions where tests have not been performed.

(b) When the design Code requires the use of a single stress intensification factor, the largest of the directional stress intensification factors shall be used. For size-on-size branch connections or laterals, the torsional stress intensification factor may be the highest single stress intensification factor.

#### A-2.6 Variable-Amplitude Test

If the applied displacement amplitude is changed during a cyclic test as described in para. A-1.4(e), the number of cycles to leakage shall be determined by eq. (A-4). The cyclic load case  $j$  in eq. (A-4) should be associated with the displacement and cycle range that is believed to cause the most significant fatigue damage. When  $N_i < 5,000$ , the value of  $b$  used must be validated. For cantilever specimens with girth butt welds where the displacement range  $N_i < 3,200$  and the leak occurs at the weld, the value of  $b$  may be taken as 0.335. For in-plane loads through the branch where the displacement range  $N_i < 3,200$ , and where the leak occurs at the weld, the value of  $b$  may be taken as 0.2 if other guidance is not available.

The exponent  $b$  may be a function of the component geometry and the magnitude of the loading. When the cycle range that causes the most significant fatigue damage is applied for less than 3,200 cycles, the value of  $b$  may range from 0.1 to 0.7, but is likely between 0.15 and 0.38.

$$N = N_j + \sum (r_i)^{1/b} \times N_i \text{ for } i = 1, 2, \dots, n \quad (\text{A-4})$$

where

$b$  = material exponent, usually 0.2 or 0.335 for metals

$j$  = the number of the test case chosen as the base case

$N$  = equivalent number of cycles to leakage, at maximum amplitude  $X_j$

$N_i, N_j$  = number of cycles at amplitudes  $X_i, X_j$ , where all  $X_i < X_j$

$r_i = X_i/X_j; r_i < 1$

$X_i, X_j$  = amplitudes of displacement applied during cycle  $N_i, N_j$ , in. (mm)

#### A-2.7 Instrumentation

Instrumentation such as load-measuring devices and displacement indicators shall be calibrated within 12 months prior to the test and traceable to NIST standards or equivalent. The cycle counter shall be checked to be in good working condition. The force-measuring device shall be capable of measuring forces within 200-N (0.05-kip) increments or 1% of the largest load imparted during the test increments.

When possible, the applied displacement measurement point described in para. A-1.3 shall be identified clearly with respect to the fixed base of the specimen, and, when practical, the base of the test specimen shall be quantitatively rigid as described in Nonmandatory Appendix B, para. B-4.3.



### A-3 Variations in Materials and Geometry

#### A-3.1 Material Constant and Material Exponent

When using a test specimen made of Code-listed materials other than lower-strength carbon steel, a new material constant,  $C$ , shall be established using one of the two following methods unless the material constant  $C$  has already been established (see [11]):

(a) *C-Factor Determination Method Using Modulus of Elasticity Ratios.* This method of  $C$ -factor determination assumes that the  $C$ -factor for a material different from low-carbon steel is equal to the  $C$ -factor of low-carbon steel multiplied by the modulus of elasticity of the different material and divided by the modulus of elasticity of low-carbon steel.

(U.S. Customary Units)

$$C(\text{other material}) = \frac{245,000 \times E(\text{other material})}{27,800,000 \text{ psi}} \quad (\text{A-5a})$$

(SI Units)

$$C(\text{other material}) = \frac{1\,690 \times E(\text{other material})}{192\,000 \text{ MPa}} \quad (\text{A-5b})$$

where

$C$  = material constant, for use in eq. (A-3), psi (MPa)

$E$  = modulus of elasticity, psi (MPa)

(b) *C-Factor Determination Method Using Fatigue Tests*

(1) This method of  $C$ -factor determination requires that a minimum of eight butt-welded, cantilever, test specimens of the to-be-tested material are fabricated and tested in accordance with paras. A-1.1 through A-1.4.

(2) Each of the minimum of eight specimens shall be subjected to different applied displacements.

(3) The pairs of  $N$  (cycles to failure) and  $S$  (elastically extrapolated nominal stress) shall be plotted on a log-log scale. The material constant  $a$  and exponent  $b$  shall be obtained by plotting a best-estimate straight line through the  $(N, S)$  points in the form

$$a/(SN^b) = 1 \quad (\text{A-6})$$

(4) The factor  $b$  found from the best estimate should not be less than 0.1 or greater than 0.7. The value of  $a$  that results from the best-estimate fit of eq. (A-6) shall be taken as the  $C$  value for the tested material.

(5) The target number of cycles to failure for each test should be greater than 5,000 and less than 200,000. Tests in which failures occur outside of this range shall be excluded from the best estimate in subpara. (3) above.

#### A-3.2 Geometric Similarity

(a) The stress intensification factor derived from tests conducted in accordance with this Standard is applicable to geometrically similar components, where the degree of geometric similarity should be established by rational analysis.

(b) Dimensional extrapolations considered acceptable shall be identified in the Test Report, along with their technical justification. For example, a complex fitting may have multiple tests run on different diameters and thicknesses to establish a relationship between the SIF and the dimensional parameters. Alternatively, if a closed-form evaluation can be confidently performed, the technical justification can be based on a single-size fitting with the closed-form evaluation used to clearly identify how the SIF is extrapolated to other sizes.

(c) Of particular importance is the dimensional detail at intersections (radii for crotch regions, weld details, etc.).

(d) Tests on small welded components in which the fillet weld leg lengths are equal to or larger than the thicknesses of the parts joined may not be comparable to tests on larger components where the fillet weld leg lengths are smaller than the thicknesses of the parts joined.

(e) The procedure used in this Standard may be used to establish SIFs for partial-penetration welds, provided adequate geometric similarity is established.

#### A-3.3 Flanges, Compression Fittings, Ball Joints, Slip Joints, and Other Pipe-Joining Components

Where  $i$ -factors are needed for flanges or other pipe-joining components, and where recognized standards do not exist, a clear definition of failure due to the repeated displacement loading described in para. A-1.3, i.e., pressure retention, must be established.

#### A-4 Test Report

(a) To meet the requirements of this Standard, a Test Report shall be prepared and certified by a Registered Professional Engineer competent in the design and analysis of pressure piping systems, or a person of equivalent expertise as defined by national practice. The Test Report shall be complete and written to facilitate an independent review. The report shall contain

(1) description of the tested specimens.

(2) nominal pipe and piping component size and dimensions and actual cross-sectional dimensions of importance in interpreting the test results.

(3) description and photograph(s) or sketch(es) of the test equipment, including positioning of the test specimens in the loading device.

(4) calibration of the test equipment. This information may be provided by reference.



(5) certified Material Test Reports for the tested component, including, but not limited to, elongation, yield, and ultimate strength.

(6) component and component-to-pipe weld examinations where required by the construction Code. This may include a copy of the Welding Procedure Specification (WPS) and the Welding Operator Performance Qualification (WPQ) for the welding operator who welded the components. When the failure occurs in or near a welded joint, a narrative describing the visual examination of the welds shall also be included. If possible, detailed high-resolution photographs of all or a portion of the welds should be included in the report, including images from the inside of the specimen at the root of any welds where through-wall cracks or critical weld joints appear.

(7) assembly procedure used for joints.

(8) loading and unloading load-displacement points.

(9) values of material constant,  $C$ , section modulus,  $Z$ , number of cycles to leakage,  $N$ , length to leak point,  $L$ , and imposed displacement for each test.

(10) derivation of the force,  $F$ , moment,  $M$ , and stress intensification factor,  $i$ , for each test.

(11) description and photograph(s) or sketch(es) of the leak location.

(12) justification for geometric similarity, if any, in accordance with [para. A-3.2](#).

(13) description of any additional preparation or pre- or post-weld heat treatment provided to improve the quality of the weld beyond that which would be anticipated for the service conditions.

(14) value of the  $i$ -factor derived for all tests; see [section A-2](#).

(15) failure location in each test, identified by sketches and/or photographs, including the weld profile at the failure cross section so that service requirements can be specified and actual performance can be expected to exceed test behavior.

(b) When the SIF test is conducted by or for a manufacturer, the Test Report shall be maintained by the manufacturer for review by the owner, purchaser, or designer.

(c) If the specimen is of a material other than carbon steel or an already evaluated material [11], then the Test Report shall be prepared in two parts, one that describes the development of the  $a$  and  $b$  material constants, and another that describes the determination of the SIF for the component or joint.

(d) Dimensional extrapolations that are deemed appropriate shall be identified and technically justified as described in [section A-3](#). The value of  $Z$  to be used with the respective  $i$ -factor shall be clearly identified.

(e) [Paragraph A-3.2\(b\)](#) permits dimensional extrapolations provided the technical justification for such extrapolations is included in the Test Report. An example of such justification is the elastic-stress theory for elbows used by Markl ([2], [3], [4], [5]). More generally, an accept-

able justification for extrapolations would consist of a valid elastic-stress theory applicable to the type of component, e.g., a branch connection. Care would be needed to ensure that the elastic-stress theory applies over the range of dimensional extrapolations given in the Test Report. The use of finite element analysis, for example, when applied properly, is considered an applicable elastic-stress theory.

## A-5 Nonmandatory Commentary on SIF Test Procedure

ASME B31J is based on the work by E. C. Rodabaugh in WRC Bulletin 392 [1]. The following commentary provides a synopsis of the discussion in [Ref. \[1\]](#). Readers who desire more detail are urged to see [Refs. \[1\]](#) through [5].

### A-5.1 General

The Codes for pressure piping (e.g., ASME B31.1; ASME B31.3; ASME Boiler and Pressure Vessel Code (BPVC), Section III, Class 2/3) use stress intensification factors ( $i$ -factors) for various piping components and joints as a measure of their fatigue performance relative to girth butt welds. Occasionally, a need arises to establish  $i$ -factors for components not included in the Codes, such as a branch connection in an elbow or some proprietary piping component. This Standard provides a set of requirements that will ensure that newly developed  $i$ -factors are consistent with existing  $i$ -factors.

(a) Papers by Markl and George ([2], [3], [4]) provided the basis for most of the  $i$ -factors in the Codes. Key aspects of the testing and interpretation of test results are as follows:

(1) a preliminary load-deflection plot is developed (see [Figure A-1.3-1](#))

(2) cyclic bending tests are run with controlled displacements

(3) failure is defined as a through-wall crack

(4) the  $i$ -factor is calculated by [eq. \(A-3\)](#)

(b) Markl [5] discusses "allowable stress range" and, in Appendices 1 and 2 of [1], describes rules that were eventually incorporated into ANSI B31.1-1955. [Reference \[5\]](#) discusses the following concepts that are fundamental to the use and interpretation of  $i$ -factors as a control of fatigue failure:

(1) the  $i$ -factors are dependent on dimensions and are independent of the material

(2) as a consequence of [subpara. \(1\)](#) above,  $i$ -factors developed by Markl using ASTM A106 Grade B material are presumed to be applicable to components made of any of the metallic materials listed in the piping Codes

(3) the Code stress limits [e.g.,  $f[1.25(S_c + S_h)]$ ] are proportional to the fatigue strength of materials used in the components

(4) the dependent variable should always be the logarithm of the cycle life and the independent variable should always be the logarithm of the stress to failure when curve fitting fatigue life test results; see pages 97–99 of Ref. [12].

(c) *Materials and Material Extrapolations.* Markl ran tests on specimens made of ASTM A106 Grade B material. Paragraph A-1.2 prescribes analogous Grade B materials. It is recognized that some components may not be made from Grade B materials, e.g., copper tubing and fittings. For example, if tests were run on a copper elbow, then a different  $C$  constant would be needed so that the concept that  $i$ -factors are independent of the material is preserved. Section A-1 is written to allow for this eventuality, with rules provided in section A-3.

(d) *Dimensions and Dimensional Extrapolations.* Markl's tests were run on NPS 4 test specimens. Markl's broad extrapolations to other dimensions were based on elbow theory. For elbows, Markl's in-plane tests led to

$$i = 0.90/h^{2/3}$$

where

$$h = tR/r^2$$

$R$  = bend radius

$r$  = mean radius of elbow cross section

$t$  = wall thickness

Paragraph A-3.2 permits dimensional extrapolations provided the technical justification for such extrapolations is included in the Test Report. An example of such justification is the elastic-stress theory for elbows used by Markl. More generally, an acceptable justification for extrapolations would consist of a valid elastic-stress theory applicable to the type of component, e.g., a branch connection. Care is needed to ensure that the elastic-stress theory is applicable over the range of dimensional extrapolations given in the Test Report. For example, the use of finite element analysis, when applied properly, is considered an applicable elastic-stress theory.

Appropriate dimensional extrapolations for branch connections, their theory, testing, and the effect of weld profiles on  $i$ -factors are exhaustively treated in Refs. [6] and [7]. The need to ensure and quantify the weld geometry used in the testing is clearly shown in Ref. [6], where a subtle change in weld profile produced a change in the SIF value by a factor of 2. Thus, paras. A-4(b), 6(b), and 6(f) require the weld contour to be representative of the installation.

## A-5.2 Basis for Requirements

The paragraphs from the Standard that are considered self-explanatory do not have a corresponding commentary.

(a) Section A-1

(1) Paragraph A-1.1. Markl's test was a cantilever test with the specimen oriented as shown in Figure A-1.1-1. Removing the component from the mounting by two diameters is to prevent end effects (stiffening) from affecting the SIF results even though end results are strongly influenced by the deformed shape of the component being tested. If the tested component is axisymmetric where ovalization is not introduced during loading, the boundary effect is influenced by either the extent of any plastic zone at the boundary or a length from the boundary equal to  $(RT)^{0.5}$ . If the component being tested is not axisymmetric, such as a branch connection or bend, then ovalization will likely be introduced at the component due to bending, and the effective boundary distance will be either the extent of any plastic zone size or a length from the boundary equal to  $0.5D^{1.4}T^{0.4}$ . A quantitative discussion of the effect of boundary condition lengths adjacent to branch connections can be found in Ref. [8]. The basis for requiring that a numerical evaluation be performed when  $D/T > 40$  is that the ovalization interaction length found by  $0.5D^{1.4}T^{0.4}$  is greater than  $2D$ , and some extra precaution might be warranted.

Calibration of instrumentation is necessary to ensure repeatability by independent organizations.

(2) Paragraph A-1.2. Markl's test specimens were ASTM A106 Grade B material, or equivalent in the case of forgings, castings, and plate. Use of different materials requires a new  $C$  constant to be developed since materials such as copper, aluminum, or very high-strength steels exhibit a fatigue life that may be different from plain carbon steel. The intent of the test is to develop a SIF that is geometry dependent and not material dependent. Cantilever fatigue test data on different piping materials is reported in Ref. [11].

Nominal dimensions and wall thicknesses should be reported carefully for branch connections to ensure that extrapolation of results to other sizes is done correctly. The importance of the weld profile is clearly demonstrated in Ref. [6].

(3) Paragraph A-1.3. Markl's tests were based on linear elastic equivalent moments, i.e., a constant displacement or rotation was applied and the moment at the failure location was based on extrapolation of the  $M - \theta$  (or  $F - \delta$ ) elastic curve. This allows agreement with the way linear elastic thermal expansion analyses are used in typical pipe stress analysis programs, even though predicted stresses may be above yield.

(4) Paragraph A-1.4. The use of a nominal pressure is to ensure a ready means of detecting a through-wall crack. The use of 5,000 cycles as a minimum is to ensure correlation with the lower bounds of Markl's work and to remove the need to determine if Markl's equation or Hinnant's [9] equation should be used to determine the  $i$ -factor. From Ref. [4], it can be seen that the preponderance of tests are for failures above 1,000 cycles. The few test failures that

occur below 1,000 cycles show significant deviation from the proposed straight line. Some high  $D/T$  configurations in the test record are artificially fabricated pipe specimens that may not represent typical installed components. Until more work is done in the very low cycle and the very high  $D/T$  ranges, the lower limit of 5,000 cycles should remain.

(b) **Section A-2**

(1) **Paragraph A-2.3.** The equation in **para. A-2.3** is taken directly from the work by Markl [i.e., [4], eq. (4)]. Since Markl's tests formed the basis of the current  $i$ -factors and Code rules, use of Markl's equation is appropriate for correlation, especially when  $200,000 > N > 5,000$  cycles, where the Markl and Hinnant equations agree.

(2) **Paragraph A-2.4.** The factors in **Table A-2.4-1** provide for scatter in the test results. The basis for the  $R_t$  factor in **Table A-2.4-1** is the assumption that a single test point is likely to be one standard deviation shifted from the mean. An approximation that gives a two-standard-deviation shift from the mean can be found in ASME BPVC, Section III, Appendix II, II-1520 (f) as given below.

$K_{ss} = 1.47 - 0.044 \times \text{number of replicate tests (may be zero for only a single test)}$

$$K_{ss} \geq 1.0$$

$K_{ss}$  replaces  $R_t$  from **Table A-2.4-1**.  $K_{ss}$  should be used when a higher probability of success is desired. It is assumed that one standard deviation is equal to 0.27, where the standard deviation is based on the logarithm to the base 10 of the cycles to failure. If some other value for standard deviation arises from the test data of a particular part, alternate values for  $R_t$  can be used.

(3) **Paragraph A-2.6. Equation (A-4)** for variable-amplitude tests is the same equation incorporated in ANSI B31.1-1955 to convert different operating condition stress ranges, typically thermal stress ranges, to a single base stress range, and is based on Miner's rule and Markl's relation for stress and cycles when  $b = 0.2$ .

(c) **Section A-3**

(1) **Paragraph A-3.1.** Based on the work in *Thermal Stress and Low Cycle Fatigue* by Manson [14], the material exponent,  $n$ , for metals stays fairly constant at 0.2 and has been set to that value in the Standard. Based on work done by Koves et al. [8], the material constant,  $C$ , can be found from a ratio of the moduli of elasticity, i.e.

$$C \left( \text{other material} \right) = \frac{245,000 E(\text{other material})}{E(\text{carbon steel} = 27.8E6 \text{ psi})}$$

(2) **Paragraph A-3.2.** Dimensional extrapolations need to be justified based on either elastic-stress theory or tests of additional sizes. Elastic theory was the basis of Markl's extrapolation work in elbows and straight pipe.

(d) **Section A-4.** The Test Report should assure owners that the testing was carried out in compliance with this Standard. Since the Test Report must also describe any

weld profiles, the owner can also ensure that such profiles/procedures are incorporated into the welding program for any actual installations of the component. The basis for the  $i$ -factor must be reviewable by the owner or his agent.

## A-6 References

- [1] WRC Bulletin 392, "Part 1: Standardized Method for Developing Stress Intensification Factors for Piping Components," E. C. Rodabaugh (June 1994).
- [2] Markl, A. R. C., "Fatigue Tests of Welding Elbows and Comparable Double-Mitre Bends," *Trans. ASME*, Volume 69, 869-879 (1947).
- [3] Markl, A. R. C., and George, H. H., "Fatigue Tests on Flanged Assemblies," *Trans. ASME*, Volume 72, 77-87 (1950).
- [4] Markl, A. R. C., "Fatigue Test of Piping Components," *Trans. ASME*, Vol. 74, 287-303 (1952).
- [5] Markl, A. R. C., "Piping-Flexibility Analysis," *Trans. ASME*, 77, 127-149 (1955).
- [6] WRC Bulletin 392, "Part 2: Effects of Weld Metal Profile on the Fatigue Life on Integrally Reinforced Weld-on Fittings," G. E. Woods and E. C. Rodabaugh (June 1994).
- [7] WRC Bulletin 329, "Accuracy of Stress Intensification Factors for Branch Connections," E. C. Rodabaugh (1987).
- [8] WRC Bulletin 497, "Large Diameter Ratio Shell Intersections," Koves, W. J., Mokhtarian, K., Rodabaugh, E. C., and Widera, G. E. O. (December 2004); "Parts 1, 2, and 3: Parametric Finite Element Analysis of Large Diameter Shell Intersections (External Loadings)," G. E. O. Widera and Z. Wei (December 2004).
- [9] Hinnant, C., and Paulin, T., "Experimental Evaluation of the Markl Fatigue Methods and ASME Piping Stress Intensification Factors," PVP2008-61871 (July 2008).
- [10] Wais, E., and Rodabaugh, E. C., "Stress Intensification Factors and Flexibility Factors for Unreinforced Branch Connections," TR 110996 Final Report (November 1998).
- [11] Hinnant, C., Paulin, T., Becht, V. C., Becht IV, C., and Lock, W. S., "Experimental Evaluation of the Markl Fatigue Methods and ASME Piping Stress Intensification Factors Part 2," PVP2014-28268, Anaheim, CA (July 2014).
- [12] Conway, J. B., and Sjodahl, L. H., "Analysis and Representation of Fatigue Data," ASM International, Materials Park, OH (1991).
- [13] Paulin, A., "Stress Intensity Factor and  $k$ -Factor Alignment for Metallic Pipes," ASME STP-PT-073-2014.
- [14] Manson, S. S., *Thermal Stress and Low Cycle Fatigue*, McGraw-Hill, Inc. (1966).

## NONMANDATORY APPENDIX B

### Test Method for Determining Branch Connection Flexibility Factors

#### B-1 General

Flexibility factors for moment loading are defined in Table 1-1 and are expressed as the in-plane, out-of-plane, and torsional flexibility factors  $k_i$ ,  $k_o$ , and  $k_t$ , respectively, and may be unique for both the branch and run sides of a component; see Figures B-1-1 and B-1-2. The basis for  $k$ -factor development can be found in Ref. [1]. The procedure defined here improves on that method by incorporating an iterative approach using results from a linear elastic beam analysis computer program to determine  $k$ -factors, eliminating the need to develop a mathematical stiffness model of the test configuration.

The  $k$ -factor test should be conducted prior to any SIF or sustained load test because

(a) The  $k$ -factor test uses low loads with elastic displacements that do not damage or plastically deform the specimen. Tested specimens can subsequently be used in a SIF or sustained load test.

(b) The mounting required for the  $k$ -factor test is identical to the mounting required for either the SIF test described in Nonmandatory Appendix A or the sustained load test described in Nonmandatory Appendix D.

(c) For the sustained load test, the  $k$ -factor is used to estimate the required dimensions needed for the test.

(d) All six  $k$ -factors can be determined from a single mounted test configuration.

Flexibility factors ( $k$ -factors) for multiple loading directions can usually be evaluated with a single assembled position and test specimen. The number of load-deflection pairs possible for a single assembled and mounted specimen will typically depend on the availability of fixed points in the vicinity of the assembly that can be used to apply the relatively small required loads. Figure B-1-2 shows a single assembled position and six loading configurations that can be used to determine each  $k$ -factor. The procedure designer must determine the appropriate loads and directions for the component evaluated and the facility where the test is conducted. Table B-1-1 contains recommended load and degree of freedom directions. In all cases, applied forces are used as the loads. Measured values are displacements or rotations as identified for each load in Table B-1-1.

Once the assembled position, pipe sizes, and lengths are established, load magnitudes can be determined in accordance with para. B-1.3 and the practicality of applying loads in the necessary directions determined. Locations selected for the force application should be removed from the specimen of interest by straight pipe having a length at least  $0.5D^{1.4}T^{-0.4}$  and applied at a section on the pipe that has an essentially rigid cross section, i.e., a flange, end cap, or clamp.

The displacement measurement can be taken at the same point as the force is applied, provided the cross section where the force is applied is rigid.

#### B-1.1 Types of $k$ -Factors

There are two types of components for which  $k$ -factors are determined, as follows:

(a) components where added flexibility is provided over a finite-length centerline, e.g., the curved centerline of bends

(b) components where added flexibility is provided at a single point, e.g., branch connections, reducers ( $D/T < 50$ ), or nozzles in heads

The procedure in this Appendix addresses only branch connection components described in para. (b) above. When flexibility factors must be calculated for finite-length components, the procedure outlined in WRC

**Figure B-1-1 Branch Connection Specimen**

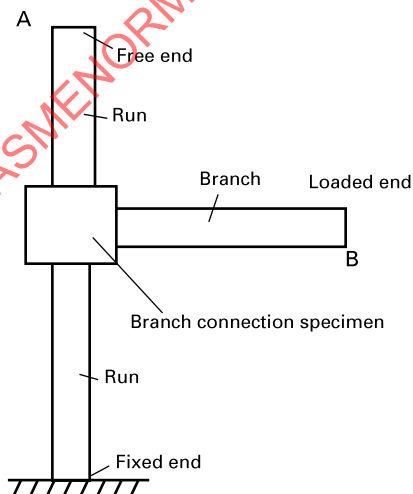


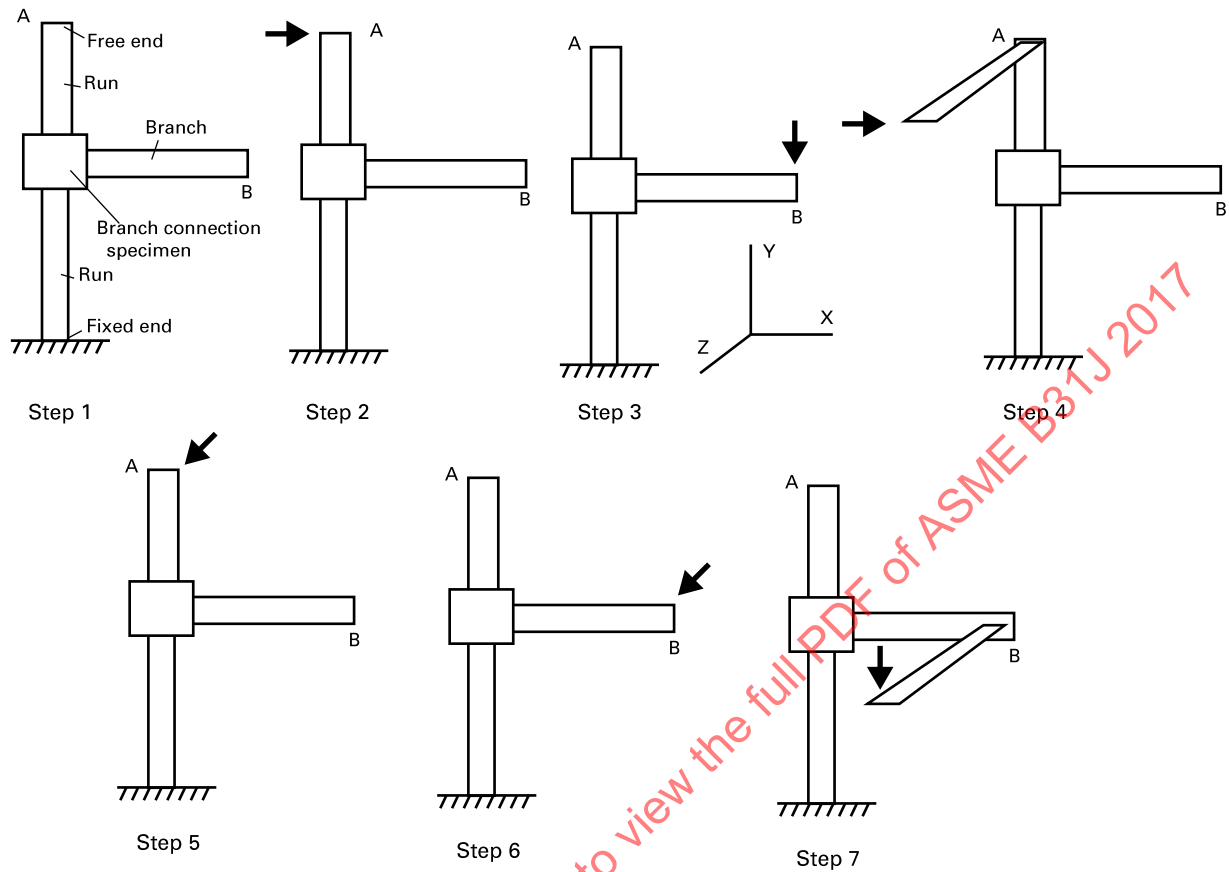
Figure B-1-2 Multiple *k*-Factor Tests on Single Assembled Position

Table B-1-1 Load-Deflection Pairs for Single Assembled Orientation Shown in Figure B-1-2

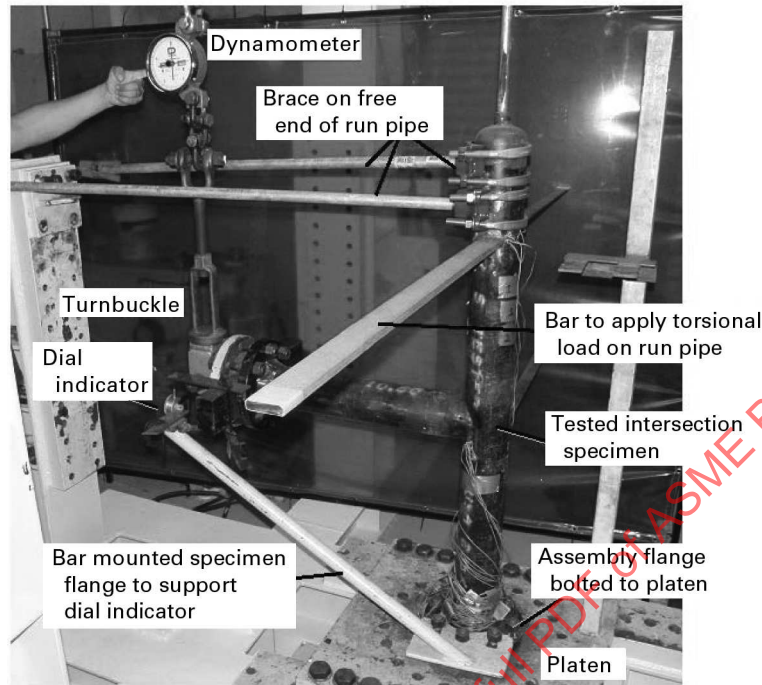
Load Step No.	Force	Displacement	<i>k</i> -Factor	Description and Degrees of Freedom
2	$F_{ri}$	$d_{ri}$	$k_{ri}$	Force, <u>displacement</u> pair for in-plane run loading. Load at point A in the x direction. Displacement measured at point A in the x direction. Run stiffness $k_{ri}$ is along the z-axis.
3	$F_{bi}$	$d_{bi}$	$k_{bi}$	Force, <u>displacement</u> pair for in-plane branch loading. Load at point B in the y direction. Displacement measured at point B in the y direction. Branch stiffness $k_{bi}$ is along the z-axis.
4	$F_{rt}$	$d_{rt}$	$k_{rt}$	Force, <u>rotation</u> pair for torsional run loading. Load at end of cantilever attached to point A in the x direction. Rotation measured at point A about the y-axis. Run stiffness $k_{rt}$ is along the y-axis.
5	$F_{ro}$	$d_{ro}$	$k_{ro}$	Force, <u>displacement</u> pair for out-of-plane run loading. Load at point A in the z direction. Displacement measured at point A in the z direction. Run stiffness $k_{ro}$ is along the x-axis.
6	$F_{bo}$	$d_{bo}$	$k_{bo}$	Force, <u>displacement</u> pair for out-of-plane branch loading. Load at point B in the z direction. Displacement measured at point B in the z direction. Branch stiffness $k_{bo}$ is along the y-axis.
7	$F_{bt}$	$d_{bt}$	$k_{bt}$	Force, <u>rotation</u> pair for torsional branch loading. Load at end of cantilever attached to point B in the y direction. Rotation is measured at point B about the x-axis. Run stiffness $k_{bt}$ is along the x-axis.

Bulletin 463 can be used. The main characteristic of para. (b) branch connections is that the flexibility is provided by a local deformation of the shell at the point of pipe or nozzle attachment. A “point spring” approach is used in these cases to simulate local flexibilities in a piping analysis. The point spring flexibility factor approach is discussed in more detail in Nonmandatory Appendix C.

### B-1.2 Effect of Attached Straight Pipe

To determine the flexibility factor of a branch connection, the effect of the straight pipe, flanges, rigid clamps, etc. must be removed from the cumulative rotation of the test assembly so that only the local rotation of the component remains. Estimating the effect of multiple straight pipe lengths with flanges, lugs, etc., is performed most effectively by a computer program designed for that



**Figure B-2-1 Example Flexibility Factor Branch Load Assembly Orientation**

purpose. Manual formulation and solution of the elastic equations relating forces and displacements in multisection test assemblies is prone to error and should be avoided.

### B-1.3 Magnitude of Loading

The maximum applied load in the  $k$ -factor test must

(a) produce a measured displacement at least 10 times larger than the smallest measurable displacement and be at least 10 times larger than the smallest measurable force

(b) not cause plastic deformation or stiffening due to large rotation

The smallest measurable displacement should not be less than 0.001 in. (0.025 mm).

The magnitude of the applied force appears in both the numerator and the denominator of the  $k$ -factor expression and so low-magnitude loads may be used if they satisfy the criteria in para. B-4.6(j).

### B-2 Definitions

**assembled position:** orientation of the piping assembly to be tested where one or more legs are rigidly fixed to a platen or heavy frame and where one or more load-deflection tests can be conducted. A single assembled position is shown in Figure B-1-2 along with five possible load-deflection tests that could be conducted on the specimen while it is in the single assembled position. An example fabricated tee test specimen is shown in the assembled position in Figure B-2-1.

**load-deflection pair:** a force and corresponding displacement or rotation that can be used to compute a  $k$ -factor for a given direction. The force and displacement or rotation for any flexibility factor test is measured along the same line of action, but not necessarily at the same cross section in the test assembly. Load-deflection pairs that can be used for branch connection tests are shown in Table B-1-1.

**measurement point:** point on the test assembly along the direction of interest where displacement measurements are taken.

### B-3 Nomenclature

$D$  = mean diameter of matching run pipe, in. (mm)

$d$  = mean diameter of matching branch pipe, in. (mm)

$D_L$  = mean diameter of loaded matching pipe, in. (mm)

$d_m$  = test displacement measurement, in. (mm)

$d_{\min}$  = smallest measurable displacement, in. (mm)

$d_n$  = displacement at the measurement point calculated from the elastic model of the test assembly with rigid intersections for both the run and branch sides of the branch connection, in. (mm)

$E_e$  = elastic slope of load-displacement diagram [see Figure D-3-1 and eq. (D-8) in Nonmandatory Appendix D]

$F_m$  = test force measurement

$F_{\min}$  = smallest measurable force

$F_n$  = nominal load applied to elastic beam model of the test assembly used to find elastic constant  $G$

- $I$  = moment of inertia of the elbow, matching pipe for tees and other components, in.<sup>3</sup> (mm<sup>3</sup>)  
 $k_i$  = in-plane flexibility factor  
 $k_o$  = out-of-plane flexibility factor  
 $k_t$  = torsional flexibility factor  
 $L$  = distance from centerline of specimen to point where displacements are measured when  $d/D \geq 0.5$  and distance from surface of branch connection to point where displacements are measured when  $d/D < 0.5$ , in. (mm)  
 $L_{\min}$  = minimum length of nominal pipe attached to the tested specimen, in. (mm)  
 $M_m$  = moment found from  $M_m = L \times F_m$   
 $T$  = thickness of matching run pipe, in. (mm)  
 $t$  = thickness of matching branch pipe, in. (mm)  
 $T_L$  = thickness of the attached pipe on the loaded pipe leg, in. (mm)  
 $S_y$  = yield stress of tested component from the Material Test Record  
 $S_{yp}$  = yield stress of the pipe attached to the tested component from the Material Test Record

## B-4 Test Procedure

### B-4.1 Test Setup

Select the test specimen material, pipe size, and lengths for the applied displacement calibration test described in [Nonmandatory Appendix A, para A-1.3](#). The length of straight pipe attached to the loaded specimen should be greater than the length  $L_{\min}$  calculated by [eq. \(B-1\)](#) and should match the schedule or wall thickness of the component tested.

$$L_{\min} = 0.5D_L^{1.4}T_L^{-0.4} \quad (\text{B-1})$$

### B-4.2 Smallest Measurable Loads and Displacements

The smallest measurable displacement  $d_{\min}$  and load  $F_{\min}$  can be estimated as one-half the value between indications on the ruler or dial indicator used to measure displacement, or the precision of the digital readout, or the manufacturer's reported instrument precision, whichever is greatest. The smallest measurable displacement should not be less than 0.001 in. (0.025 mm).

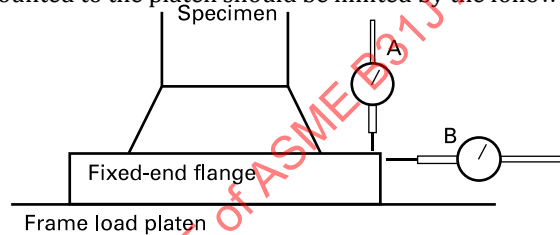
Displacements should be measured with respect to a fixed location at the base of the piping test assembly to eliminate any influence due to the rotation of the pipe assembly flange. Displacement measurements are often taken using a dial indicator mounted on an extended rod whose base is fixed to the top of the assembly flange that is bolted to the test machine platen. See [Figure B-2-1](#).

Rotations are generally measured with lasers mounted to the centerline of the pipe, e.g., point A in [Figure B-1-2](#) Step 4 and point B in [Figure B-1-2](#) Step 7. The rotation is the linear trace of the laser pointer divided by the distance between the laser and the trace.

Criteria are provided in [para. B-4.6\(j\)](#) when additional restraint or bracing of the pipe assembly free end is required. Example additional bracing is shown in [Figure B-2-1](#). Additional temporary restraint or bracing of the test specimen is required when the pipe stiffness are lower than the branch connection stiffness in the load direction of interest.

### B-4.3 Load Platen Base Fixity

Load platens must be rigid with respect to the loaded specimen. When the relative stiffness of the load platen is unknown, the displacements of the component flange mounted to the platen should be limited by the following:



The measured displacement at A and B during any phase of the  $k$ -factor test must be less than 0.00025 in. or 1/50 of the maximum displacement measured in the test, whichever is less.

### B-4.4 Elastic Beam Model Construction

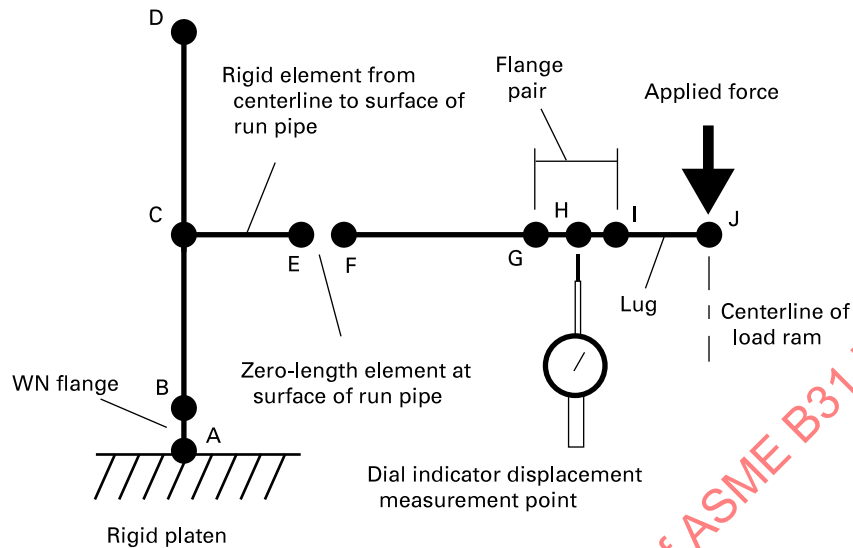
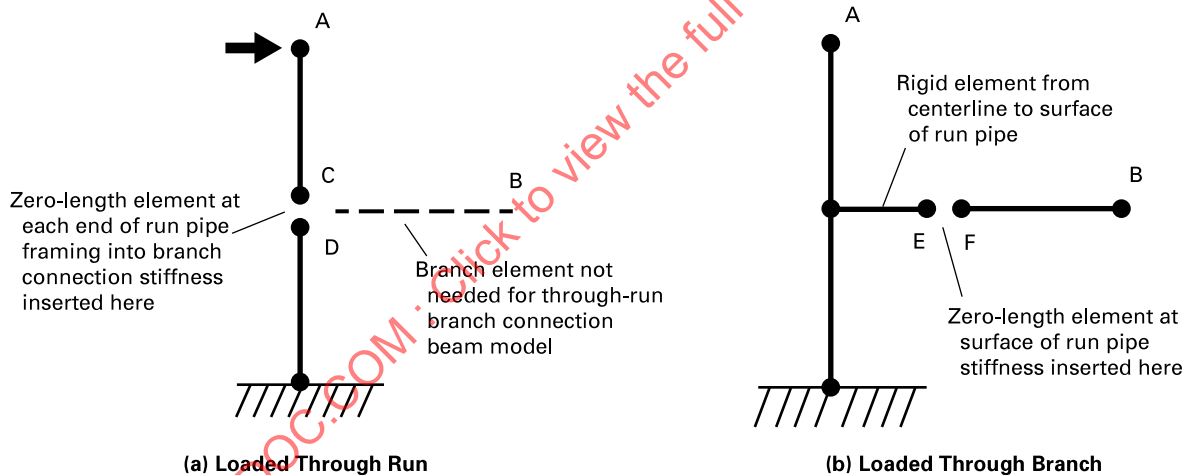
Once the assembly dimensions and load directions are selected, the following hold:

(a) A six-degree-of-freedom beam model of the test piping assembly, as shown in [Figure B-4.4-1](#), should be constructed that includes all piping components in the assembly beginning with the rigid base and going up to and including the points where

- (1) loads are applied
- (2) displacements are measured
- (3) additional braces may be needed [see [para. B-4.6\(j\)](#) and [Figure B-2-1](#)]

When developing flexibility factors for through-run loadings, the [Figure B-4.4-1](#) beam model should be adjusted to accommodate the zero-length element between points C and D in [Figure B-4.4-2](#), illustration (a). A rigid element should be placed between points C and D to start the  $k$ -factor iteration process. The rigid element between points C and D should fix point C to point D so that their displacements and rotations are equal.

When developing flexibility factors for through-branch loadings, the [Figure B-4.4-1](#) beam model should be adjusted to accommodate the zero-length element between points E and F in [Figure B-4.4-2](#), illustration (b). A rigid element should be placed between points E and F to start the  $k$ -factor iteration process. The rigid element between points E and F should fix point E to point F so that their displacements and rotations are equal.

**Figure B-4.4-1 Detailed Beam Model for Through-Branch  $k$ -Factor Test****Figure B-4.4-2 Beam Model**

(b) Any weld neck flange in the test piping assembly should be simulated in the beam analysis using a rigid element followed by a nominal matching pipe section. The rigid element length should be equal to  $\frac{3}{4}$  of the total length of the weld neck flange from the weld line to the flange face.

(c) The load applied to the beam model should produce measurable displacements at least equal to 10 times the minimum measurable displacement determined in para. B-1.3.

(d) For each load-deflection location and direction evaluated (see Figure B-1-2 Steps 2 through 7), apply the loads from para. (c) to the beam model, and record the magnitudes of the calculated displacements or rotations. These results define the overall stiffness of the beam

model in all applicable directions and will be used to compute the  $k$ -factors for each direction.

(e) The multiple  $k$ -factor calculation procedure involves making measurements of the displacements or rotations in accordance with paras. B-4.5 and B-4.6 for each of the load Steps 2 through 7 as defined in Table B-1-1. Once the displacements or rotations are recorded, the flexibility factors for each respective direction can be determined by applying the iterative procedure in para. B-4.7. Table B-1-1 identifies the measurement point location and whether the recorded parameter is displacement or rotation.



### B-4.5 Test Mounting and Loading

Mount the piping test specimen in the assembled position. Gaskets are generally not needed between the mounting flange and the platen.

### B-4.6 Initial Step for a Single Load-Deflection Test

(a) For each load and measurement direction, attach the load mechanism and dial indicator or laser. Be sure that any dial indicator in contact with the pipe is not influenced by local ovalization induced in the pipe cross section by the applied load.

(b) Record the displacement and load at the zero displacement location.

(c) Apply the load chosen in para. B-4.4(c) to the model in five even intervals. Record the displacement or rotation and force at each interval.

(d) Remove the load in approximately the same five load intervals, recording the displacement or rotation and force at each interval.

(e) Record the displacement or rotation and load at the zero displacement location. Repeat steps (c) through (e) in the opposite direction when practical or when opposite-direction loading may produce a different load-deflection diagram.

(f) Repeat the entire loading sequence at least one time, recording the displacement or rotation and load at each interval.

(g) Produce a load-deflection diagram as shown in Figure B-4.6-1. The displacement or rotation should be along the horizontal axis and the load should be along the vertical axis. The diagram should show a constant slope on loading and unloading for both load cycles. All points should fall along the same straight line. Any discrepancy should be resolved before proceeding. An example linear load-deflection diagram is shown in Figure B-4.6-1.

(h) Return to step (a) and repeat the procedure for each load-deflection direction.

(i) Compute the displacement or rotation ratio from eq. (B-2) for each load-deflection direction.

$$\text{displacement ratio} = |d_m - d_n|/d_m \quad (\text{B-2})$$

(j) If the displacement ratio is less than 0.30, then consider temporarily increasing the stiffness of the assembly for the direction and loading evaluated. The value of the computed displacement or rotation,  $d_n$ , must include the effect of any included temporary stiffness. Clamps and braces as shown in Figure B-2-1 have been used successfully to increase the stiffness of the test assembly.

(k) When validated load-deflection curves are produced, the  $k$ -factor can be calculated using the iterative procedure in para. B-4.7.

(l) When additional stiffness or bracing is added in accordance with para. (j), the beam model must be modified and the additional bracing added accurately in the beam model. If the bracing is not rigid (see Figure B-2-1), then the displacement at the bracing should be measured during the loading test and the bracing stiffness adjusted in the beam model. When bracing or additional stiffnesses are added, the displacement ratio found in eq. (B-2) should be less than 0.3 before proceeding.

### B-4.7 Iterative Computation of $k$ -Factor for Branch Connections

For each load-deflection pair, estimate the flexibility factor from existing data, literature, numerical analysis, or eq. (B-3).

$$k_{\text{est}} = (R/T)(d/D) \quad (\text{B-3})$$

Compute the estimated rotational branch stiffness  $K_{\text{estb}}$  using eq. (B-4) or the estimated rotational run stiffness  $K_{\text{estr}}$  using eq. (B-5). The rotational run stiffness is needed for Figure B-1-2 Steps 2, 4, and 6. The rotational branch stiffness is needed for Figure B-1-2 Steps 3, 6, and 7.

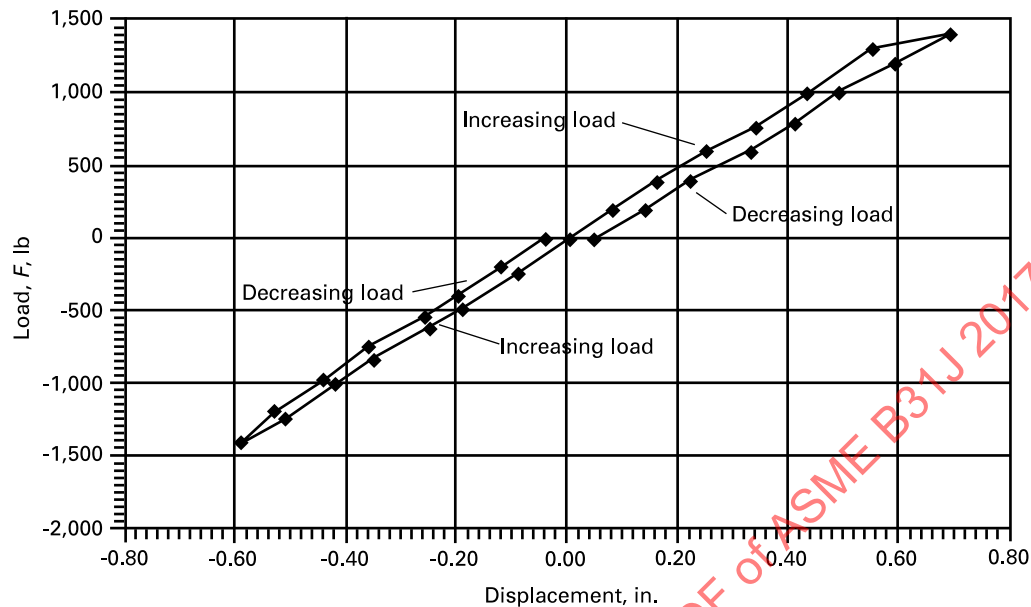
$$K_{\text{estb}} = EI_b/(k_{\text{est}} \times d) \quad (\text{B-4})$$

$$K_{\text{estr}} = EI_r/(k_{\text{est}} \times D) \quad (\text{B-5})$$

Enter the computed estimated rotational stiffness into the beam model configuration in Figure B-4.4-2, illustration (b) for loads through the branch and in the beam model configuration in illustration (a) for loads through the run. Apply the maximum test load in the beam model used for the appropriate load-deflection test in para. B-4.6. Stiffnesses calculated using eqs. (B-4) and (B-5) are in units of length multiplied by force per radian. When the correct component stiffness is used, the predicted displacements or rotations from the computer program should be within 1% of the measured displacements or rotations as calculated in eq. (B-6). When the calculated displacements or rotations are too low, the procedure in para. B-4.7.1 can be used to find an improved value for the local stiffness. When the calculated displacements or rotations are too high, the procedure in para. B-4.7.2 can be used to find an improved value for the local stiffness. When the calculated displacements or rotations are within 1% of the measured displacements or rotations, the flexibility factor used in the last iteration is sufficiently accurate.

**B-4.7.1 Convergence When Predicted Displacements or Rotations Are Too Low.** If the calculated displacement or rotation at the measurement point is too low, divide the local stiffness in the current beam model by 2. Replace the original local stiffness in the beam model with the modified stiffness and rerun the analysis. If the newly calculated displacement or rotation at the measurement point is still

Figure B-4.6-1 Load-Displacement Diagram



too low, halve the stiffness again and rerun the analysis. Continue halving the stiffness and rerunning the calculation until the computed displacement is too high. When the computed displacement is too high, multiply the stiffness by 1.5 and rerun the analysis. On subsequent runs, continue increasing or decreasing the stiffness by smaller amounts until the calculated displacement or rotation is equal to the measured displacement or rotation within 1% as found from eq. (B-6).

$$\text{Percent Difference} = 100 \times |\text{Calculated} - \text{Measured}| / \text{Measured} \quad (\text{B-6})$$

**B-4.7.2 Convergence When Predicted Displacements or Rotations Are Too High.** If the calculated displacement or rotation at the measurement point is too high, multiply the local stiffness in the current beam model by 2. Replace the original local stiffness in the beam model with the modified stiffness and rerun the analysis. If the newly calculated displacement at the measurement point is still too high, then double the stiffness again and rerun the analysis. Continue doubling the stiffness until the calculated displacement is too low. When the calculated displacement is too low, divide the stiffness by 1.5 and rerun the analysis. On subsequent runs, continue increasing or decreasing the stiffness by smaller amounts until the calculated displacement or rotation is equal to the measured displacement or rotation within 1% as found from eq. (B-6).

#### B-4.7.3 Iteration for Each Load-Deflection Direction

(a) Use the iterative solution described in para. B-4.7.1 or para. B-4.7.2 to find the flexibility factors for each load-deflection direction.

(b) If continuing to increase the stiffness does not significantly reduce the calculated displacement or rotation, then the  $k$ -factor is 1 or zero.

(c) When converged stiffnesses are found, record the calculated displacement or rotation  $d_c$  that corresponds with the converged stiffness for each load-deflection direction.

(d) For through-run loaded branch connections, compute the flexibility factor from eq. (B-7).

$$k = EI_r / (KD) \quad (\text{B-7})$$

(e) For through-branch loaded branch connections, compute the flexibility factor from eq. (B-8).

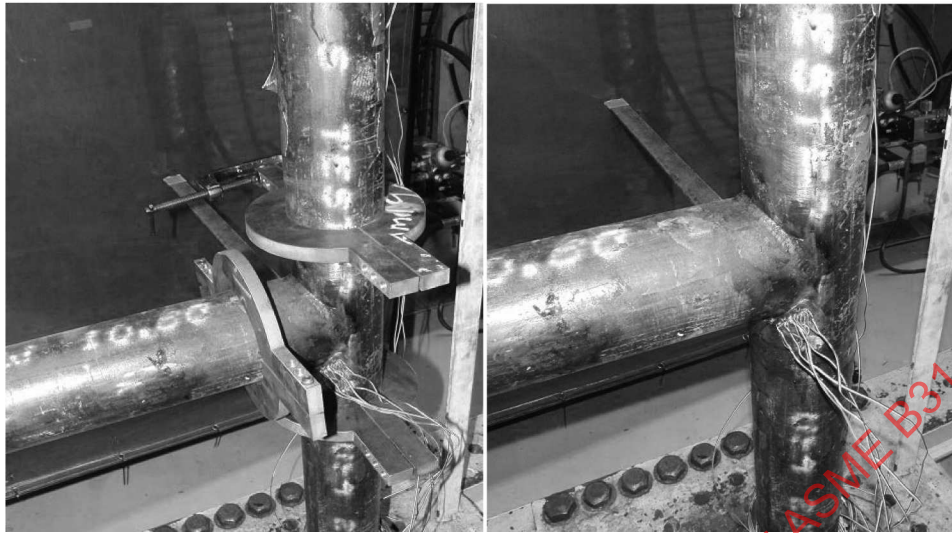
$$k = EI_b / (Kd) \quad (\text{B-8})$$

### B-5 Number of Test Specimens

The number of repeated tests required to determine the flexibility factor,  $k$ , and any increase factor needed for a smaller number of tests shall be determined by a rational analysis included in the Test Report.

### B-6 Variations and Similarities in Materials and Geometry

(a) The flexibility factors derived from tests conducted in accordance with this Standard are applicable to geometrically similar components, where the degree of

**Figure B-6-1 Unreinforced Branch Connection With (Left) and Without Ovalization Restraint Plates in Place**

geometric similarity should be established by rational analysis.

(b) Dimensional extrapolations considered acceptable shall be identified in the Test Report, along with their technical justification. For example, a complex fitting may have multiple tests run on different diameters and thicknesses to establish a relationship between the flexibility factors and dimensional parameters. Alternatively, if a closed-form evaluation or a finite element analysis can be performed, the technical justification can be based on a single-size fitting with the closed-form evaluation or the finite element analysis showing how the flexibility factor is extrapolated to other sizes.

(c) Where ovalization in the vicinity of the branch connection should be evaluated, some ovalization restraint can be provided and the test repeated. Ovalization restraint as shown in Figure B-6-1 is intended to simulate the effect of attached flanges, valves, clamps, or other rigid components.

## B-7 Test Report

(a) To meet the requirements of this Standard, a Test Report shall be prepared and certified by a Registered Professional Engineer competent in the design and analysis of pressure piping systems, or a person of equivalent expertise as defined by national practice. The Test Report shall be complete and written to facilitate an independent review. The report shall contain

- (1) description of the tested specimens
- (2) nominal pipe and piping component sizes and dimensions and actual cross-sectional dimensions of importance in interpreting the test results
- (3) description and photograph(s) or sketch(es) of the test equipment, including positioning of the test specimen in the machine

(4) calibration of the test equipment; this information may be provided by reference

(5) certified Material Test Reports for the tested component and attached pipe, including mill-test values of yield, ultimate strength, and elongation

(6) descriptions of all beam or finite element models

(7) tables clearly identifying the results of each iterative  $k$ -factor solution

(8) optionally, component and component-to-pipe weld examinations where they are required by the construction Code with certification of weld Code compliance, a copy of the Welding Procedure Specification (WPS) and the Welding Operator Performance Qualification (WPQ) for the welder, along with a narrative describing the visual examination of the welds used in the test pieces and photographs of the weldment

(9) assembly procedure used for the joints where joint assembly is required

(10) details of the bracing, and descriptions of the beam models used to simulate the bracing stiffness, along with results before and after the bracing is applied, where bracing or other temporary restraint is used

(b) When the  $k$ -factor test is conducted by or for a manufacturer, the Test Report shall be maintained by the manufacturer for review by the owner, purchaser, or designer.

## B-8 Ovalization Restraint Plates

Figure B-6-1 shows ovalization restraint plates on a branch connection used to simulate the close proximity of flanges, clamps, or other rigid components. The attachment of these plates results in a lowering of the flexibility factor.

**B-9 References**

- [1] WRC Bulletin 463, "Report 1: Standardized Method for Developing Flexibility Factors for Piping Components," E. C. Rodabaugh and E. A. Wais (2001).
- [2] Wais, E. A., Rodabaugh, E. C., and Carter, R., "Stress Intensification Factors and Flexibility Factors for Unreinforced Branch Connections," *Pressure Vessel and Piping Codes and Standards*, Vol. 383, ASME (1999).
- [3] Moore, S. E., Rodabaugh, E. C., Mokhtarian, K., and Gwaltney, R. C., "Review and Evaluation of Design Analysis Methods for Calculating Flexibility of Nozzles and Branch Connections," NUREG/CR-4785, ORNL-6339 (December 1987).
- [4] Rodabaugh, E. C., and Moore, S. E., "Stress Indices and Flexibility Factors for Nozzles in Pressure Vessels and Piping," NUREG/CR-0778 (June 1979).
- [5] Khan, A. S., "A Study of Flexibility Factors of Integrally Reinforced Shell to Shell Intersections," *International Journal of Pressure Vessels and Piping*, Vol. 29, 23-31 (1987).
- [6] Rodabaugh, E. C., and Atterbury, T. J., "Report on Flexibility and Stress Intensification Factors for Piping Components with Moment Loadings" to American Gas Association, Battelle Memorial Institute Columbus Laboratories, Columbus, OH (1966).
- [7] Xue, L., Widera, G. E. O., and Sang, Z., "Flexibility Factors for Branch Pipe Connections Subjected to In-Plane and Out-of-Plane Moments," *Journal of Pressure Vessel Technology*, Vol. 128 (February 2006).

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# NONMANDATORY APPENDIX C

## Use of Branch Connection Flexibility Factors in Piping System Analysis

### C-1 Nomenclature

The following symbols are used in this Appendix:

- $b$  = subscript indicating branch
- $D$  = mean run pipe diameter
- $d$  = mean branch pipe diameter
- $D_o$  = outside diameter of matching run pipe
- $d_o$  = outside diameter of matching branch pipe
- $E$  = elastic modulus
- $I$  = pipe moment of inertia
- $I_b$  = branch pipe moment of inertia
- $I_r$  = run pipe moment of inertia
- $K$  = rotational stiffness
- $k$  = flexibility factor
- $K_b$  = branch leg rotational stiffness (see Leg 3 in [Figure 1-1](#))
- $K_i$  = in-plane rotational stiffness
- $k_i$  = in-plane flexibility factor
- $K_o$  = out-of-plane rotational stiffness
- $k_o$  = out-of-plane flexibility factor
- $K_r$  = run leg rotational stiffness (see Legs 1 and 2 in [Figure 1-1](#))
- $K_t$  = torsional rotational stiffness
- $k_t$  = torsional flexibility factor
- $r$  = subscript indicating run

### C-2 General

Flexibility factor equations for branch connections in piping systems are given in [Table 1-1](#). These flexibility factors are intended to be used with beam models of piping systems where each point, or node, in the piping system model has at least six degrees of freedom defined about a local or global orthogonal axis. Three of these degrees of freedom are translational and three are rotational.

The flexibility factors for branch connections in [Table 1-1](#) Sketches 2.1 through 2.6 are used with [Table 1-2](#) moment-rotation relationships, or “rotational stiffnesses.” These rotational stiffnesses are a function of the flexibility factor, modulus of elasticity, moment of inertia, and branch or run diameter given in [eq. \(C-1\)](#) and in [Table 1-2](#).

$$K = (EI)/(kd) \quad (C-1)$$

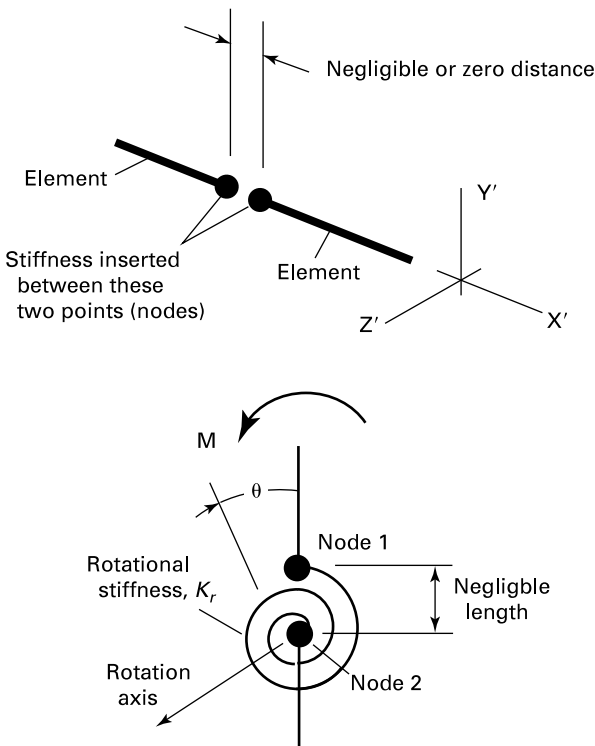
The calculated rotational stiffnesses define the rotation of one point on a branch or run pipe element in the piping system with respect to another. The ends of these elements are located at the same point in space or at very nearly the same point in space so that there is a negligible distance between the two points. This is shown in [Figure C-2-1](#).

The coordinate system in [Figure C-2-1](#) is a local  $X'$ ,  $Y'$ , and  $Z'$  coordinate system where  $X'$  is along the element centerline axis,  $Y'$  is normal to the element axis, and  $Z'$  is orthogonal to  $X'$  and  $Y'$ . The  $Y'$  and  $Z'$  coordinate axes are generally aligned along a defined in-plane or out-of-plane orientation for the branch connection. For elements that make up branch connection models in a piping system, the in-plane orientation is shown in [Figure C-2-2](#). The out-of-plane orientation is different for branch and run elements and is found by crossing the branch element axial direction vector into the in-plane orientation vector shown in [Figure C-2-2](#) to find the branch out-of-plane orientation. The out-of-plane orientation is found by crossing the run element axial direction vector into the in-plane orientation vector.

There may be up to three moment-rotation node-pair relationships for each location on the branch connection in a beam model of a piping system depending on whether the branch, run, or both the branch and run flexibilities are considered; see [Figures C-2-3](#) through [C-2-5](#). There are three branch moment-rotation relationships (rotational stiffnesses) and three run moment-rotation relationships (rotational stiffnesses) for each intersection in [Table 1-1](#) Sketches 2.1 through 2.6. The run rotational stiffnesses may be used as shown in [Figures C-2-4](#) and [C-2-5](#). The branch rotational stiffnesses may be used as shown in [Figures C-2-3](#) and [C-2-5](#).

The piping designer must determine which of the branch connection models in [Figures C-2-3](#) through [C-2-5](#) is most appropriate. In all cases, the run and branch rotational stiffnesses may be used together in series to provide an accurate simulation. Including the branch stiffness relationships only ([Figure C-2-3](#)) is reasonable when the run stiffness relationships are rigid or approximately rigid. When the  $d/D$  ratio for a branch connection is less than 1, the run flexibility factors will often be less than 1 and need not be used. When a flexibility factor is not used, the corresponding



**Figure C-2-1 Rotational Stiffness Location Between Two Nodes**

rotational stiffness should be rigid. [See Table 1-1, Note (1).] The three combinations available for each branch connection are

- (a) only branch flexibilities included
- (b) only run flexibilities included
- (c) branch and run flexibilities included together (in series)

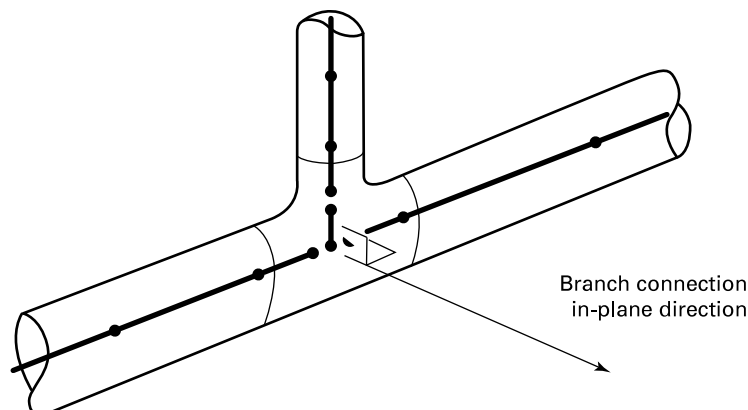
When both the branch and run flexibilities are used simultaneously for the same branch connection, the interaction of loads through the branch and run must be considered and the branch connection symmetry maintained. These requirements are satisfied by the model shown in Figure C-2-5.

Rotational stiffnesses should be inserted between the two indicated nodes so that the rotation of one node relative to the other is given by the branch connection moment-rotation relationship in the appropriate direction. This is illustrated in Figure C-2-6, where the rotational stiffnesses are given with respect to the nodes numbered 1 and 2. Translational stiffnesses between nodes are rigid.

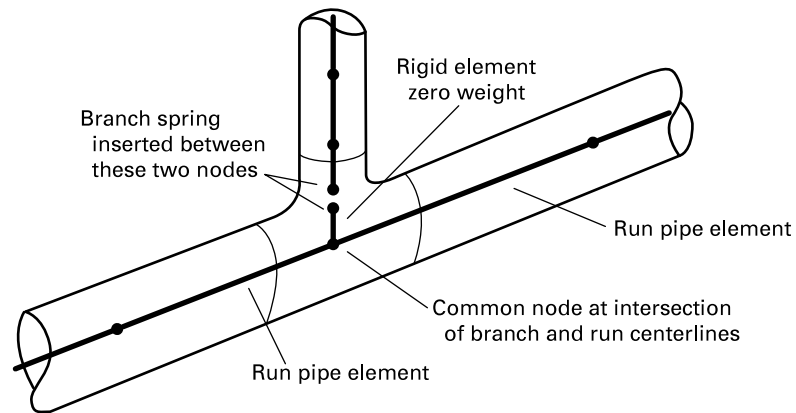
For the branch moment-rotation relationship, the moment of inertia,  $k$ -factor, and mean diameter ( $d$ ) of the branch pipe should be used. For the run moment-rotation relationship, the moment of inertia,  $k$ -factor, and mean diameter ( $D$ ) of the run pipe should be used. Branch and run node locations are shown in Figure C-2-7.

### C-3 References

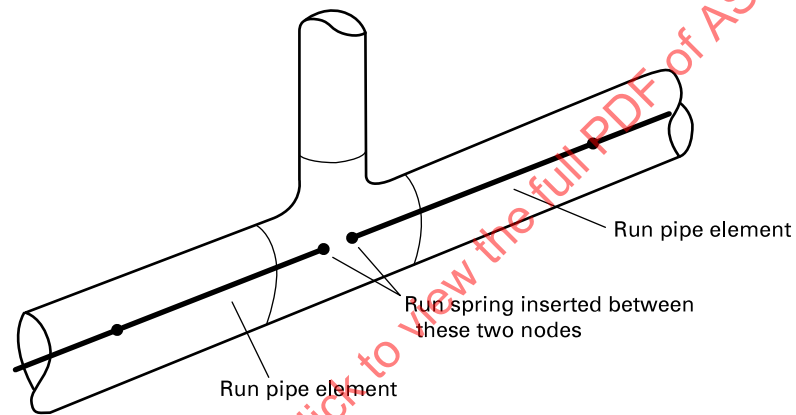
- [1] WRC Bulletin 463, "Report 1: Standardized Method for Developing Flexibility Factors for Piping Components," E. C. Rodabaugh and E. A. Wais (2001).
- [2] Wais, E., Rodabaugh, E. C., "Stress Intensification Factors and Flexibility Factors for Unreinforced Branch Connections," TR 110996 Final Report (November 1998).
- [3] Moore, S. E., Rodabaugh, E. C., Mokhtarian, K., and Gwaltney, R. C., "Review and Evaluation of Design Analysis Methods for Calculating Flexibility of Nozzles and Branch Connections," NUREG/CR-4785, ORNL-6339 (December 1987).

**Figure C-2-2 Branch Connection In-Plane Direction**

**Figure C-2-3 Branch Connection Flexibilities Used on Branch Side Only**



**Figure C-2-4 Branch Connection Flexibilities Used on Run Side Only**



**Figure C-2-5 Branch and Run Flexibilities Used Together (in Series)**

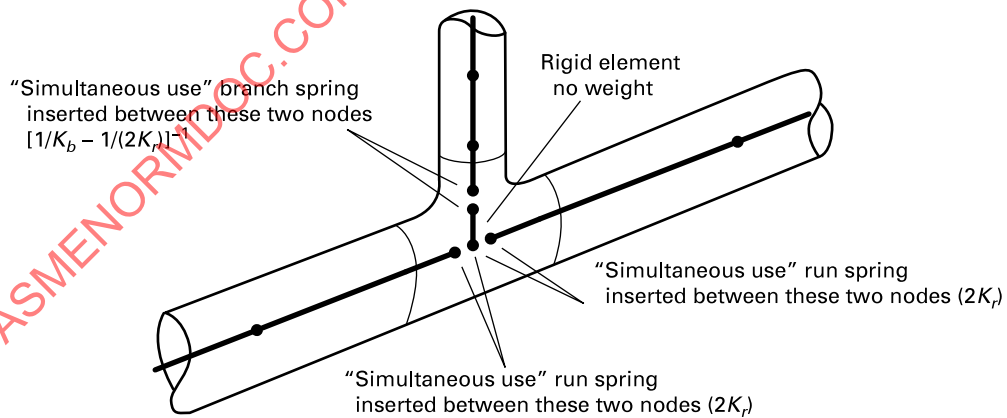
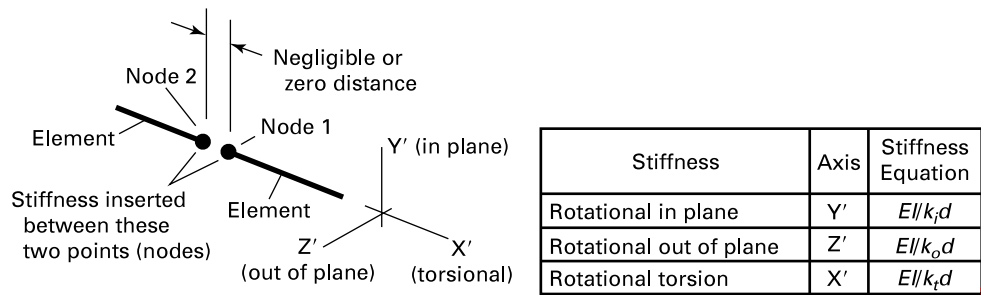
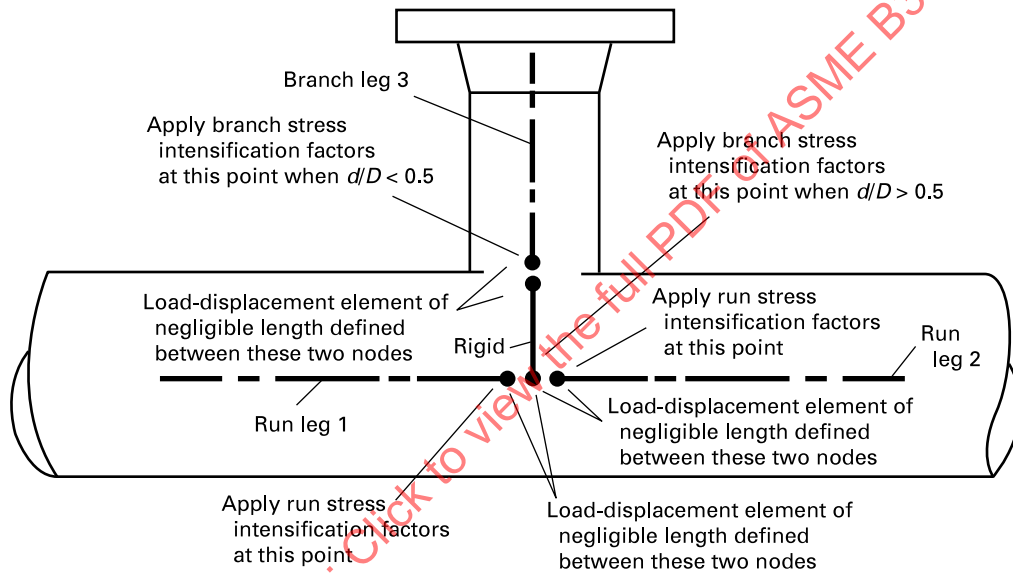




Figure C-2-6 Rotational Flexibility Definitions

Figure C-2-7 Branch and Run SIF and  $k$ -Factor Intersection Orientations

## NONMANDATORY APPENDIX D

### Sustained Load Test Procedure

#### D-1 General

The twice elastic slope (TES) test procedure may be used to evaluate sustained loads on individual piping components with geometries similar to those shown in Figures D-1-1 and D-1-2 and is the collapse load criteria required in ASME Boiler and Pressure Vessel Code (BPVC), Section III, Mandatory Appendix II-1430.

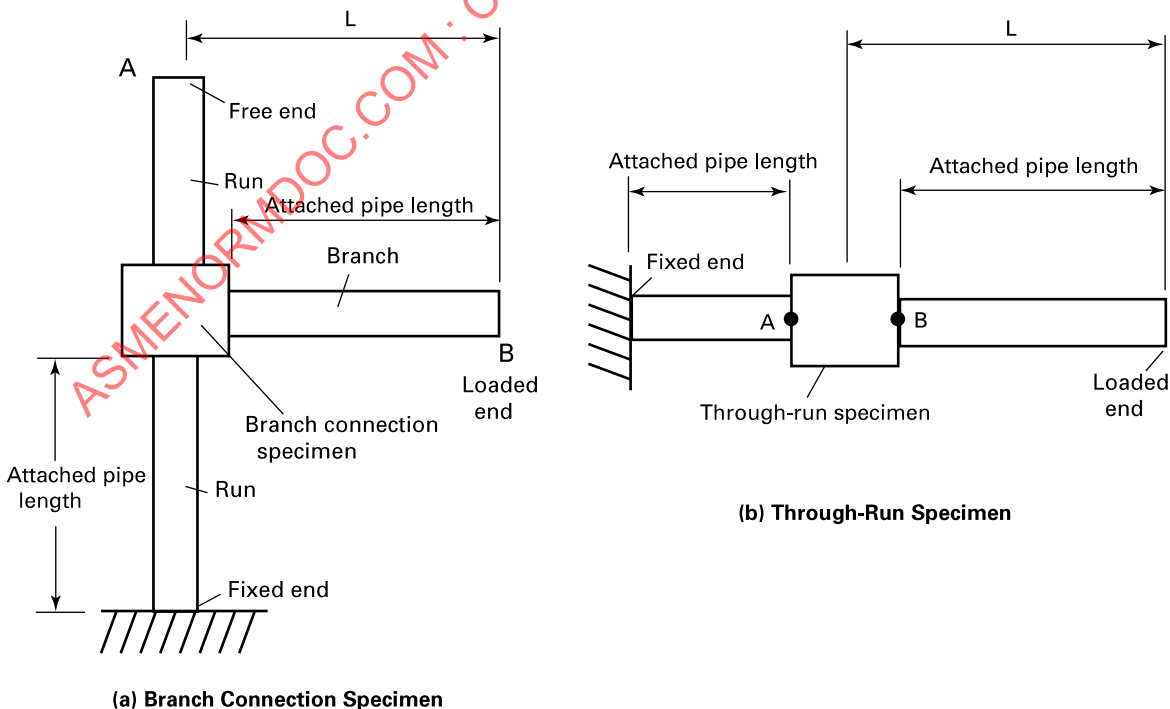
The TES test is equivalent to the twice elastic displacement test described in detail in Refs. [1], [2], [3], [4], [6], [7], [8], and [15]. The critical load in the TES test is the load that causes a displacement in the test specimen at the measurement point equal to twice the elastically computed displacement. Strains associated with the TES critical load are usually in the range of 1% to 2%.

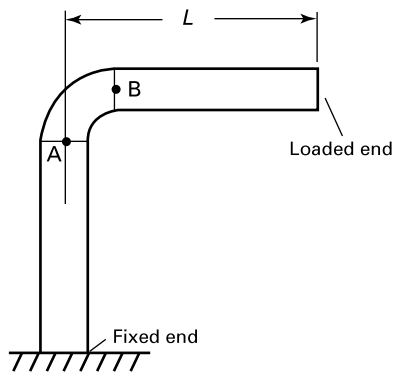
The TES-test-measured displacement  $d_m$  should reflect the movement of the centerline of the pipe in the direction of the applied force,  $F_m$ . The point on the pipe where  $d_m$  is measured should be within one diameter of the applied force. (Generally,  $d_m$  and  $F_m$  are measured and applied at

different points on the same rigid, or flanged, assembly.) The point on the load-deflection curve ( $F_2, d_2$ ) satisfies the “unsatisfactory performance” criteria described in ASME BPVC, Section VIII, Division 2, para. 5.2.4.3(b) and the collapse load criteria in ASME BPVC, Section III, Mandatory Appendix II-1430 and defines the force where the computed elastic displacement ( $d_e$ ) is equal to one-half the measured displacement ( $d_m$ ).

Components are generally not pressurized when a TES test is conducted, even though design internal pressure may affect the test results. The application of the design pressure during a TES test usually increases the maximum load capacity for ductile steel bends and tees. Other components may be “pressure sensitive,” in which case the application of the design pressure during the TES test will reduce the load capacity. It should be clearly stated in the Test Report when a tested component is pressure sensitive. The test designer, and owner or manufacturer, must consider the intended use of the test results and determine if the design pressure,

Figure D-1-1 Standard Sustained Load Setup



**Figure D-1-2 Bend Sustained Load Setup**

or some other pressure, should be included in the test loading procedure described in [para. D-4.3](#) or [para. D-4.4](#). The Test Report should identify all loading conditions present during any test sequence and the rationale used to establish that loading condition.

When pressure is included in a TES test, it should be held constant throughout the test while external loads are applied and removed. Pressure in a closed, liquid-filled system should be monitored at all times during any load application.

When pressure is included in a TES test, the safety of personnel or equipment in the vicinity must be considered. [Section D-8](#) provides safeguarding guidelines for pressurized tests.

### D-1.1 Collapse Mechanism

Two types of collapse mechanisms are identified, as follows:

- (a) where load-carrying capacity drops rapidly as  $M_2$  is approached, or where flow restriction or loss of pressure (leak) is imminent as  $M_2$  is approached
- (b) where load-carrying capacity is constant or increasing as  $M_2$  is approached, and where flow restriction or loss of pressure (leak) is not imminent as  $M_2$  is approached

### D-1.2 Sustained Load Test Safeguarding

When the collapse mechanism of the test specimen is characterized by [para. D-1.1\(a\)](#), or where a pressure sensitivity may exist, safeguarding must be provided when the component is used in an operating environment. Safeguarding requirements should be highlighted in the Test Report. Safeguarding might include any or all of the following:

- (a) additional limits for allowed loads or displacements when primary loads are present.
- (b) inclusion of a pressure reduction in the sustained stress evaluation.
- (c) additional vertical pipe supports to limit pipe movement in the vicinity of the component.

(d) additional gapped horizontal pipe supports when a component is pressure sensitive to limit externally applied bending strains in the vicinity of the component.

(e) recommended flexible modeling of the coupling for use in an elastic beam analysis with appropriate limits for sustained, expansion, operating, occasional, range, and combination load cases. The maximum permitted displacements, rotations, and/or loads on the component for all possible load combinations should be identified.

### D-1.3 Simplified Approach

The approach described in this Appendix is intended to produce a lower bound estimate of the sustained load capacity. A designer capable of determining a more accurate sustained load capacity is permitted to do so.

### D-2 Definitions

*critical load direction*: direction along the selected line of action that usually corresponds with the direction of the applied force.

*flow restriction*: any damage to the pipe or reduction of the pipe cross section that prevents the necessary conveyance of the pipe contents from one location to another.

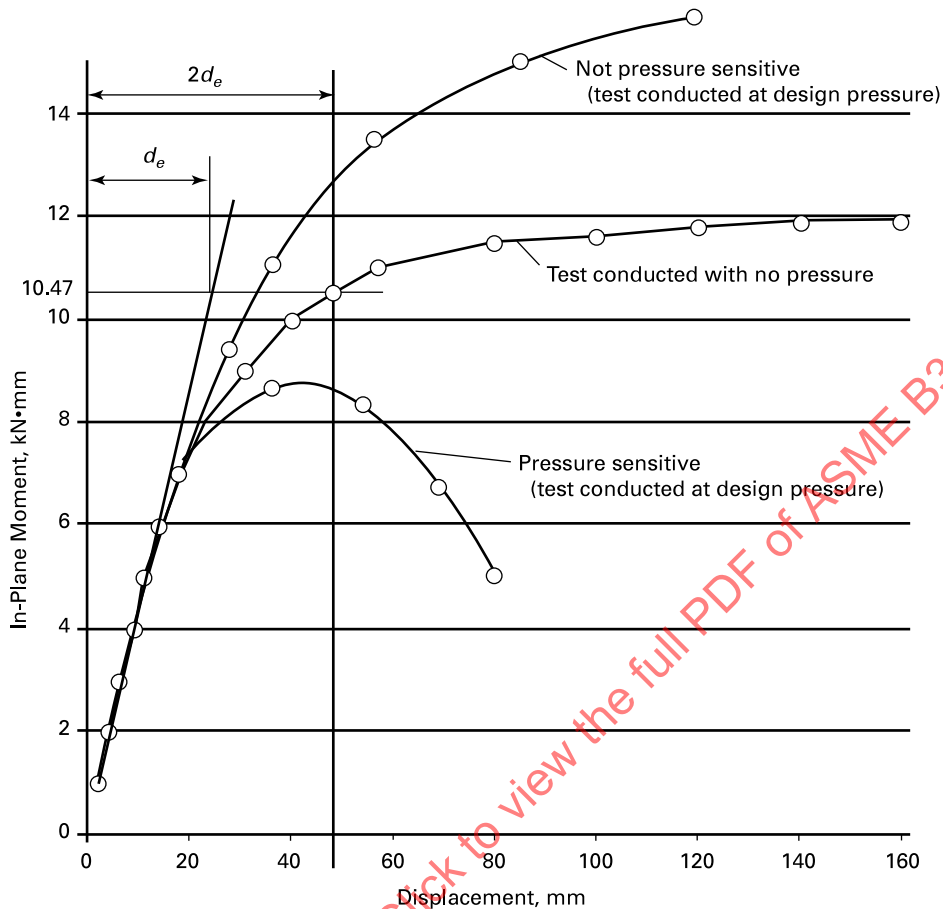
*measurement point*: point on the test assembly where displacement measurements are taken. The measurement point may be at the same point on the test specimen where the load is applied, but if not, it should be within one diameter of the point on the test specimen where the load is applied. Once the measurement point is selected, it must remain at the same location throughout the test.

*pressure sensitive*: a condition where the presence of an internal pressure (most likely the design pressure) reduces the load capacity of the tested component or otherwise limits its ductility, flow behavior, tightness, or performance. (See [Figure D-2-1](#).)

*repeated displacement test*: test conducted after the initial TES test and repeated 100 times to demonstrate the component's ability to undergo repeated displacements to a TES displacement magnitude. This test is generally run when the component must undergo repeated dynamic displacements caused by seismic or fluid transient events.

*test pressure*: usually atmospheric air pressure is maintained in the test specimen, although a constant pressure greater than atmospheric may be applied to the test specimen using an incompressible liquid such as water during the TES loading and unloading procedure. When a test pressure higher than atmospheric pressure is used, it is most often the component design pressure. Any other pressure may be used by agreement of the owner or manufacturer. Atmospheric nitrogen at a cryogenic temperature is also used if the cryogenic properties of the component are to be tested. The test pressure is

**Figure D-2-1 Load-Displacement Diagram Illustrating Typical Pressure-Sensitive, Not Pressurized, and Not-Pressure-Sensitive Load-Deflection Behavior**



applied before the external load is applied and is held constant throughout any loading and unloading sequences.

*unsatisfactory performance:* behavior due to excessive primary loading as described in ASME BPVC, Section VIII, Division 2, para. 5.2.4.3(b).

### D-3 Nomenclature

The following symbols are used in this Appendix:

- $d$  = mean diameter of matching pipe
- $d_2$  = measured displacement at load  $F_2$
- $d_e$  = elastic specimen displacement calculated from eq. (D-2)
- $d_i$  = measured displacement at point  $i$  on the TES load-deflection curve corresponding to the force  $F_i$ ; the next point on the load-deflection curve is  $(F_{i+1}, d_{i+1})$
- $D_L$  = mean diameter of the loaded attached pipe
- $d_m$  = measured displacement at the selected point
- $d_n$  = displacement at the measurement point calculated from the elastic model of the test assembly used to find the elastic constant  $G$

$d_{\text{onset}}$  = estimated displacement at the measurement point that will initiate plastic deformation in the test specimen  
 $= L^2 S_y / (Ed)$

$d_{rd}$  = reversed load-displacement amplitude used in para. D-4.4.4 and selected for the extended displacement test by the manufacturer, owner, or test engineer responsible for establishing the extended test failure criteria, typically either  $d_s$  or  $2d_s$  depending on the need for the component to survive unpredictable, repeated displacement extremes

$d_s$  = total specimen displacement calculated from eq. (D-4)

$d_\Delta$  = displacement increment used to develop the plastic portion of the load-deflection diagram

$E$  = modulus of elasticity of the attached straight pipe

$E_e$  = elastic slope of load-displacement diagram [see Figure D-3-1 and eq. (D-8)]